

invert 3×3 Matrix

$$A^{-1} = B$$

$$\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}$$

$$\text{inverse} \begin{matrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{matrix}$$

$$\sum_j a_{kj} b_{ji} = \delta_{ki} \quad \text{def of inverse}$$

for $i=1$ get j eqs for b_{j1} 's etc. $\sum a_{kj} b_{j1} = \delta_{k1}$

$$a_{11} b_{11} + a_{12} b_{21} + a_{13} b_{31} = 1$$

$$a_{21} b_{11} + a_{22} b_{21} + a_{23} b_{31} = 0$$

$$a_{31} b_{11} + a_{32} b_{21} + a_{33} b_{31} = 0$$

& same for b_{12}, b_{13} etc

$$b_{11} = \frac{\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}}{D_1}$$

$$b_{21} = \frac{-\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}}{D_1}$$

$$b_{31} = \frac{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}{D_1}$$

$$D_1 = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$b_{11} = \frac{a_{22} a_{33} - a_{23} a_{32}}{a_{11} (a_{22} a_{33} - a_{32} a_{23}) - a_{12} (a_{21} a_{33} - a_{31} a_{23}) + a_{13} (a_{21} a_{32} - a_{22} a_{31})}$$

$$\text{Take } \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \end{vmatrix}$$

$$b_{11} = \frac{\frac{1}{5} \cdot \frac{1}{9} - \frac{1}{6} \cdot \frac{1}{8}}{1 \left[\frac{1}{5} \cdot \frac{1}{9} - \frac{1}{6} \cdot \frac{1}{8} \right] - \frac{1}{4} \left[\frac{1}{2} \cdot \frac{1}{9} - \frac{1}{3} \cdot \frac{1}{8} \right] + \frac{1}{7} \left[\frac{1}{2} \cdot \frac{1}{6} - \frac{1}{3} \cdot \frac{1}{5} \right]}$$

$$= \frac{\frac{1}{45} - \frac{1}{48}}{1 \left[\frac{1}{45} - \frac{1}{48} \right] - \frac{1}{4} \left[\frac{1}{18} - \frac{1}{24} \right] + \frac{1}{7} \left[\frac{1}{12} - \frac{1}{15} \right]}$$

$$\frac{\frac{1}{15} - \frac{1}{16}}{1 \left[\frac{1}{15} - \frac{1}{16} \right] - \frac{1}{4} \left[\frac{1}{6} - \frac{1}{8} \right] + \frac{1}{7} \left[\frac{1}{4} - \frac{1}{5} \right]}$$

$$= \frac{\frac{1}{15 \times 16} - \frac{1}{3 \cdot 5 \cdot 4 \cdot 4}}{\frac{1}{3 \cdot 5 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 2} + \frac{1}{2 \cdot 5}} = \frac{\frac{1}{4}}{\frac{1}{4} - \frac{5}{4 \cdot 2} + \frac{3}{7}}$$

$$= \frac{1}{1 - \frac{5}{2} + \frac{12}{7}} = \frac{1}{1 - \frac{11}{14}} = \frac{1}{\frac{3}{14}} = \frac{14}{3} = 4 \frac{2}{3}$$

$$\frac{11}{14} = .7857142857$$

$$1 - \frac{11}{14} = .2142857143$$

$$b_{11} = 4.666666 = 4 \frac{2}{3}$$

Decimal case

using 4 figures

| | 1 | 2 | 3 |
|---|-------|-------|-------|
| 1 | 1.000 | .5000 | .3333 |
| 2 | .2500 | .2000 | .1667 |
| 3 | .1429 | .1250 | .1111 |

$$\textcircled{1} a_{22}a_{33} - a_{32}a_{23} = .02222 - .02084 = .00138$$

$$\textcircled{2} a_{12}a_{33} - a_{13}a_{32} = .05555 - .04166 = .01389$$

$$\textcircled{3} a_{12}a_{23} - a_{22}a_{13} = .08335 - .06666 = .01669$$

$$\textcircled{4} a_{11}\textcircled{1} - a_{21}\textcircled{2} + a_{31}\textcircled{3} = .001380 - .003473 + .002385 = .000292$$

$$\textcircled{5} b_{11} = \frac{\textcircled{1}}{\textcircled{4}} = 4.726$$

$$\text{Error} = .05933 \quad (1.255\%)$$

using 5 figures

| | 1 | 2 | 3 |
|---|--------|--------|--------|
| 1 | 1.0000 | .50000 | .33333 |
| 2 | .25000 | .20000 | .16667 |
| 3 | .14286 | .12500 | .11111 |

$$\textcircled{1} a_{22}a_{33} - a_{32}a_{23} = .022222 - .020834 = .001388$$

$$\textcircled{2} a_{12}a_{33} - a_{13}a_{32} = .055555 - .041666 = .013889$$

$$\textcircled{3} a_{12}a_{23} - a_{22}a_{13} = .083335 - .066666 = .016669$$

$$\textcircled{4} a_{11}\textcircled{1} - a_{21}\textcircled{2} + a_{31}\textcircled{3} = .0013880 - .0034723 + .0023813 = .0002970$$

$$\textcircled{5} b_{11} = \frac{\textcircled{1}}{\textcircled{4}} = 4.6734$$

$$\text{Error} = .006733 \quad (0.1449\%)$$

$$\text{Compared vs. 4 fig case } 4.726 - 4.6734 = .0526 \text{ difference}$$

"Noisy Mode": Round by 5 in last place
 (sub 5 if ^{not equal} greater than 5)
 (add 5 if less than 5)

