3 × 3 Matrix using different expansion

9 bits

$b_{11}' = a_{22}a_{33} - a_{32}a_{23} = .0^5\underline{101101100}$

$-b_{21}' = a_{21}a_{33} - a_{31}a_{23} =$

$b_{31}' = a_{21}a_{32} - a_{22}a_{31} =$

$a_{11}b_{11}' =$

$-a_{12}b_{21} =$

$a_{13}b_{31} =$

1.42 12

1.0921042

1

$$\begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad \text{transpose} \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

$(.013302)_8 = .000001011011000010$

$= .012526 = .0000010101001001$

(1) $b'_{11} = a_{22}a_{33} - a_{32}a_{23} = \frac{1}{5}\cdot\frac{1}{9} - \frac{1}{8}\cdot\frac{1}{6} = \left(\frac{1}{3}\right)\left(\frac{1}{15}\right)\left(\frac{1}{16}\right) = \frac{1}{720} = .0013888 = (.000555)_8 = .0000101101101$

$(.0161616)_8 \quad (.014142)$

(2) $-b'_{21} = a_{21}a_{33} - a_{31}a_{23} = -\left(\frac{1}{4}\cdot\frac{1}{9} - \frac{1}{7}\cdot\frac{1}{6}\right) = -\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{7}\right) = -.00396825$

(3) $b'_{31} = a_{21}a_{32} - a_{22}a_{31} = \frac{1}{4}\cdot\frac{1}{8} - \frac{1}{5}\cdot\frac{1}{7} = \left(\frac{3}{32}\right)\left(\frac{1}{35}\right) = .00267857$

$d = a_{11}b'_{11} + a_{12}b'_{21} + a_{13}b'_{31} = \left(\frac{1}{3}\right)\left(\frac{1}{15}\right)\left(\frac{1}{16}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{32}\right)\left(\frac{1}{35}\right)$

$= \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{5}\cdot\frac{1}{2}\cdot\frac{1}{8} - \frac{1}{8}\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{7} + \frac{1}{8}\cdot\frac{1}{4}\cdot\frac{1}{5}\cdot\frac{1}{7}$

$= -\frac{1}{3}\cdot\frac{1}{8}\cdot\frac{1}{10}\cdot\frac{1}{7} + \frac{1}{8\cdot4\cdot5\cdot7} = \frac{1}{8\cdot7\cdot5}\left[-\frac{1}{6} + \frac{1}{4}\right]$

$= -\frac{1}{4}\cdot\frac{1}{7}\cdot\frac{1}{5}\cdot\frac{1}{6}\cdot\frac{1}{4} = .0002976$

$= .00138888 - .001984125 + .00089286 = .00029760$

$b_{11} = \cancel{\frac{1}{3}}\cdot\cancel{\frac{1}{2}}\cdot\cancel{\frac{1}{5}}\cdot\cancel{\frac{1}{2}}\cdot\cancel{\frac{1}{8}}\cdot\cancel{4}\cdot7\cdot\cancel{5}\cdot\cancel{3}\cdot\cancel{8} = 7.0$

$b_{21} = -\cancel{\frac{1}{6}}\frac{1}{6}\cancel{\frac{1}{7}}\cdot4\cdot\cancel{7}\cdot5\cdot\cancel{6}\cdot4 = -\frac{1}{3}\cdot2\cdot20 = -\frac{40}{3}$

$b_{31} = \frac{3}{32}\cdot\frac{1}{35}\cdot4\cdot\cancel{7}\cdot\cancel{8}\cdot6\cdot4 = 9.0$

$b'_{12} = a_{12}a_{33} - a_{32}a_{13} = \frac{1}{2}\cdot\frac{1}{9} - \frac{1}{8}\cdot\frac{1}{3} = \frac{1}{6}\cdot\frac{1}{3}\cdot\frac{1}{4} = .0138888 = (.00707)_8$

$b'_{22} = -a_{11}a_{33} + a_{31}a_{13} = -1\cdot\frac{1}{9} + \frac{1}{7}\cdot\frac{1}{3} = -\frac{4}{3\cdot3\cdot7}$

