COMPANY CONFIDENTIAL

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STRETCH CIRCUIT MEMO #9

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SUBJECT: Y-Parameter Equivalent Circuits of IBM Drift Transistors

SUMMARY: From measured curves $y = f(\omega)$ with the operating point (Ie, Ve) as parameter, equivalent circuits have been designed which duplicate the measured curves.

Changing of the operating point of the transistor affects only the values of the components of the respective equivalent circuits of the y parameters but does not require changing of the composition of the equivalent circuits.

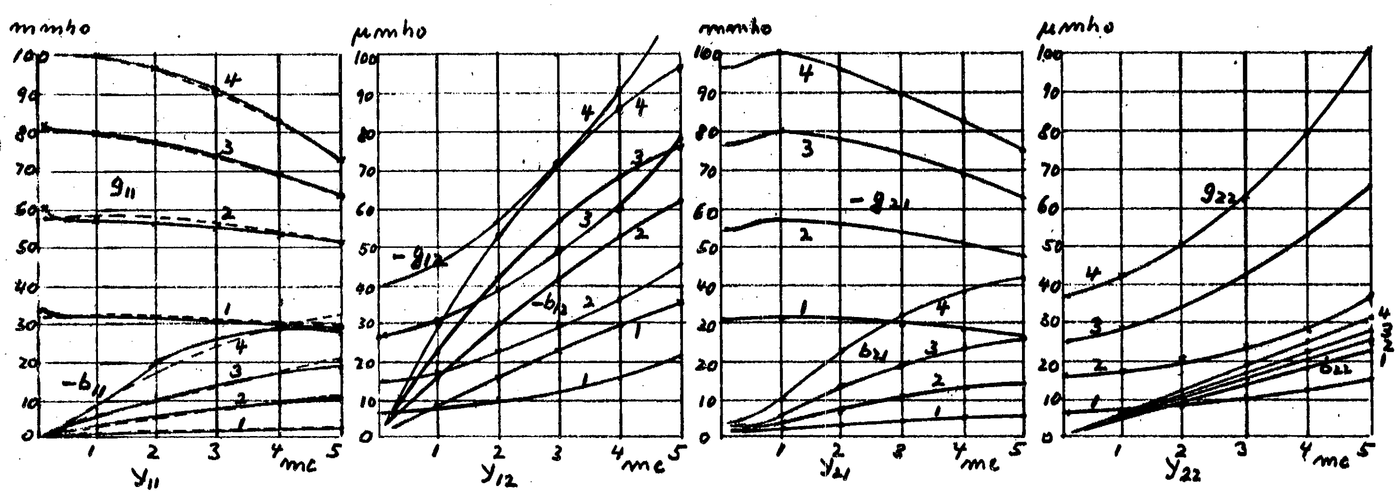
1. Equivalent Circuits and Correlation

In Stretch Circuit Memo #8 results of measurements on the y-parameters of IBM drift transistors as function of the frequency with the operating point (Ie) as parameter has been presented.

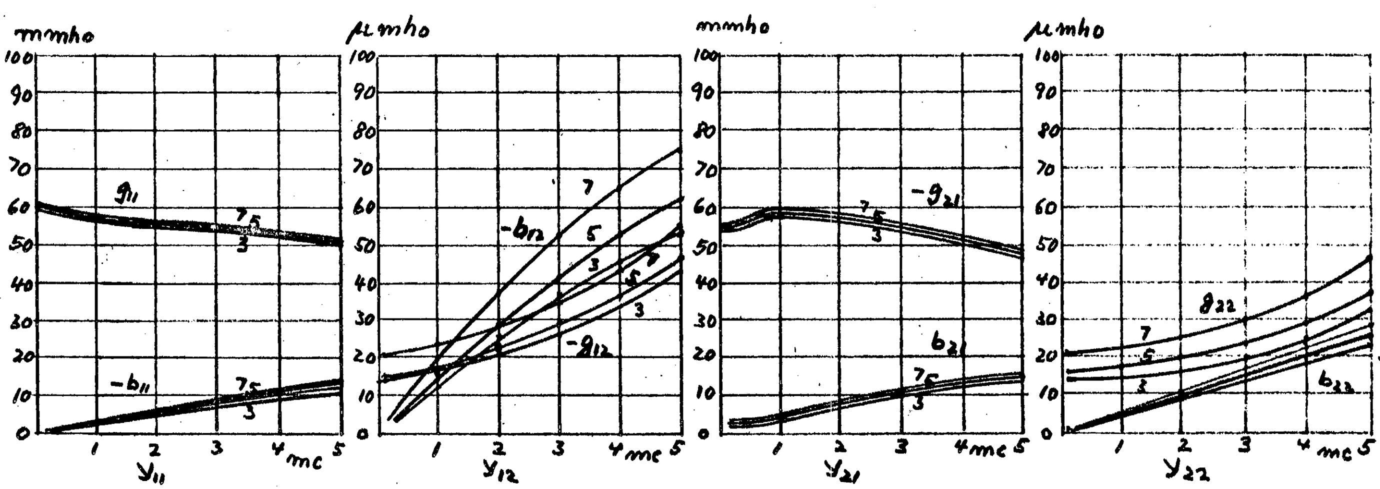
Equivalent circuits have been designed to duplicate the measured curves. These equivalent circuits are shown on Fig. 10 of the present memo together with the curves ro, r, ℓ, C = f (Ie) vc representing the variation of the values of the components of the equivalent circuits with Ie.

