A Study of Threshold Detection

Thomas Allan Froeschle

A STUDY OF THRESHOLD DETECTION

by

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ABSTRACT

An extremely versatile, high speed, tunnel diode stabilized threshold detector is described. The device is d.c. coupled, and uses negative feedback to control the hysteresis. The hysteresis is adjustable from less than 1 millivolt to 5 volts. When the hysteresis is set below 1 millivolt, the device serves as a stable zero crossing detector. The existing experimental threshold detector operates from d.c. to 500 kc/s.; the switching speed at the output is 30 ns. Operation at frequencies above 1 mc/s. is feasible.

Two tunnel diodes set the thresholds of the device. The thresholds depend only on the peak currents and peak voltages of the tunnel diodes. The valley currents and voltages have no effect on the transfer characteristic. Since the peak currents and peak voltages have a high degree of stability, the transfer characteristic is very stable. A static analysis yields theoretical expressions for the transfer characteristic.

The tunnel diode stabilized threshold detector is the most sophisticated device presented in the thesis.

The report begins with a discussion of some basic threshold detectors. A Schmitt trigger, a flip-flop, a four transistor complementary flip-flop, and a differentially driven flip-flop are included. Analytical expressions yielding the static transfer characteristics of these devices are presented.

Another more complex threshold detector which utilizes negative feedback is included. This device uses two flip-flops in conjunction with a latching circuit to set the thresholds. It is not as stable as the tunnel diode circuit, and is intended to have a hysteresis of .1 volt to 10 volts. The experimental model can have as little as 1 millivolt of hysteresis; but, the thermal stability is not good. The device operates from d.c. to 200 kc/s.; the output is capable of switching 1 ampere in 30 ns. A static analysis of this device is presented.

Thesis Supervisor: John S. MacDonald
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CHAPTER I

Introduction

1.1 The General Switching Threshold Detector

Threshold detectors and limiters have been used for phase measurement and for analog to digital conversion. Now, with the development of new switching systems, there is an increased demand for fast, stable threshold detectors.

In general, a threshold detector is a two port device having an input and an output. The output has two states. There is a range of input voltages for which either output state may occur depending on the history of the input; this is the hysteresis range. For input voltages above and below the hysteresis range, only one output state is possible.

A special threshold detector, having an output voltage which changes states whenever the input voltage crosses zero, is called a zero crossing detector. Such devices are realizable with simple electronic circuits. However, all practical zero crossing detectors have a transfer characteristic which fluctuates with time, temperature, and frequency, so that they can achieve the ideal transfer characteristic only for short periods of time. Thus, practical zero crossing detectors are useful only if the input signal exceeds the long term fluctuations of their hysteresis and detection point. The fluctuations limit the useful sensitivity of a zero crossing detector.

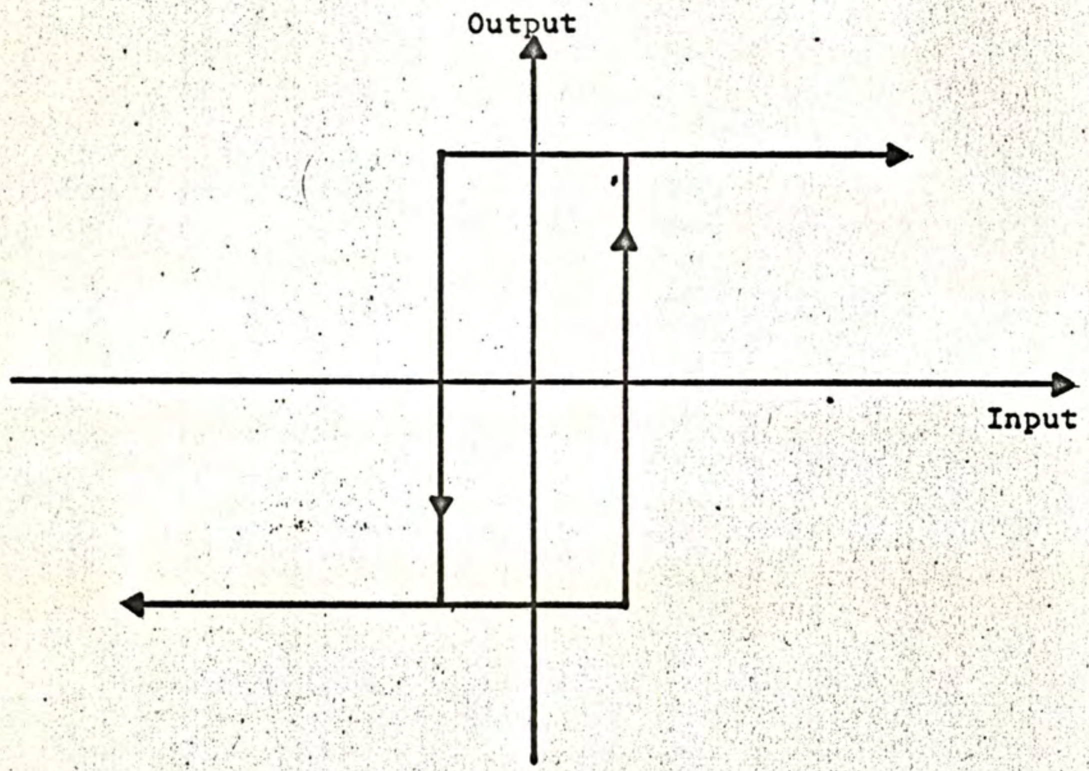

Fig. 1 Transfer Characteristic of a Threshold Detector

The purpose of this thesis is to describe some techniques and circuits which are useful for threshold detection. All the circuits are d.c. coupled; that is, all the circuits presented in this paper are designed to work at arbitrarily low frequencies. Zero crossing detection and feedback control of hysteresis are given special emphasis.

Chapter 1 is a collection of circuits and concepts used in threshold detection. The limiter and threshold detector are introduced and compared. A number of threshold detectors are discussed briefly. Expressions yielding their theoretical static transfer characteristics are included. The differentially driven flip-flop, a circuit which uses preamplification to reduce the hysteresis width, is discussed. Analytical expressions for the transfer characteristic are, again, included. Finally, the concept of feedback control of the hysteresis width is introduced.