- ① $.022227 - .020839 = .001388$ ^{cancel} = $.001383$
- ② $.055550 - .041661 = .013889$ ^{cancel} = $.013884$
- ③ $.083330 - .066661 = .016669$ ^{cancel} = $.016664$
- ④ $.0013835 - .0034715 + .0023801 = .0037631 - .0034715 = .0002916$ ^{cancel} (2916)
- ⑤ $\frac{1383}{2911} = 4.7504$

compared to no noise $4.6734 - 4.7504 = .077$ difference

"Noisy Mode": Round by 5 on left shift only

- ① $.022222 - .020834 = .001383$ ^{no noise}
- ② $= .013889$ ^{no noise}
- ③ $= .016669$ ^{no noise}
- ④ $.0013830 - .0034723 + .0023813 = .0037643 - .0034723 = .0002920$ ^{no noise} $+5$
- ⑤ $\frac{.001383}{.0002925} = 4.7282$

compared to no noise $4.6734 - 4.7282 = .055$ difference

Binary case

| | | | | |
|-------|---|--------|------------|---------|
| octal | 1 | 1.000 | .4000 | .252525 |
| | 2 | .2000 | .146314... | .125252 |
| | 3 | .11111 | .1000 | .070707 |

True answer $4\frac{2}{3} = 4.52525...$

100.101010101...

keep 10 bits

① $a_{22}a_{33} - a_{32}a_{23} = .0^5 101101000 - .000001010101011 = .0^5 0000101101$

② $a_{12}a_{33} - a_{13}a_{32} = .10001110001110 - .00001010101011 = .100001001100011$

③ $a_{12}a_{23} - a_{22}a_{13} = .0001010101011 - .0001000100010 = .0000010001001$

④ $a_{11}① - a_{21}② + a_{31}③ = .0^9 1011010000 - .0^8 1110001100 + .0^8 1001110010 = .0^8 0001001110$

⑤ $b_{11} = \frac{.0^9 101101}{.0^{11} 1001110} + \frac{4}{39} = 4.61538$ no noise 10 bits
 $4.47305 = 100.1001111$ error = .707011

keeping 9 bits

① $a_{22}a_{33} - a_{32}a_{23} = .0^5 101101011 - .0^5 101010101 = .0^5 000010110$

② $a_{12}a_{33} - a_{13}a_{32} = .0^4 111000111 - .0^4 101010101 = .0^4 001110010$

③ $a_{12}a_{23} - a_{22}a_{13} = .0^3 101010101 - .0^3 100010001 = .0^3 001000100$

④ $a_{11}① - a_{21}② + a_{31}③ = .0^9 10110000 - .0^8 111001000 + .0^8 100110111 = .0^8 000011111$

⑤ $= \frac{.0^9 10110}{.0^{12} 11111} = 8\frac{22}{31} = 5.6774$ no noise 9 bits
 $5.53265 = 101.101011$ error = 1.000

let case noise input 10^{25} bit every operation

① $a_{22}a_{33} - a_{32}a_{23} = .0^5 1011011001 - .0^5 1010101010 = .0^5 0100101111 = .0^9 101110$

② $a_{12}a_{33} - a_{13}a_{32} = .0^4 1110001111 - .0^4 1010101010 = .0^4 0011100101 = .0^6 11100100$

③ $a_{12}a_{23} - a_{22}a_{13} = .0^3 1010101010 - .0^3 1000100011 = .0^3 0010001111 = .0^5 10000110$

④ $a_{11}① - a_{21}② + a_{31}③ = .0^9 1011100011 - .0^8 1110010001 + .0^8 1001100111 = .0^8 000100010$

⑤ $\frac{.0^9 10111}{.0^{11} 10001} = 4\frac{23}{17} = 5.47057$ noise difference between main & no noise

2 case of noise insert on left side only

$$\textcircled{1} a_{22}a_{33} - a_{32}a_{23} = .0^9 101100 \checkmark$$

$$\textcircled{2} a_{12}a_{33} - a_{13}a_{32} = .0^5 11100010$$

$$\textcircled{3} a_{12}a_{23} - a_{22}a_{13} = .0^5 10001000 \checkmark$$

$$\textcircled{4} a_{11}\textcircled{1} - a_{21}\textcircled{2} + a_{31}\textcircled{3} = .0^9 101100_{0000} - .0^8 11100010_{000} + .0^8 1001101110_{000}$$
$$= .0^8 0001000111$$

$$\textcircled{5} = \frac{.0^9 101100}{.0^9 1000111} = \frac{14}{21} = 4.9576 = 4.952 = 100.11101010$$

difference from memory .21011