The values of all components vary monotonous with Ie except the shunt capacitance of $-y_{21}$, which shows a maximum at about 3 MA. This rather unexpected behavior has been confirmed by repeating measurements and design of the equivalent circuit of another drift transistor.

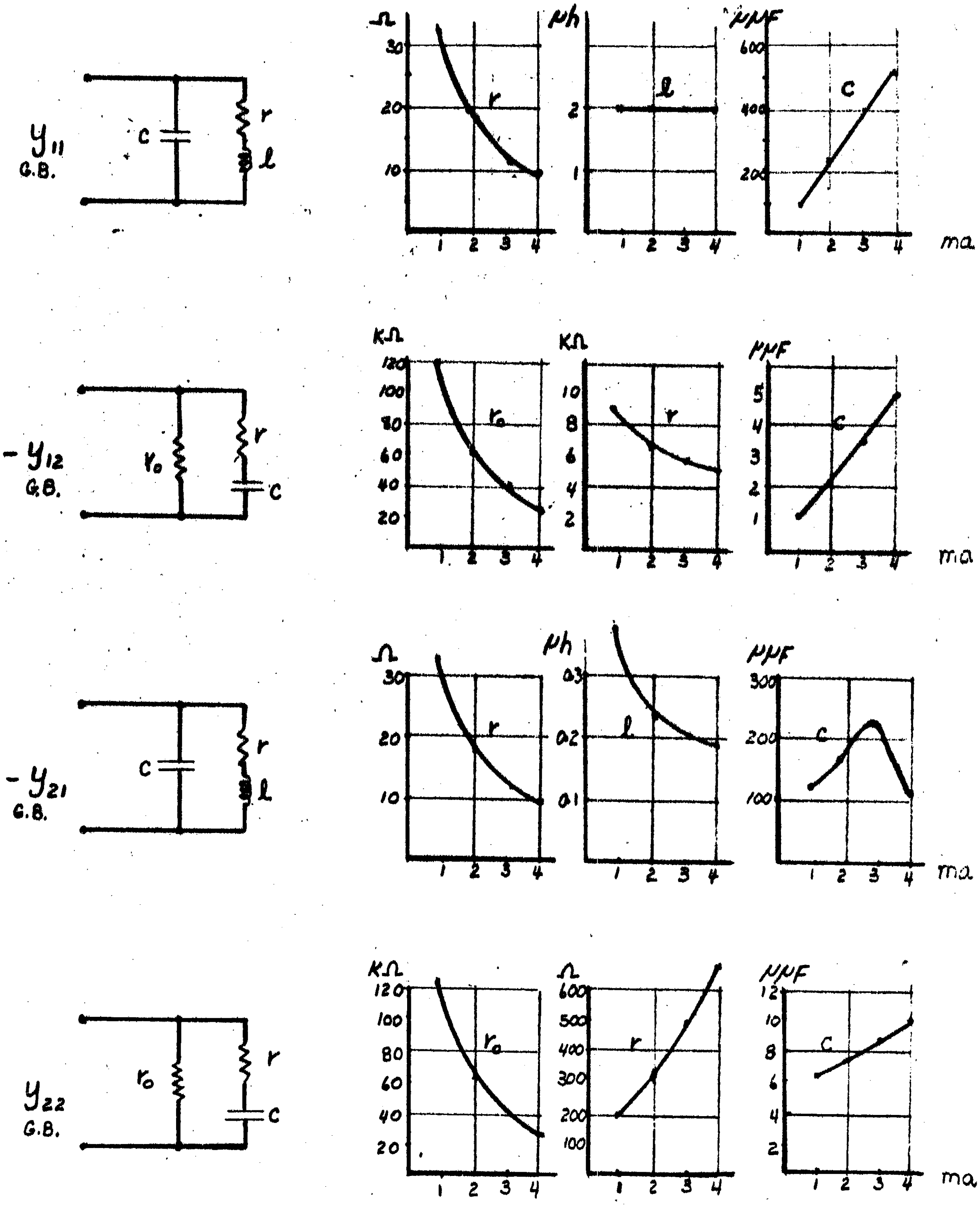
The degree of correlation between the measured and the simulated curves is shown on Fig. 8 and 9.