Chapter 1 provides a background and extended introduction for chapter 2. Two more sophisticated circuits utilizing negative feedback are presented in chapter 2. The operation of these circuits is discussed in detail, and the theoretical transfer characteristics are calculated. The second circuit presented in the chapter, the tunnel diode stabilized circuit, is extremely versatile and stable. Th-s versatile and stable. This circuit has a hysteresis width which can be varied from 5 volts to less than 1 millivolt. It operates at speeds up to 500 kc/s; improved circuits may operate at speeds greater than 1 mc/s.

1.2 Switching Devices vs. Limiting Devices

A limiter is another device which is commonly used as a zero crossing detector. This device has a transfer characteristic which is similar to a threshold detector except that the hysteresis range is replaced by a region where the limiter acts as an amplifier.

Consider a threshold detector with a hysteresis region of width W which is centered about zero. Let the input signal be A sin ωt. The output of the threshold detector will have a phase lag, θ, where

$$\theta = \sin^{-1}\left(\frac{W}{2A}\right)$$

The switching time of the output of the threshold detector is constant regardless of the amplitude of the input signal.

If the same signal is used as the input to a limiter with an amplification region of width W which is centered about zero; then the output will have a rise time, T_r, where

$$T_r = \frac{2}{W}\sin^{-1}\left(\frac{W}{2A}\right)$$

The limiter does not have any phase shift from the input to the output.

When the hysteresis width

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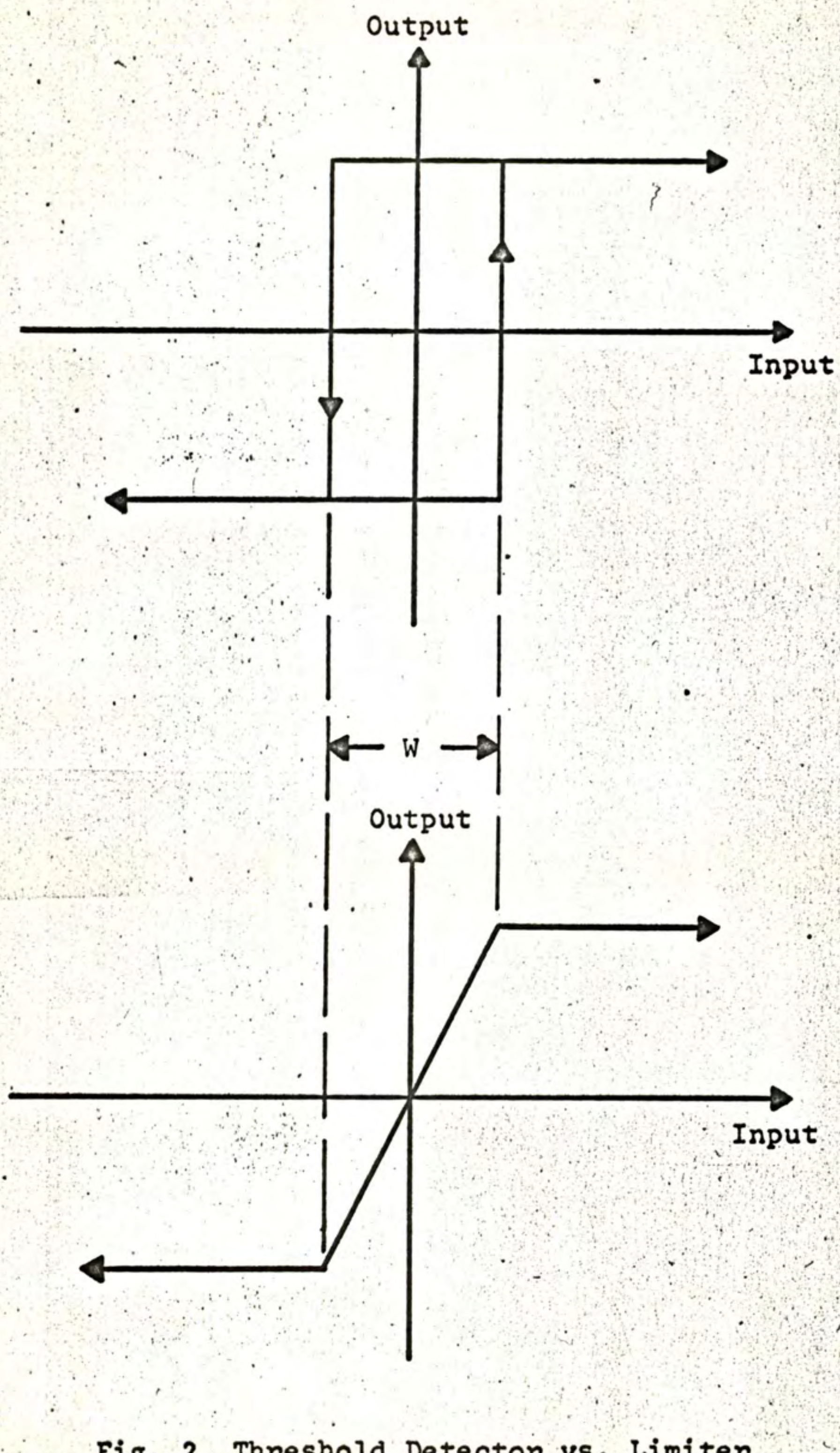


Fig. 2 Threshold Detector vs. Limiter

When the hysteresis width of the threshold detector is set to zero, the device becomes a zero crossing detector. Similarly, when the gain of the limiter in its amplifying region becomes infinite, it becomes a zero crossing detector.

In practice, stable limiters appear to be more difficult to realize than stable threshold detectors. Furthermore, limiters do not have the useful hysteresis region which is the identifying characteristic of threshold detectors. The hysteresis region is an integral part of certain two state modulation systems.(1)

1.3 Basic Threshold Circuits

1.31 The Schmitt Trigger

The Schmitt trigger or emitter coupled multivibrator is probably the most common threshold detector.(2) A typical circuit is pictured in fig. 3. The Schmitt trigger is useful whenever temperature stability is not important. It is usually acceptable when moderate or large hysteresis is required since temperature effects are not important for such operation. If, however, the hysteresis width is very critical, the Schmitt trigger cannot be used unless the temperatures of the transistors are regulated.

