

F U Z Z

Richard W. Hamming, Bell Labs computer pioneer, spent a year at Stanford as visiting professor around 1960. He completed his book on numerical analysis, taught classes using it, and in office hours explicated computational wonders like the Hamming error-correcting code and the workings of the Bell Interpreter.

The Bell Interpreter on the IBM 650 was a groundbreaking floating-point development environment. A calculation that, if written in machine code, wouldn't fit within the 650's 2000-word drum memory, would fit in half that space when interpreted; and it would execute faster.

The Bell Interpreter's comparison operations were "fuzzy": an equality comparison ignored differences in the least significant digit(s?) and so saved users from familiar floating-point anomalies, such as $1/3+1/3+1/3$ not equaling 1.0 . This appealed to Larry Breed, who introduced fuzzy relational functions to the initial specs for APL. He extended fuzziness to include floating-to-integer conversions, both implicit (as, for instance, in an index value, or the argument of monadic iota) and explicit (the floor and ceiling functions).

Defining and implementing fuzziness for these functions was troublesome. No floating point arithmetic can fulfill the expectations of users who think in terms of real numbers and who expect, for example, a value displayed as "0.1" to be exactly one-tenth. Trying to prevent obvious anomalies by using fuzz inevitably introduces other anomalies.

The definition and implementation of fuzzy floor (also ceiling) in the early releases of APL/360 had anomalies around zero and above $1E13$. A redefined fuzzy floor was shipped in a software release around 1974, without announcement. It exhibited different anomalies, and it changed results produced by programs of major financial customers. Customers, and their IBM marketing offices, were not pleased.

In 1977 Larry redesigned floor and ceiling, formalized their characteristics using methodology introduced by Michael Jenkins of Queens College, and presented the new definitions in an IBM Technical Report, "Definitions for Fuzzy Floor and Ceiling." These definitions survived scrutiny and became part of the ANSI/ISO APL standard. An illustration in the Tech Report showed a pinstriped IBM salesman inspecting (with understandable skepticism) a fuzzy floor being proffered by an APL backroom nerd. One of Eugene McDonnell's identical twin cartoonist sons, Luke and Pete, created the illustration, but Larry doesn't remember which.