y as function of frequency with Vc = -5 V, ie = 1, 2, 3, 4 mA Fig. 8



y as function of frequency with ie = 2mA, Vc = -3, -5, -7 Volts Fig. 9



Equivalent circuits of the y parameters (G.B.) and the values of their components as $f(i_e)$. $(V_c = 5\text{ v.})$

Fig. 10

2. Method of Design of the Equivalent Circuits

The equivalent circuits have been designed by application of the Smith chart, which is very convenient for this purpose. The method will be discussed on two examples; the design of an equivalent circuit for y_{11} G. B. and $-y_{12}$ G. B. (Fig. 11 and 14)

2.1 Design of an Equivalent Circuit for y_{11} G.B.

(See Fig. 11)

The measured values of y11 = f (ω) (for Ie = 4 MA, Vc = -5V in the present example), y11 = g -jb, are transformed into the relative values g/go and -jb/go, where go is a suitable reference value. go is chosen to allow the curves y/go = g/go -jb/go to be located in the vicinity of the center part of the chart.

The values then are:

Frequency mc	Measured g	Measured -jb	Reference Value	Relative Values g/go	Relative Values -jb/go
0		0			0
1	100×10^{-3}	-9×10^{-3}	go=	2	-.18
2	97	-20	50×10^{-3}	1.94	-.4
3	91	-26.7	zo=	1.82	-.53
4	82	-28	20	1.64	-.56
5	73	-28.5		1.46	-.57

The y/go = f (ω) now is drawn in the Smith Chart as curve 1. This curve 1 when transferred into the impedance plane by shifting the curve 180° around the centerpoint 1 of the chart yields the curve $\frac{z}{zo} = \frac{go}{yo} = f(\omega)$, the curve 2.

We see that this impedance curve 2 would follow a circle of constant resistance of approximately $\frac{r}{zo} = .5$, where $zo = \frac{1}{go}$, when the curve 2 could be suitably stretched. This can be done by stretching curve 1 by deducting a corresponding amount of capacitance.

Deduction of capacitance from curve 1 means splitting a parallel capacitor from the remaining network as shown in the following fig.