The circuit of fig. 3 has the transfer characteristic shown in fig. 4 where:

The transistor model used to obtain the theoretical expressions is discussed in the appendix.

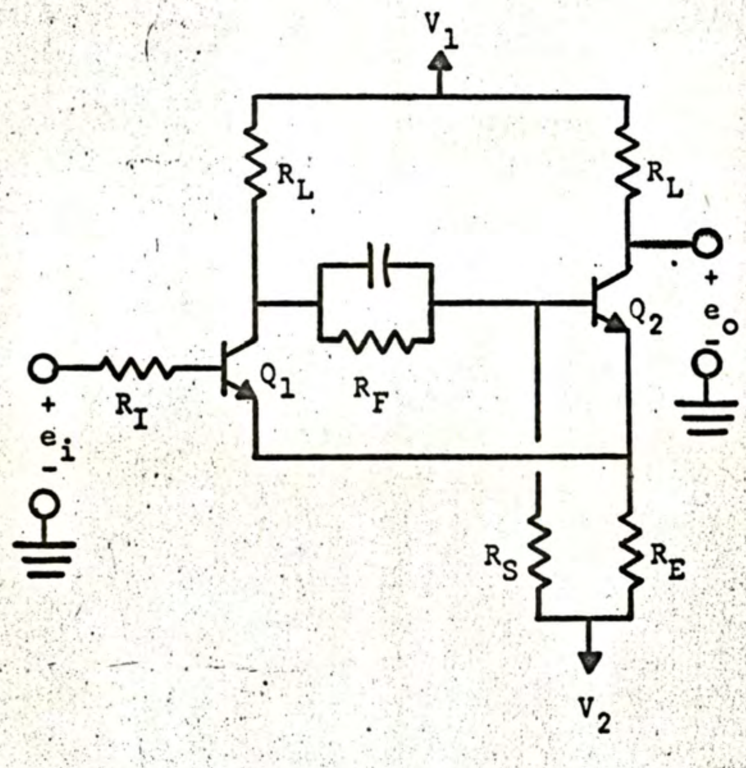


Fig. 3 A Typical Schmitt Trigger

$$e_1 = \frac{V_1\{R_S[(\beta_1+1)R_E+R_I][(\beta_2-1)R_L-R_F] + (\beta_2+1)R_E\beta_1R_LR_S - (\beta_2+1)R_ER_I(R_F+R_L)\}}{D}$$

$$+ \frac{V_2\{(R_F+R_L)[(\beta_1+1)R_E\beta_2R_L + (\beta_2+1)R_ER_I] + \beta_1\beta_2R_L^2R_S - R_IR_S[(\beta_2-1)R_L-R_F]\}}{D}$$

$$+ \frac{V_{BE}\{\beta_1\beta_2R_L^2R_S - \beta_2R_LR_I(R_F+R_L) - R_IR_S[(\beta_2-1)R_L-R_F]\}}{D}$$

$$D = (\beta_1+1)R_ER_S[(\beta_2-1)R_L-R_F] + (\beta_1+1)R_E\beta_2R_L(R_F+R_L) + \beta_1\beta_2R_L^2R_S + (\beta_2+1)R_E\beta_1R_LR_S$$

$$e_2 = V_2 + \frac{(V_1-V_2)[R_IR_S + (\beta_1+1)R_ER_S] + V_{BE}[\beta_1R_LR_S - R_I(R_L+R_F+R_S)]}{\beta_1R_LR_S + (\beta_1+1)R_E(R_L+R_F+R_S)}$$

$$e_3 = V_1$$

$$e_4 = \frac{V_1R_E + V_2R_L}{R_L + R_E}$$

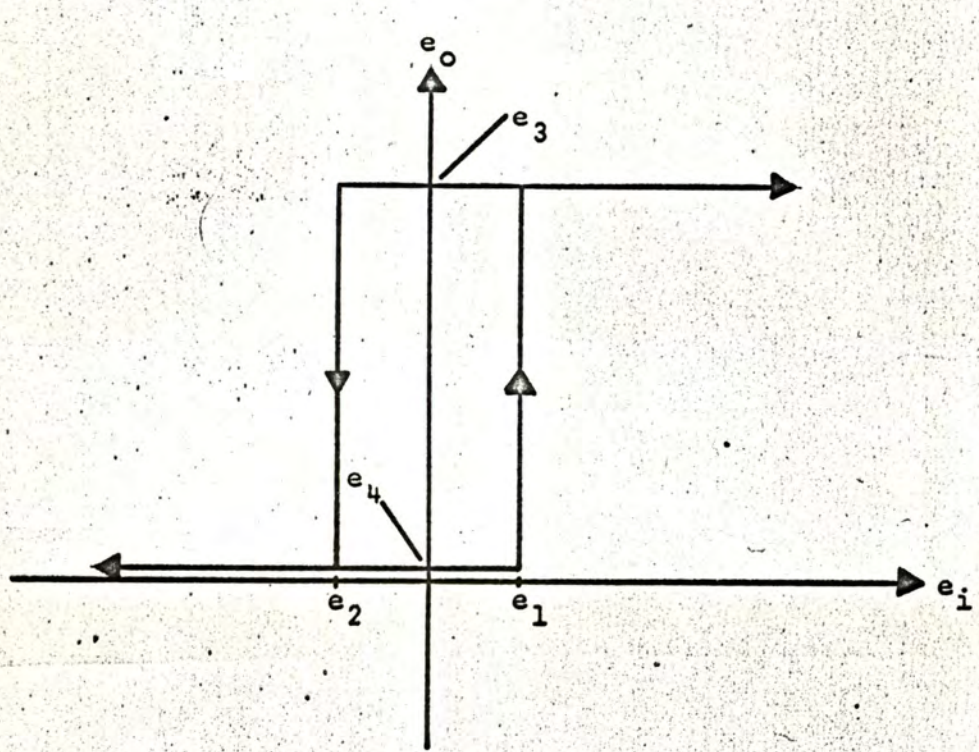


Fig. 4 Transfer Characteristic

If the voltage drop across R_I (due to the base current of Q_1) is small compared to the hysteresis width, e_1-e_2, then R_I can be neglected. Generally, R_I can be eliminated if:

$$R_I \ll \frac{(\beta_1+1)R_E R_L}{R_E + R_L}$$

If R_I is negligible, then:

$$e_1 = \frac{V_1\{(\beta_1+1)R_E R_S[(\beta_2-1)R_L-R_F] + (\beta_2+1)R_E\beta_1 R_L R_S\}}{D} + \frac{V_2[\beta_1\beta_2 R_L^2 R_S + (\beta_1+1)R_E\beta_2 R_L(R_F+R_L)] + V_{BE}\beta_1\beta_2 R_L^2 R_S}{D}$$

where D is the same as before.

