PROJECT STRETCH
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SUBJECT: Significant Digits Required In The Word Length

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There seem to be two considerations for determining how many significant figures a word should have in the Stretch machine.

The first of these factors is the fact that the major use of the Stretch machine at Los Alamos will be for hydro-dynamics problems. At present, they have the choice on the 704 of using 8 significant figures floating point or 10 significant figures fixed point. I think they have demonstrated to their own satisfaction that 8 significant figures, floating point, is not sufficient and that it will require 10 significant figures, floating point, to do an adequate job so far as round-off accuracy goes. Consider this as a starting point then, and consider what this means to future one-dimensional calculations.

Their objective, of course, is to increase the number of space mesh points in a problem so as to be able to get more detailed information about how things behave. Let us suppose, for example, that they increased the mesh points by a factor of 10. The first differences of the various hydro-dynamic functions would be decreased 1 significant figure thereby. Similarly the second differences would be decreased by 2 significant figures. Therefore, it would require 2 more decimal digit places in the word length in order to be able to do a calculation of the required accuracy.

Now let us consider the time it would take. At present, we can assume the problems will be done implicitly, and under such an assumption we know that as $\triangle x$ decreases, $\triangle t$ also must decrease in order to ensure stability. For the equations involved, $\triangle t$ must vary as $(\triangle x)^2$. Now we had assumed that $\triangle x$ was decreased by a factor of 10 when we increased the mesh points by this factor, and we therefore must decrease $\triangle t$ by a factor of 100. Hence we see that 2 significant figures must be added to those quantities in the equations involving time difference in order to preserve the required accuracy. We also see that 100 times as much computing will be required to reach the same point in real time due to $\triangle t$ being 100^{th} of its previous value. In addition to this, we have 10 times as much data to process, since we have made the mesh points 10 times as fine, and this will add another factor of 10 to the time it takes to do a problem. To summarize, if we wish to do a problem 10 times as well, it will take 1000 times as long to do it. This means, so far as a Stretch machine goes, that such a problem will require 5 to 10 times as long to solve as do the present hydro-dynamics problems

on the 704, assuming Stretch is 100 or 200 times speedier than a 704. I think it not likely that anyone will wish to spend more time than that on one hydro-dynamics problem and therefore, I believe that this time consideration determines a practical limit on the fineness of mesh for such problems, and this in turn tells us that two extra significant digits is sufficient. The conclusion is that the fraction part of the Stretch word should be 12 digits in length so far as single dimensional hydro-dynamics problems are concerned. Time factors will be even more constricting for multi-dimensional hydro-dynamics problems.

The other factor I mentioned in the beginning is the fact that general purpose computers will more and more be called upon to invert high order matrices. If one limits this inversion to those taking no more than 5 to 10 hours, this includes matrices approaching 1500 by 1500 elements. I recently wrote a letter to many of my friends throughout the computing field, asking their opinion about significant digits for matrix inversion. I have received replies from General Electric, Douglas Aircraft Corporation, University of California Radiation Lab and the Data Processing Center in New York. They point out the following facts concerning inversion of high order matrices. In the first place, a serious problem exists in that it is difficult to maintain linear independence in a matrix of this size unless one has 20 or 30 significant digits. Secondly, there is no general method which is best for all matrices and it seems unlikely that there will be enough mathematical talent available to seriously examine every matrix before it is put on a machine in sufficient detail to choose the appropriate method from the viewpoint of conserving significance. The people in the Data Processing Center think that we should have 16 or 20 significant figures. Other people think it is hopeless to try to outguess the matrix originators and that the only thing one can possibly do is make multiple precision operations extremely easy to program. They all point out however, that the greater the significance of a word, the greater will be the number of matrices that can be handled without resorting to multiple precision. All the matrix arguments lead one to believe that the word size should be as large as possible.

The only other thing required in order to be able to determine the Stretch word length is the size of the exponent. It is my belief that $10^{\pm 38}$ is too small, based on 704 experience. CPC experience tells me that $10^{\pm 50}$ is also too small. It is my belief that anything under $10^{\pm 100}$ is too small for a machine of this size and speed. (Also some very clever way of treating the overflow-underflow detection will be required.)

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