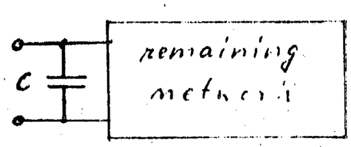


Fig. 12

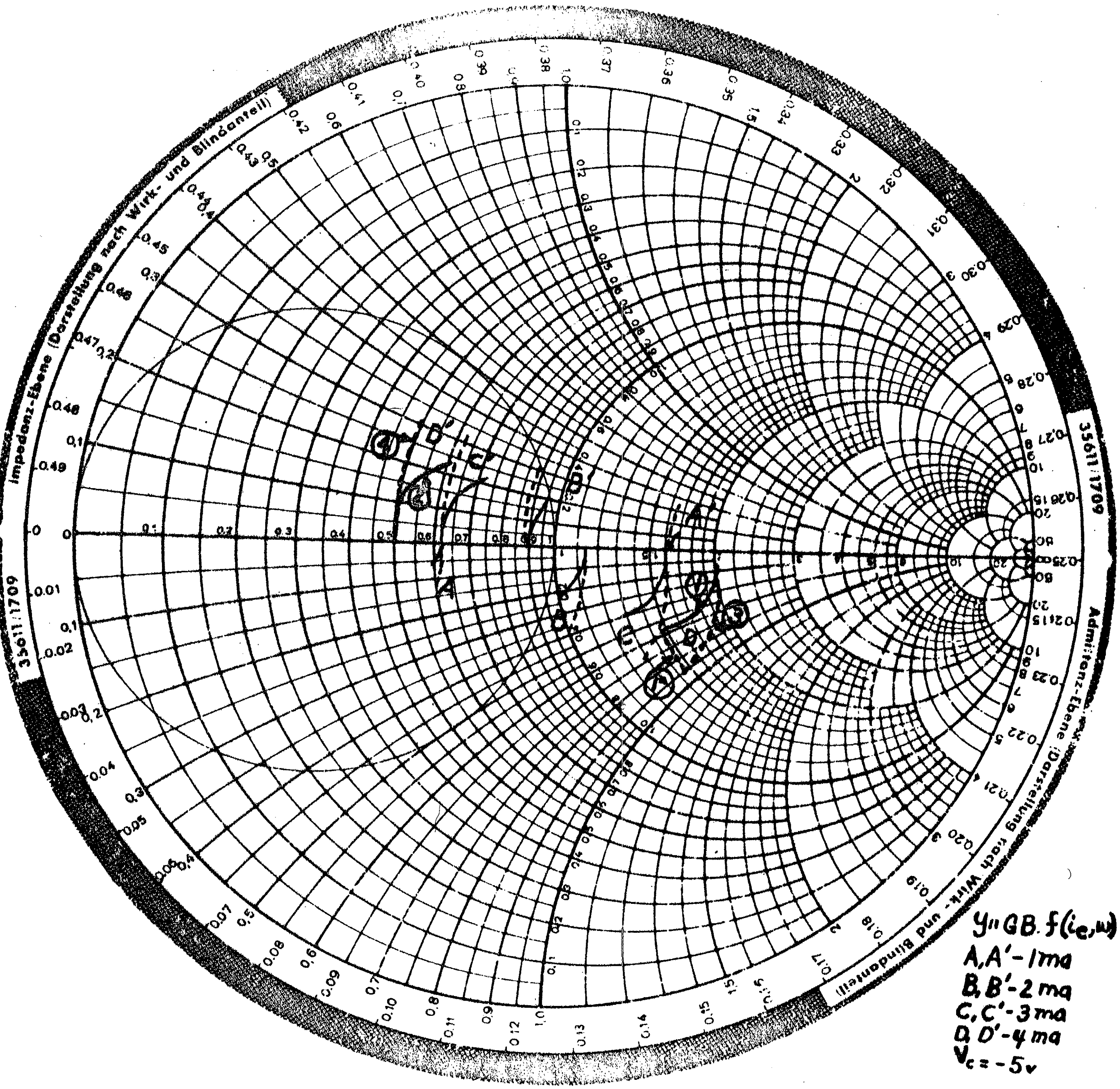

Impedanz-Ebene (Darstellung nach Wirk- und Blindanteil)
Admittanz-Ebene (Darstellung nach Wirk- und Blindanteil)
35611/1709
y_{11} GB. $f(i_e, \omega)$
A, A' - 1 ma
B, B' - 2 ma
C, C' - 3 ma
D, D' - 4 ma
$V_c = -5$ v