$$e_2 = V_2 + \frac{(V_1-V_2)(\beta_1+1)R_E R_S + V_{BE}\beta_1 R_L R_S}{(\beta_1+1)R_E(R_L+R_F+R_S) + \beta_1 R_L R_S}$$

e_3 and e_4 are unchanged.

If $R_S \gg R_F$, then the expressions can be simplified in another way. If R_I is <u>not</u> neglected, the expressions for e_1 and e_2 become:

$$e_1 = \frac{V_1\{[(\beta_1+1)R_E+R_I][(\beta_2-1)R_L-R_F] + \beta_1 R_L(\beta_2+1)R_E\}}{D'} + \frac{(V_2+V_{BE})\{\beta_1\beta_2 R_L^2 - R_I[(\beta_2-1)R_L-R_F]\}}{D'}$$

where

$$D' = (\beta_1+1)R_E[(\beta_2-1)R_L-R_F] + \beta_1\beta_2 R_L^2 + \beta_1 R_L(\beta_2+1)R_E$$

$$e_2 = \frac{V_1[(\beta_1+1)R_E+R_I] + (V_2+V_{BE})[\beta_1 R_L-R_I]}{(\beta_1+1)R_E + \beta_1 R_L} .$$

e_3 and e_4 are, again, unchanged.

The expressions, above, are correct if the equilibrium states of Q_2 are the cut off state and the saturated state.

The temperature dependence of this threshold detector is introduced through $\beta_1(T)$, $\beta_2(T)$, and $V_{BE}(T)$. β_1 and β_2 are the current gains of the transistors Q_1 and Q_2 respectively. V_{BE} is the base to emitter voltage of the transistors when they are active or saturated. V_{BE} has been approximated as a constant; it is in reality a logarithmic function of the emitter current.(3) For a fixed emitter current,

$$\frac{dV_{BE}}{dT} = -\frac{0.65}{T} \text{ mv./°C for T in °K.}$$

For T = 300 °K,

$$\frac{dV_{BE}}{dT} = -2 \text{ mv./°C.}$$

Another problem associated with the Schmitt trigger is the input impedance. the input impedance is not constant; it depends on the state of the circuit. If Q_1 is off, the input impedance is very large; if Q_1 is saturated, the input impedance is much lower. If the impedance of the source which is driving the Schmitt trigger is not sufficiently low, then the varying input impedance may cause a d.c. level shift at the input. This, of course, results in a shift of the thresholds.

1.32 Flip-Flops

Another circuit which can be used as a threshold detector is the flip-flop. Fig. 5 illustrates a typical circuit. Using the notation of fig. 4, the transfer characteristic of the flip-flop is:

$$e_1 = V_2 + V_{BE} + (V_1 - V_2 - V_{BE}) \frac{R_E[(2\beta_1\beta_2+\beta_2-1)R_L-(\beta_1+1)R_F] + R_I[(\beta_2-1)R_L-R_F]}{R_E[(2\beta_1\beta_2+\beta_2-1)R_L-(\beta_1-1)R_F] + \beta_1\beta_2R_L^2}$$

$$e_2 = V_2 + V_{BE} + (V_1 - V_2 - V_{BE}) \frac{R_E[(\beta_1+1)(R_L+R_F)] - R_I[(\beta_1-1)R_L-R_F]}{R_E[(\beta_1+1)(R_L+R_F)] + \beta_1R_L(R_L+R_F)}$$