$b'_{32} = a_{11}a_{32} - a_{12}a_{31} = 1\cdot\frac{1}{8} - \frac{1}{2}\cdot\frac{1}{7} = \frac{3}{2\cdot4\cdot7}$

$b_{12} = \frac{1}{3}\cdot4\cdot7\cdot5 = \frac{140}{3} =$

$b_{22} = \frac{4}{3}\cdot4\cdot5\cdot8 = -\frac{640}{3}$

$b_{32} = 3\cdot5\cdot3\cdot4 = 180.$

$= \frac{1}{60} = (.010655)_8$

$b'_{13} = a_{12}a_{23} - a_{13}a_{22} = \frac{1}{2}\cdot\frac{1}{6} - \frac{1}{3}\cdot\frac{1}{5} = \frac{1}{3\cdot4\cdot5}$

$b'_{23} = -a_{11}a_{23} + a_{21}a_{13} = -1\cdot\frac{1}{6} + \frac{1}{4}\cdot\frac{1}{3} = -\frac{1}{2\cdot6}$

$b'_{33} = a_{11}a_{22} - a_{12}a_{21} = 1\cdot\frac{1}{5} - \frac{1}{2}\cdot\frac{1}{4} = \frac{3}{5\cdot8}$

$b_{13} = 56.$

$b_{23} = -280.$

$b_{33} = 1512.$

$$a_{11} b_{11} = (1)\left(\tfrac{1}{3}\right)\left(\tfrac{1}{15}\right)\left(\tfrac{1}{16}\right) = \frac{1}{720} = .00138888$$
$$a_{21} b_{12} = -\left(\tfrac{1}{4}\right)\left(\tfrac{1}{6}\right)\left(\tfrac{1}{3}\right)\left(\tfrac{1}{4}\right) = -\frac{1}{288} = -.00347222$$
$$a_{31} b_{13} = \left(\tfrac{1}{7}\right)\left(\tfrac{1}{3}\cdot\tfrac{1}{4}\right)\left(\tfrac{1}{5}\right) = \frac{1}{420} = .00238045$$

.00029761

$$\frac{1}{3}\cdot\frac{1}{4}\left[\frac{1}{15}\cdot\frac{1}{4} - \frac{1}{4}\cdot\frac{1}{6} + \frac{1}{7}\cdot\frac{1}{5}\right]$$
$$\left[\frac{1}{4}\left(\frac{1}{15}-\frac{1}{6}\right) + \frac{1}{35}\right]$$
$$\frac{1}{4}\cdot\frac{-1}{5\cdot 2}$$
$$\frac{1}{12}\left[-\frac{1}{40}+\frac{1}{35}\right] = \frac{1}{12}\cdot\frac{1}{5}\left[\frac{-1}{8}+\frac{1}{7}\right] = \frac{1}{12}\cdot\frac{1}{5}\cdot\frac{1}{56} = \frac{1}{3360}$$

$D = .00029761$

$2^{12} N = 1.2190476$

$= 1.160116$

1.001110 000001001110

$= .000116)_8$

.000 000 000 001 001 110

$.0^{11}\,1001110$

$2^{3} N = \ldots 238095$

.001160

3X3 Matrix

in octal

1	1.000	.4000	.01010101010 .252525
2	.010 .2000	.001100110011001 .146314...	.00101010101 .125252...
3	.001001001001 .1111111	.0010000000 .1000	.000111000111000111 .070707.

$$b_{11} = \frac{a_{22}a_{33} - a_{23}a_{32}}{a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{12}a_{33} - a_{13}a_{32}) + a_{13}(a_{12}a_{23} - a_{22}a_{13})}$$

keeping 9 bits

b_{11}' ① $a_{22}a_{33} - a_{32}a_{23} = .0^5 101101100 - .0^5 101010101 = .00000\,00010111 = .0^9 10111xxxx$