FIG. 11

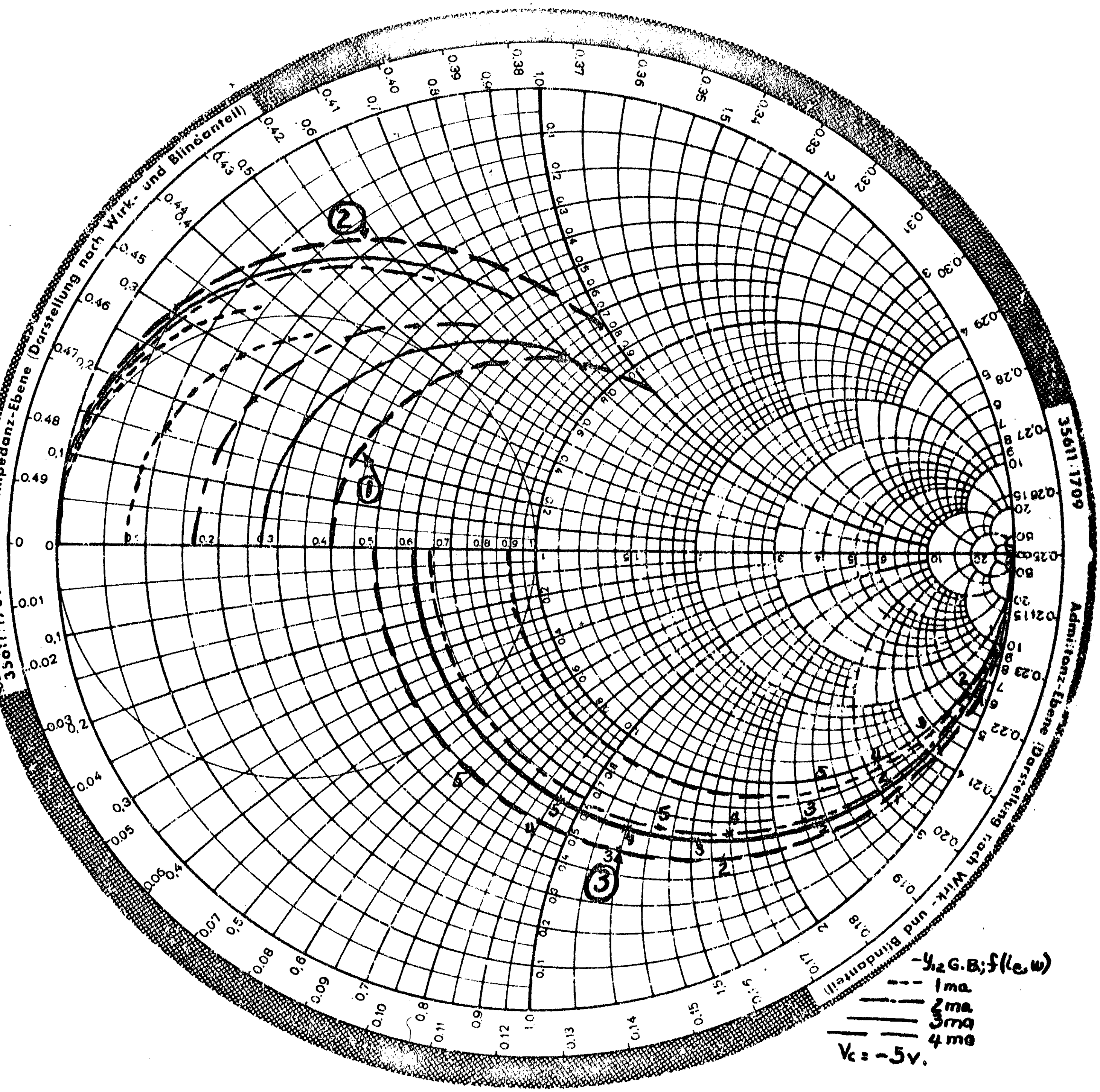

Impedanz-Ebene (Darstellung nach Wirk- und Blindanteil)
Admittanz-Ebene (Darstellung nach Wirk- und Blindanteil)
35611.1709
$-y_{12}$ G.B.; f(Ie, w)
1ma
2ma
3ma
4ma
Vc = -5v.

FIG. 14

Splitting off a parallel capacitance from the network in terms of the Smith chart means shifting the points of curve 1 of Fig. 11 along circles of constant conductance g/go in counterclockwise direction.

The new stretched curves 3 and 4 are found in the following way:

1) we construct the circles r/zo = constant = .5 (curve 4) and g^5/go = constant (curve 1*), where g^5/go is the conductance at 5 mc.

2) we then have to find a circle, the center of which is the point 1 of the chart, intersecting the curves 4 and 1* at two points which are simultaneously located on a diameter of this circle.

The points where the circle and its diameter intersect the curves 4 and 1* are the new 5 mc points $\frac{ro}{zo}$, $j\,\frac{x'5}{zo}$ and g^5/go, $-jb'5/go$ of the stretched curves 4 and 3 respectively.

The amount of capacitance to be deducted from curve 1 at 5 mc then is the difference $-j\,\Delta\, b5/go$ between the new susceptance $-jb5/go$.