$$e_3 = V_1$$

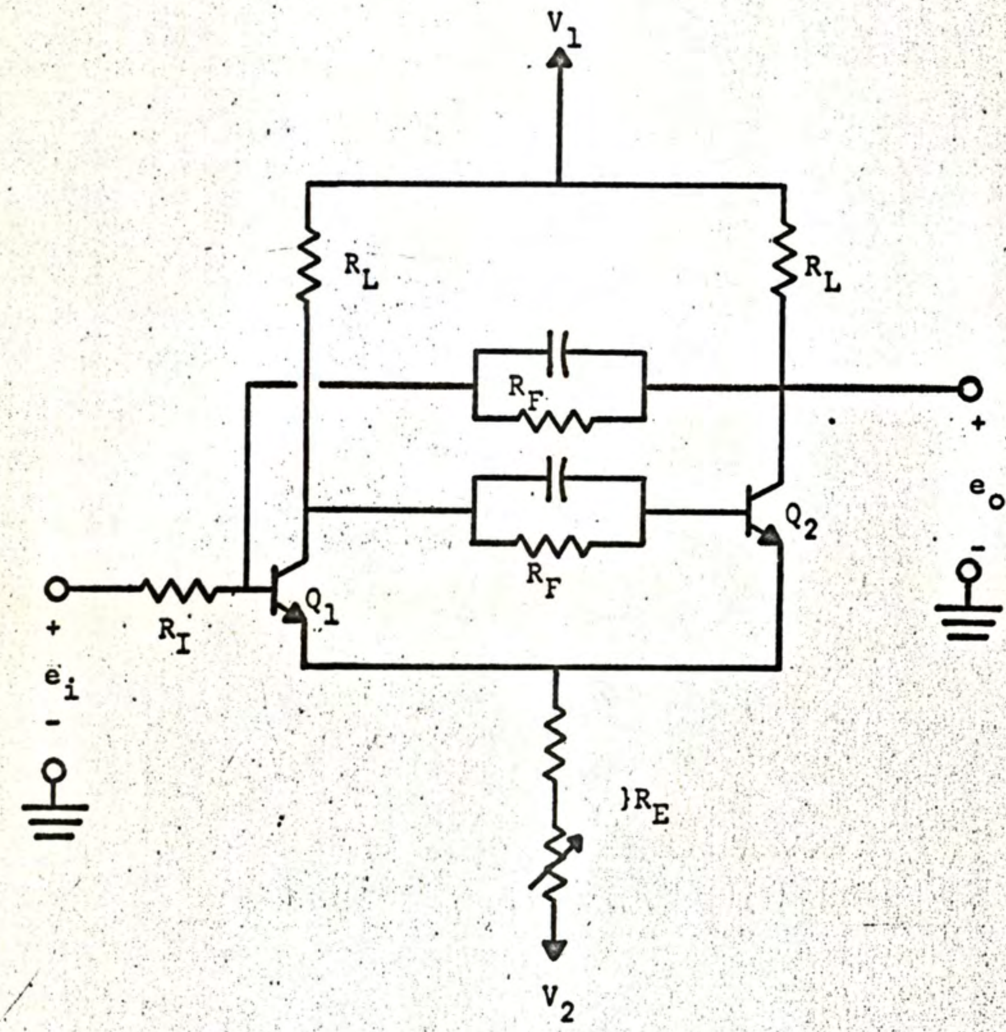


Fig. 5 A Typical Flip-Flop

$$e_4 \approx \frac{V_1R_ER_I + V_2R_LR_I + \beta_2e_iR_ER_L}{R_ER_I + R_LR_I + R_ER_L} \qquad e_i > e_4$$

$$e_4 \approx \frac{V_1R_E + V_2R_L}{R_E + R_L} \qquad e_i < e_4$$

The expressions, above, are correct providing that the circuit is adjusted so that the equilibrium states of the transistor, Q_2, are the cut off state and the saturated state.

In fig. 5, a resistor running from the base of Q_2 to the lower supply, V_2, is necessary if germanium transistors are used. The resistor eliminates the problems caused by the leakage current flowing from the collector to the base of Q_2. If silicon transistors are used, the resistor is not needed.

The temperature dependence of this threshold detector is, again, introduced through β_1, β_2, and V_{BE}.

It is interesting to note at this point that R_E can have a temperature stabilizing effect on the threshold voltages. To illustrate this fact, let

(1) $\beta_1 = \beta_2 = \beta >> 1 \rightarrow \beta+1 \approx \beta$

(2) $R_E[(\beta+1)(R_L+R_F)] >> R_I[(\beta-1)R_L-R_F]$

(3) $R_E = R_L$

Then;

$$e_2 \approx \frac{1}{2}[V_1 + V_2 + V_{BE}(T)] \ .$$

Since $V_{BE}(T)$ has the approximate temperature dependence of a diode, the effect of $V_{BE}(T)$ can be minimized by compensating V_1 and V_2 with diodes.

Unfortunately, e_1 is dependent on temperature in spite of R_E. Nevertheless, R_E does have a stabilizing effect.

Often the flip-flop circuit is used without an emitter resistor ($R_E = 0$). Such a circuit is useful because the emitters can be grounded so that the base potentials are only slightly different from ground potential. With $R_E = 0$ and $V_2 = 0$, the transfer characteristic of the flip-flop becomes, using the notation of fig. 4:

$$e_1 = V_{BE} + (V_1 - V_{BE}) \frac{R_I[(\beta_1 - 1)R_L - R_F]}{\beta_1 \beta_2 R_L^2}$$

$$e_2 = V_{BE} - (V_1 - V_{BE}) \frac{R_I[(\beta_1 - 1)R_L - R_F]}{\beta_1 R_L (R_L + R_F)}$$

$$e_3 = \frac{V_1 R_F + V_{BE} R_L}{R_L + R_F}$$

$$e_4 = V_{CE}(\text{sat.})$$

V_{CE}(sat.) is the collector to emitter voltage of a transistor when it is saturated. V_{CE}(sat.) is a function of the collector and base currents.

With $R_E = 0$, the flip-flop is very useful in constructing symmetric complementary circuits. It is an important "building block".

Like the Schmitt trigger, the flip-flop has a switching input impedance. When Q_1 is active or saturated, the input impedance is R_I ($R_E = 0$); when

Q_1 is off, the input impedance is infinite. When $R_E \neq 0$, the fluctuation of the input impedance is reduced.

1.33 Symmetric Threshold Detectors Using Complementary Flip-Flops.

Using complementary flip-flops as building blocks, a number of symmetric threshold detectors may be constructed. A typical circuit is illustrated in fig. 6. The advantage of this circuit is that the d.c. drift of the transfer characteristic will be very small if complementary transistors are used. Temperature fluctuations will not cause a d.c. offset in this device. However, a temperature change may cause the hysteresis width to vary. Another advantage of the circuit is that the input impedance is approximately constant.

Using the notation of fig. 7, the transfer characteristic of the device is given by:

$$e_1 = (V-V_{BE})\frac{R_I}{R_{L2}+R_{F2}}\cdot\frac{\beta_{1P}\beta_{2N}R_{L1}R_{L2}-(R_{L1}+R_{F1})(R_{L2}+R_{F2})}{\beta_{1P}\beta_{2N}R_{L1}R_{L2}}$$

$$- V_{BE}[1 + \frac{2R_I}{\beta_{1P}R_{L1}}\cdot\frac{(\beta_{1P}\beta_{2N}+1)R_{L1} + \beta_{2N}R_{F2} - R_{F1}}{\beta_{2N}R_{F2}}]$$

