Colloq. Talk at Los Alamos   July 1958

Clarance Lee - Multigroup code

$$\nabla D_i(\vec{r})\nabla\phi - \overbrace{\sigma_i^{(1)}}^{K}(r)\phi_i + \sum_{j=1}^{i}\sigma_{i-j\to i}\phi_{i-j} + \nu_i\Delta\lambda_i\, 7F(r) + \Delta w_i \overset{EXT}{S}(\vec{r}) = 0$$

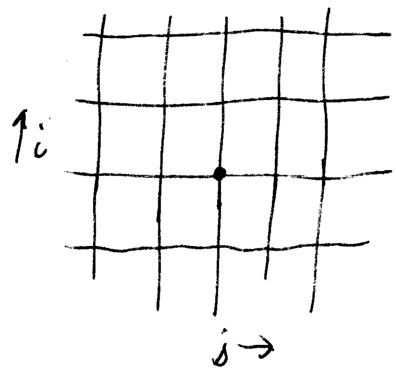
Extrapolated End Point $\phi(r_0+\Delta r_b\,\hat{n})=0$

$\phi_b = C$ $\frac{.710}{\lambda}$

$D_i\nabla\phi_i = \text{const}$

$D\nabla \ln\phi = \text{const}$

mesh 2-4000. points

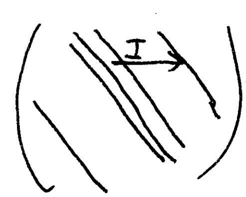


So:

$$\{A''\phi+S\} = a_{ij}\phi_{i+1j} + b_{ij}\phi_{i-1j} + c_{ij}\phi_{ij-1} + d_{ij}\phi_{ij+1}$$

$$-(a_{ij}+b_{ij}+c_{ij}+d_{ij}+k_{ij})\phi_{ij} + S_{ij} = 0$$

System of eq. $A''\phi + S = 0$ $\phi = \begin{pmatrix}\phi_{11}\\ \phi_{21}\\ \vdots\\ \phi_{IJ}\end{pmatrix}$

$A'' =$ 

1st. divide by (term in paren.)   have 1's on diagonals

Then $A'' = I + L + U$

$$\underset{\text{identity}}{I} \quad \underset{\text{lower}}{L} \quad \underset{\text{upper}}{U}$$

$$\phi = -(L+U)\phi - S$$

Simultaneous Iteration (due to Gauss) also called Schmidt's method, Richardson's method, method I

$$\phi^{p+1} = -(L+U)\phi^{p} - S$$

Successive Iteration (uses known new ones) called: Gauss method, Gauss-Seidel method, "ordinary Liebman" method II

$$\phi^{p+1} = -L\phi^{p+1} - U\phi^{p} - S$$

or $\phi^{p+1} = -(I+L)^{-1}U\phi^{p} - (I+L)^{-1}S$

$$E^{p} = \phi - \phi^{p}$$

Sim. $E^{p+1} = -(L+U)E^{p} = \lambda E^{p}$ can solve for eigenvalues.

Succ. $E^{p+1} = -(L+I)^{-1}UE^{p}$

can show $\max_i \sum_j |a_{ij}| \leq 1$

$$\max_i \sum_j |a_{ij}| < 1$$

for pos. definite & symmetric $a_{ij} = a_{ji}$

Then $\sum a_{ij} x_j x_i > 0$

Stein-Rosenberg Theorem:

$a_{ii} = 1$

$a_{ij} \leq 0$

Then either method will both converge (diverge.)

one can put in a parameter & add previous iterate.

Extrapolation it;

Sim: $\phi^{p+1} = \phi^p - \omega \left[A\phi^p + S \right]$

error dies as $E^{p+1} = (1-\omega A) E^p$

$E_i^{p+1} = (1-\omega\mu_i(A)) E_i^{(p)}$

could take out $\omega = \frac{1}{\mu_i(A)}$ one for each mesh pt.

usually take out dominant mode

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next order of approx: 2nd order Richardson

$$\phi^{p+1} = \phi^p + \alpha(\phi^p - \phi^{p-1}) + \omega(A\phi^p + S)$$

$$E^{p+1} = \left[1 + \alpha + \omega(1-\lambda_i) \right] E_i^p - \alpha E_i^{p-1}$$

assume $E_i^{p+1} = \theta_i E_i^p = \theta_i^2 E_i^p$

then $\theta_i = \frac{1}{2}\left[1+\alpha+\omega(1-\lambda_i) \pm \sqrt{(1+\alpha+\omega(1-\lambda_i)) - 4\alpha} \right]$

if $|\theta_i| = \sqrt{\alpha}$ all eigenvalues decay at same rate.

can require this.

$$\alpha = \left[\frac{\sqrt{1-\lambda_0} - \sqrt{1-\lambda_m}}{\sqrt{} + \sqrt{}} \right]^2$$

$$\omega = \left[\frac{2}{\sqrt{1-\lambda} + \sqrt{1-\lambda_m}} \right]^2$$

for Successive :

$$\phi^{p+1} = \phi^{p} - \omega\left[\phi^{p} + L\phi^{p+1} + U\phi^{p} + S\right]$$

$$E^{p} = (I+\omega L)^{-1}\left((1-\omega)I - \omega U\right)E^{p-1}$$

Extrapolated Liebman :

$$\phi^{p+1} = (1-\omega)\phi^{p} + \omega\left(U\phi^{p} + L\phi^{p+1} + S\right)$$

opt. ω : $\quad \omega = \dfrac{2}{1+\sqrt{1-\lambda_m^2}}$

λ_m — $-(L+U)$ max. eigenvalue

Extrapolated Richardson :

$$\phi^{p+1} = (1-\omega)\phi^{n} + \omega\left(A^{n} + S\right)$$

2nd order Richardson

$$\phi^{n*} = A\phi^{n-1}$$

$$\phi^{n} = (1-\alpha)\phi^{n-2} + \alpha\left(\phi^{n*} + S\right)$$

a variation Reilly's Eq. $\quad \phi^{n} = (1-\alpha)\phi^{n-2} + \alpha\left((L+U)\phi^{n} + FS\right)$ uses every other mesh point

has an advantage for non-linear

Comparison in Times :

assume factor of $R=100$ reduction in error.
usually $\lambda = 0.998$

		no. of iterations	no. of storages	
ord Richardson	(Succ. Relax)	= 2000	1	take 2 millisec per point on 704.
1st order Liebman	(Sim Relax)	= 1000	1	
Ext Rich		1000	1	(ADI takes 3 ms)
Ext. Lieb		70.	1	
2nd Rich.		140.	2	
Riley.		70	1	
A.D.I.		10		← special cases only, may not always converge.

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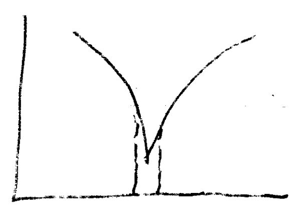
can use conjugate gradients methods
polynomial methods. somewhat better than Ext. Liebmann

Can reduce to 5 quantities per point, for storage.

very critical dependence on w

no. of it.



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