The magnitude of the parallel capacitance then is

$$C = \frac{-j\Delta\, b_5}{2\,\pi\;5\quad 10^6}$$

The complete curve 3 is designed by deducting an amount of susceptance from the points corresponding to a frequency f of curve 1 according to the equation

$$j\,\Delta\, b_f = \frac{-j\,\Delta\, b_5}{5\text{ mc}}\; f$$

Curve 3 in turn yields the corresponding curve 4 in the impedance plane which actually follows to a good degree of approximation the locus of curve 4 as presumed at the beginning of the design procedure.

From curve 4 we now read the value of the series resistance of the remaining network, in our example $r/zo = .5$, which yields $r = .5\ zo = 10\,\Omega$. The magnitude of the series inductance is found from $j\,\frac{x5}{zo} = j\,\frac{.31}{zo}$ at 5 mc to be

$$L = \frac{31\quad zo}{2\,\pi\;5\quad 10^6} = .2\ \mu\text{ Hy}$$

The total equivalent network for y_{11} then is the following:

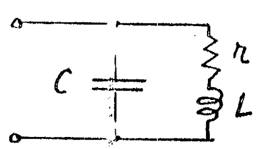


Fig. 13

The accuracy of the correlation between the measured values of y_{11} and the corresponding values as computed from the equivalent circuit then can be checked by computation according to the formulas

$$g = \frac{r}{r^2 + x^2} \qquad \text{and} \qquad jb = -j\frac{x}{r^2 + x^2}$$

where $x = \omega L$

2.2 Design of an Equivalent Circuit for $-y_{12}$ G. B.

(See Fig. 14)

Simila.. to the procedure outlined in paragraph 2.1 the measured values of $-y12 = g + jb$ are transformed into the relative values $g/go + jb/go$.
For $-y12$ for Ie = 4 ma, Vc = -5V we have:

Frequency mc	Measured g	Measured jb	Reference Value	Relative Values g/go	Relative Values jb/go
0	$40x10^{-6}$	0		.4	0
1	45	$31.5x10^{-6}$	go=	.45	.315
2	58	53	$100x10^{-6}$	.58	.53
3	71	72	zo=	.71	.72
4	89	87	10^4	.89	.87
5	117	97.5		1.17	.975

These relative values then are plotted on a Smith chart as shown in curve 1 of Fig. 14. We see that this curve transformed into the impedance plane would follow a circle of constant resistance r/zo, when a constant conductance, g/go = .4 in this example, is deducted from curve 1.

By subtracting a constant amount of g/go = .4, which means splitting off a parallel resistance ro

$$ro = \frac{1}{g/go}\, zo = 25\,K\Omega$$

from a remaining network as shown in the following fig., we get curve 2.

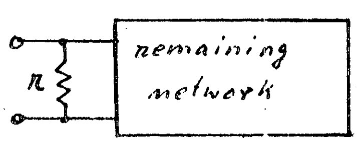


Fig 15

Curve 2 appears in the impedance plane as curve 3 which actually follows a circle of constant resistance $r/zo = .5$. The series resistance of the remaining network therefore is

$$r = z_o \times .5 \times 10^4 = 5\,K\Omega$$

Curve 3 being located in the lower half of the Smith chart determines the reactive part of the impedance to be capacitive. The circle of constant reactance $-j\,\frac{x5}{zo}$ intersecting the curve 3 at a point of impedance corresponding to a frequency of 5 mc yields

$$-j\,\frac{x5}{zo} = .64$$ and the value

of the series capacitance

$$C = \frac{1}{2 \times \pi \times 5 \times 10^6 \times \frac{x}{zo} \times zo} = \frac{1}{2\pi \times 5 \times 10^6 \times .64 \times 10^4} = 5\,\mu\mu F$$

The total equivalent circuit for $-y_{12}$ therefore is

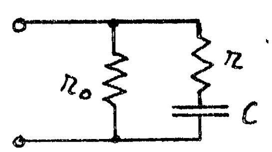


Fig. 16

with the magnitudes of the components as given by the above readings and computations.