$-b_{12}'$ ② $a_{12}a_{33} - a_{32}a_{13} = .0^4 111000111 - .0^4 101010101 = .0000\,001110010 = .0^6 1110010xx$

b_{13}' ③ $a_{12}a_{23} - a_{22}a_{13} = .0^3 101010101 - .0^3 100010001 = .000\,001000100 = .0^5 1000100xx$

$a_{11}b_{11} = .0^9 101110\,000$ ↙ Truncated to 9 bits

$a_{21}b_{12} = -.0^8 111001000$

$a_{31}b_{13} = .0^8 100110111$

(001001001001)

$.0^8 000100111$

$.0^{11} 100111xxx$

exact value

$.0^9 10110110...$

$-.0^8 111000111...$

$.0^8 100111000 1...$

$.0^{11} 1001110\,000\,001001110.$

Exact values: (octal)

$b_{11} = .013302 - .012525 = .000555...$

$-b_{12} = . \quad = .007070$

$b_{13} = \quad = .010421042...$

extremes

	smallest	largest
$a_{11}b_{11}$	$= .0^9 101110000$	$.0^9 101111111$
$a_{21}b_{12}$	$= -.0^8 111001011$	$-.0^8 111001000$
$a_{31}b_{13}$	$= .0^8 100110111$	$.0^8 100111010$
	$.0^{11} 100100\,000$	$.0^{11} 110001100$

diff: $= .0^{11} 001101100$ $\quad \frac{1}{2}$ diff $= .0^{11} 000110110$

↑ "correct" impression

Inverting last bit on left shifts only

$$b'_{11} = .0^9\,10110\,\overset{\downarrow}{0}\,xxxx$$
$$-b'_{12} = .0^6\,111\,001\,\overset{\downarrow}{1}xx$$
$$b'_{13} = .0^5\,100\,010\,\overset{\downarrow}{1}xx$$

$$(1,0) \rightarrow a_{11}\,b_{11} = .0^9\,10110\,0000$$
$$(0,1) \rightarrow a_{21}\,b_{12} = -.0^8\,111001100$$
$$a_{31}\,b_{13} \quad .0^8\,100\,111\,100$$
$$(.0010011.0001) \quad .0^{11}\,100\,000\,000$$

exact value $\quad .0^{11}\,100\,111\,000$

diff. $\quad .0^{11}\,000\,111\,000$

"correct" diff $\quad .0^{11}\,000\,110\,110$

$$\Delta^2 = .0^{11}\,000\,000\,010$$

set to zero

$$b'_{11} = .0^9\,10\,110\,\overset{\downarrow}{0}\,xxxx$$
$$-b'_{12} = .0^6\,111\,001\,\overset{\downarrow}{0}xx$$
$$b'_{13} = .0^5\,1000\,10\overset{\downarrow}{0}xx$$

$$a_{11}b_{11} = .0^9\,10110\,0000$$
$$a_{21}b_{12} = -.0^8\,111\,001000$$
$$a_{31}b_{13} = .0^8\,100\,110\,111$$
$$.0^{11}\;\;011\,111\,000$$

exact $\quad .0^{11}\,100\,111\,000$

$$.0^{11}\;\;001\,000\,000$$

"correct" $\quad .0^{11}\,000\,110\,110$

$$.0^{11}\,000\,001\,010$$

poorest

Insert ones instead of 0's

$$b'_{11} = .0^9\,10111\,1111$$
$$-b'_{12} = .0^6\,111001\,011$$
$$b'_{13} = .0^5\,100\,010\,011$$

$$a_{11}b_{11} = .0^9\,10111111$$
$$a_{21}b_{12} = -.0^8\,111\,001\,011$$
$$a_{31}b_{13} = .0^8\,100\,111\,010$$
$$.0^{11}\,101\,101\,111$$

exact $\quad .0^{11}\,100\,111\,000$

$$.0^{11}\,000\,110\,111$$

"correct" $\quad .0^{11}\,000\,110\,110$

$$.0^{11}\,000\,000\,001$$

best