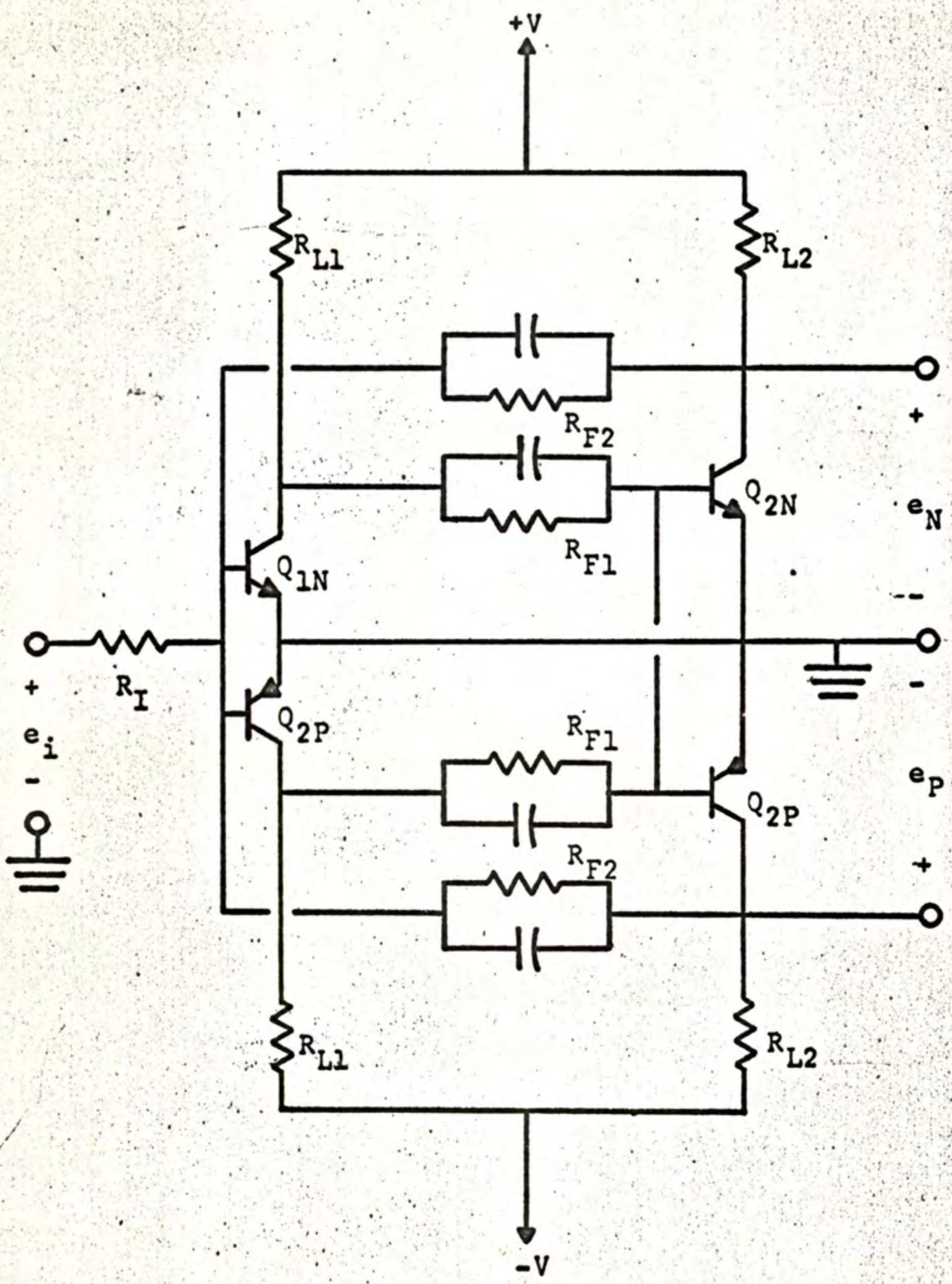


Fig. 6 A Symmetric Threshold Detector

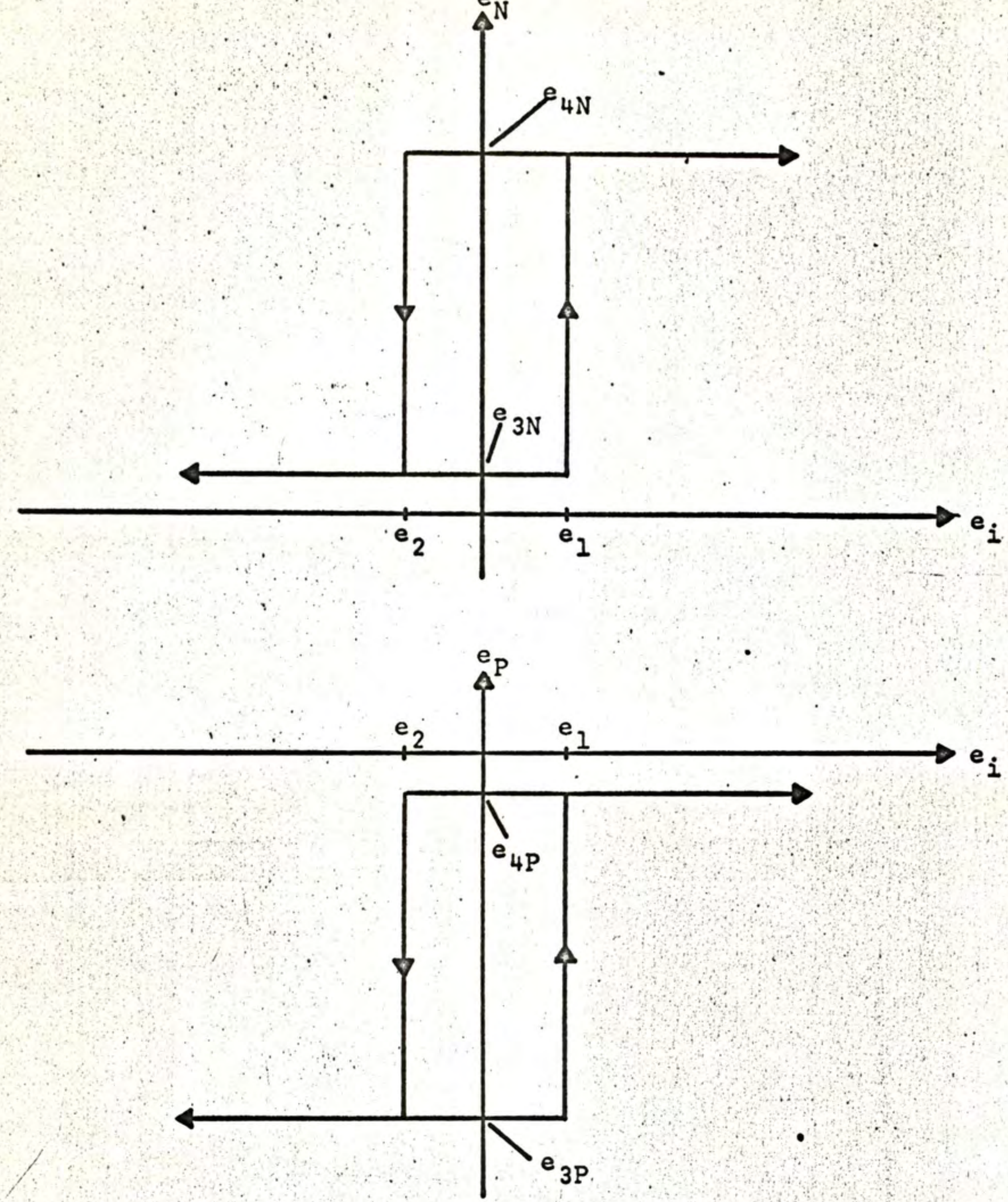


Fig. 7 Transfer Characteristic

$$e_2 = -(V-V_{BE})\frac{R_I}{R_{L2}+R_{F2}}\cdot\frac{\beta_{1N}\beta_{2P}R_{L1}R_{L2}-(R_{L1}+R_{F1})(R_{L2}+R_{F2})}{\beta_{1N}\beta_{2P}R_{L1}R_{L2}}$$

$$+ V_{BE}[1 + \frac{2R_I}{\beta_{1N}R_{L1}}\cdot\frac{(\beta_{1N}\beta_{2P}+1)R_{L1} + \beta_{2P}R_{F2} - R_{F1}}{\beta_{2P}R_{F2}}]$$

$$e_{3N} \equiv V_{CE}(sat.)$$

$$e_{4N} = \frac{VR_{F2} - V_{BE}R_{L2}}{R_{F2} + R_{L2}}$$

$$e_{3P} = -\left(\frac{VR_{F2} - V_{BE}R_{L2}}{R_{F2} + R_{L2}}\right)$$

$$e_{4P} = - V_{CE}(sat.)$$

The expressions, above, hold if the output transistors are either cut off or saturated when the circuit is in equilibrium.

Switching occurs as follows in the circuit of fig. 6. Let e_i be positive and decreasing. Initially, Q_{1N} and Q_{2P} are saturated; Q_{2N} and Q_{1P} are off. As e_i decreases:

(1) Q_{1N} becomes active.

(2) The magnitude of the collector voltage of Q_{1N} increases.

(3) Because the magnitude of the collector voltage of Q_{1N} is rising, the magnitude of the base current of Q_{2P} decreases.

(4) Q_{2P} comes out of saturation and becomes active.

(5) Regenerative switching occurs between Q_{1N} and Q_{2P}.

(6) Q_{1N} and Q_{2P} turn off.

(7) The by-pass capacitors cause Q_{1P} and Q_{2N} to become active.

(8) Regenerative switching occurs between Q_{1P} and Q_{2N}.

(9) Q_{1P} and Q_{2N} saturate. The switching is complete.

The symmetric device has three stable equilibrium states. Two of the states, the desired states, have two transistors saturated and two off. The third state has all the transistors off. This "off" state is undesirable. A relatively large swing in the input voltage may be required before the circuit "escapes" from this state. Fortunately, by-pass capacitors can be used to prevent the "off" state from occurring. When two transistors turn off, the capacitors bring the other two transistors into their active regions, and they eventually saturate.

In practice, the "off" state occurs when the circuit is energized by raising the supply voltages symmetrically. Once the circuit "escapes" from the state, it will not recur if the capacitors are large enough so that the two transistors which are initially off become active before the other transistors turn off. This is verified in practice; when the capacitances are increased beyond a certain critical value, the "off" state does not recur.

1.34 The Tunnel Diode Pair

Another basic threshold detecting circuit, built around the tunnel diode pair is illustrated in fig. 8.[4] The tunnel diodes are used to obtain an i-v characteristic with a negative resistance region which is symmetric about the origin. The analysis of this circuit is done graphically; it may be found in the reference.

The hysteresis of the tunnel diode pair can be made as small as 1 mv. However, the hysteresis varies due to temperature fluctuations. The circuit may oscillate; or, it may switch to an extraneous stable point near the origin. Furthermore, the source impedance is very critical to the operation of the device. The hysteresis is directly dependent on this impedance.

In order to avoid confusion later, it is necessary to explain why this device is not stable for fluctuating temperature. The non-linear i-v characteristic created by the tunnel diodes depends not only on the stable peak regions of the tunnel diodes, but also on the unstable valley regions. The i-v characteristic fluctuates with temperature changes because of its dependence on these valley regions.