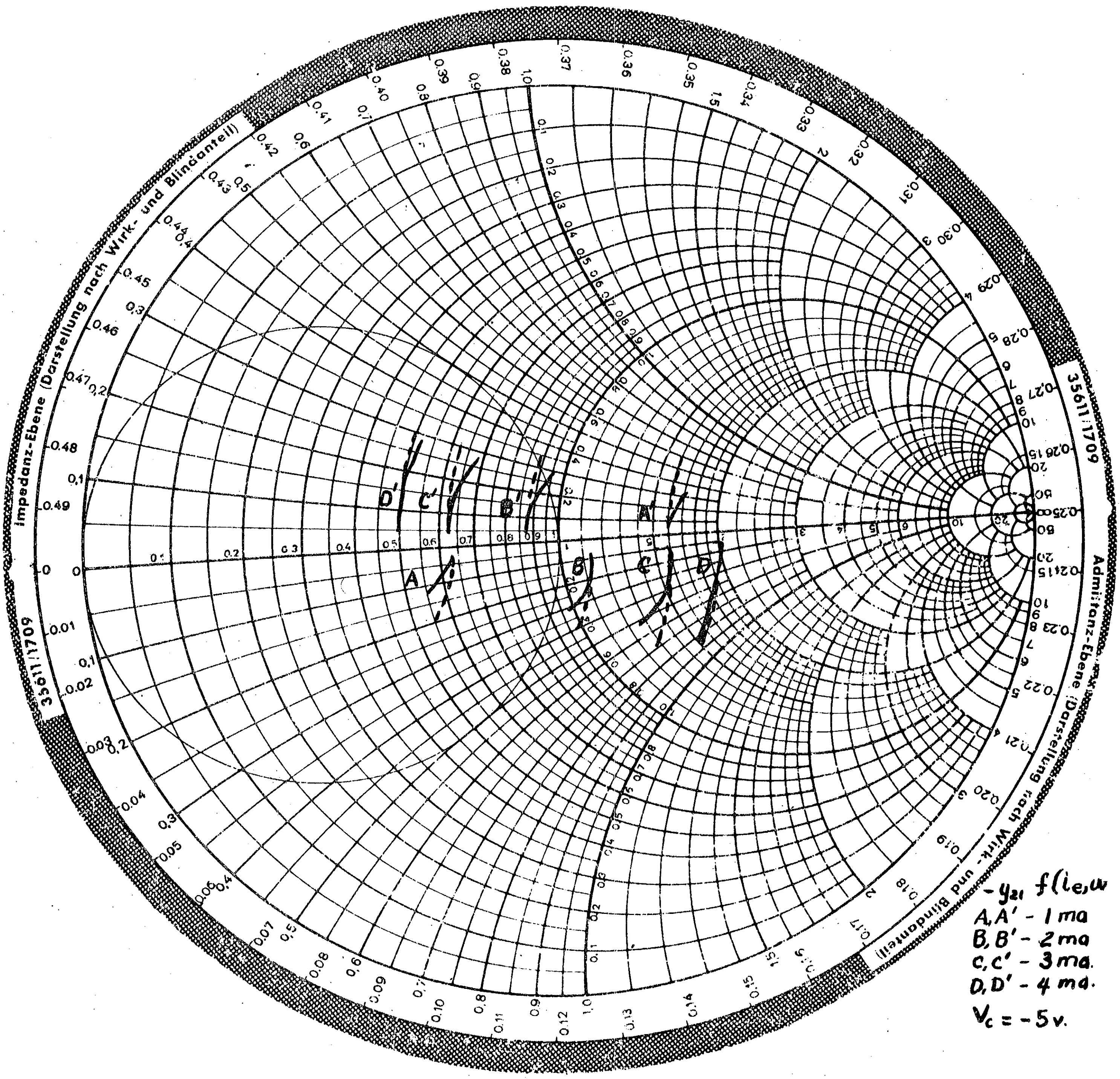
The degree of correlation between the measured data and the data as derived from the equivalent circuit can be checked by computation according to the formulas:

$$g = go + \frac{r\omega^2 C^2}{1 + r^2 \omega^2 C^2} \quad \text{and} \quad jb = \frac{C}{1 + r^2 \omega^2 C^2}$$

where $go = \frac{1}{ro}$.

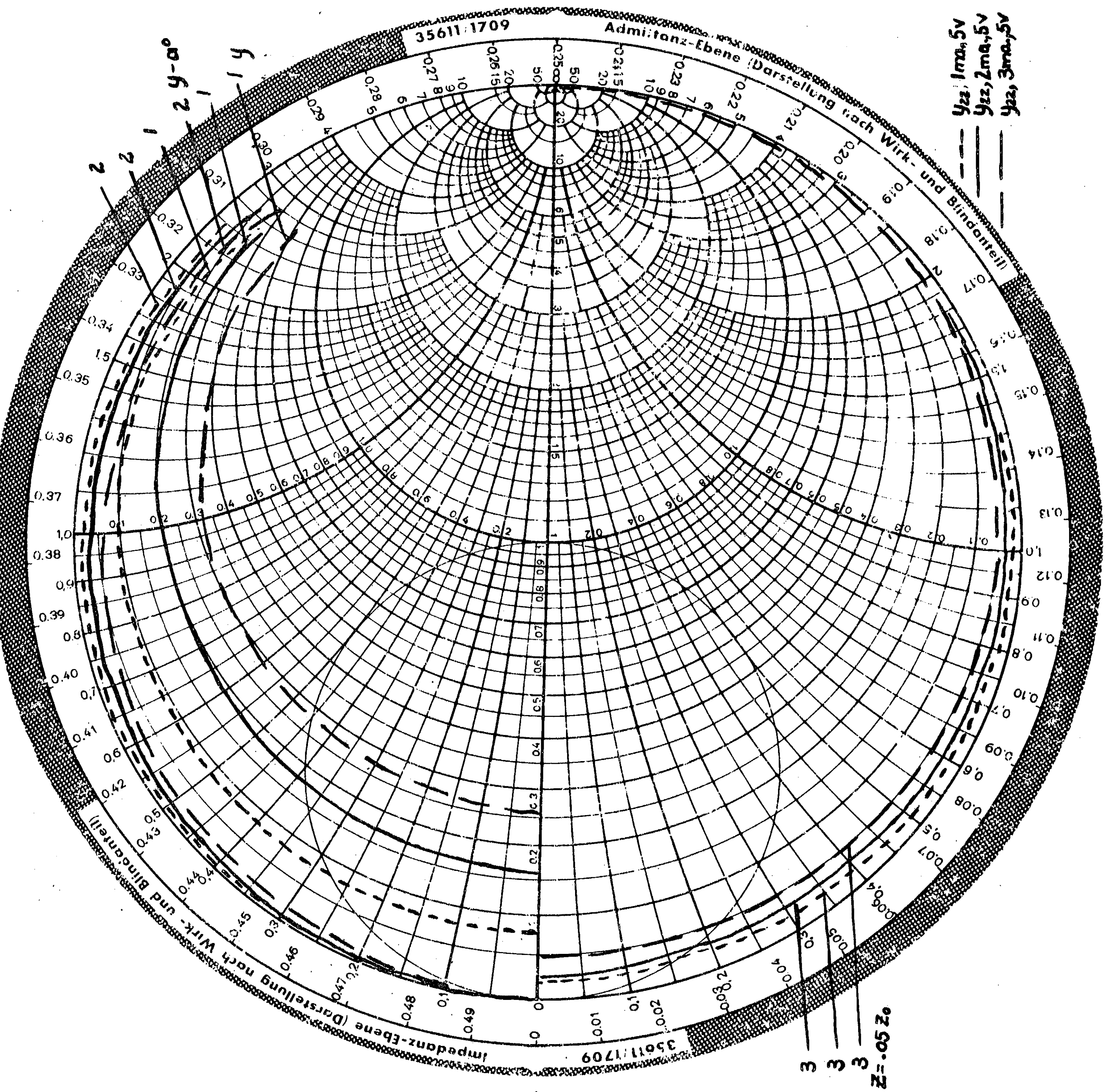
2.3 Equivalent Circuits for $-y_{21}$ and y_{22}

In a similar way as explained under paragraph 2.1 and 2.2 the equivalent circuits for $-y_{21}$ and y_{22} have been designed. The representation of $-y_{21}$ and y_{22} are shown in Fig. 17 and 18 respectively without further discussion.



$-y_{21}$ f(ie,ω)
A,A' - 1 ma
B,B' - 2 ma
C,C' - 3 ma.
D,D' - 4 ma.
$V_c = -5$ v.

FIG. 17


35611/1709
Admittanz-Ebene (Darstellung nach Wirk- und Blindanteil)
Impedanz-Ebene (Darstellung nach Wirk- und Blindanteil)
35611/1709
y22, 1ma, 5v
y22, 2ma, 5v
y22, 3ma, 5v
2
2
1
2 y-a°
1
1 y
3
3
3
Z = .05 Z0

.265 g0

FIG. 18