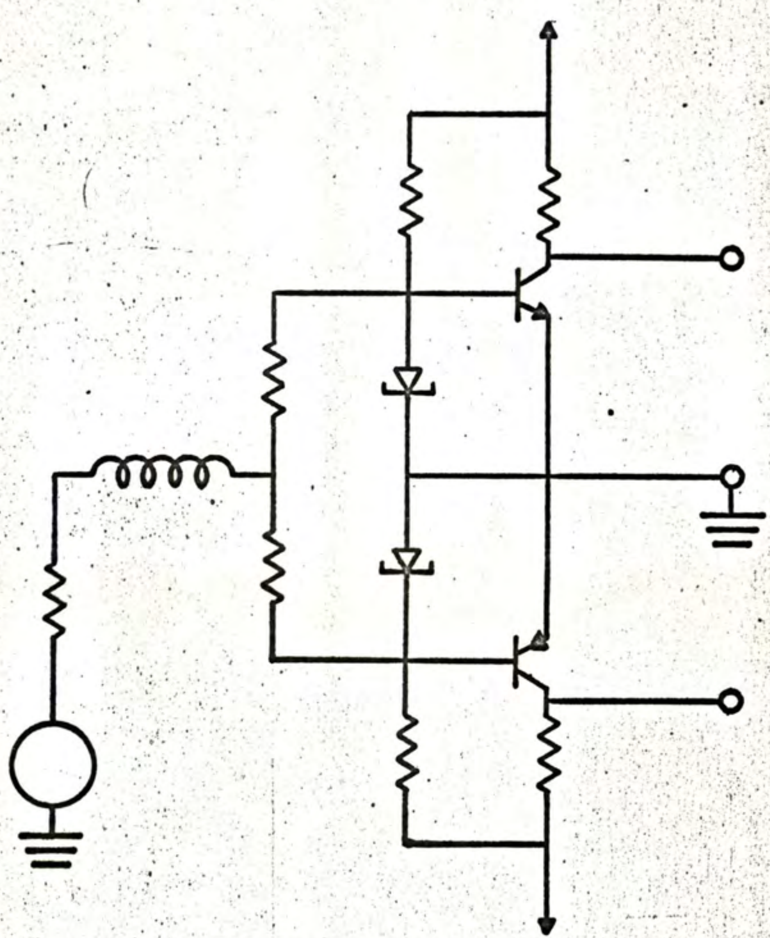


Fig. 8 The Tunnel Diode Pair

1.4 General Techniques for Reducing or Adjusting the Hysteresis

1.41 Preamplification

Preamplification is an obvious technique for hysteresis reduction. However, the design of a simple d.c. preamplifier is not so obvious. One possible circuit is the differentially driven flip-flop illustrated in fig.9.

The differentially driven flip-flop is a simple circuit. It is also inexpensive because complementary transistors are not required. However, if matched transistors in one package are used, the cost increases.

In order to analyze this circuit, the Thevenin equivalent of the differential amplifier is used. (Cf. fig. 10). The collector voltages of the transistors in the differential amplifier are modeled as a quiescent voltage, E_S, and an a.c. voltage $+e_s$ or $-e_s$. The drive resistance to the base of Q_1 and Q_2 is R_S. If the differential amplifier has some voltage gain, A, then $e_s = Ae_i$.

The circuit of figs. 9 & 10 has two distinct modes of operation. Assume that initially Q_1 is cut off and Q_2 is saturated. Switching may occur in two ways: If Q_1 becomes active before Q_2, then switching will be initiated when Q_2 becomes active. If Q_2 becomes

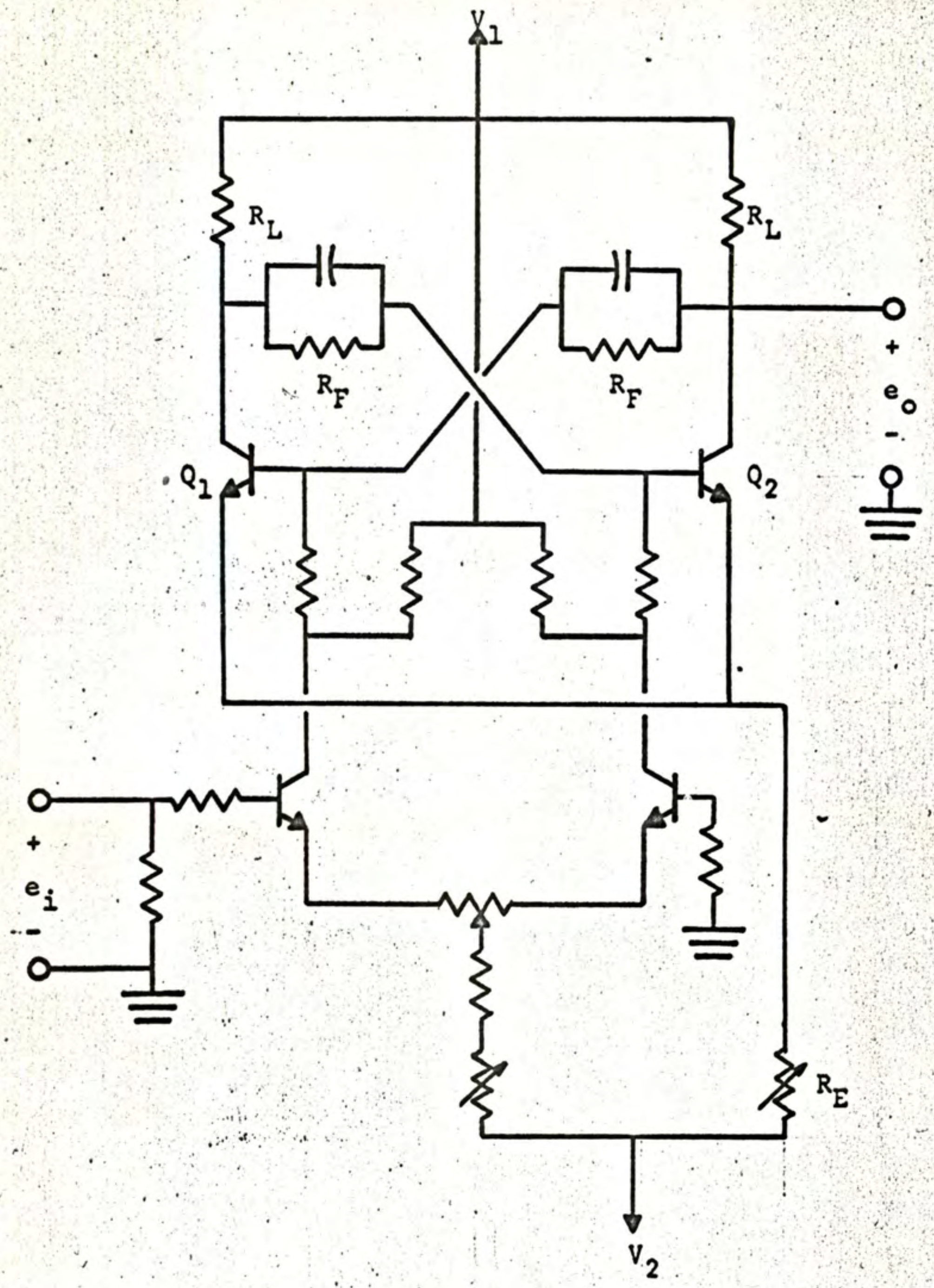


Fig. 9 A Differentially Driven Flip-Flop

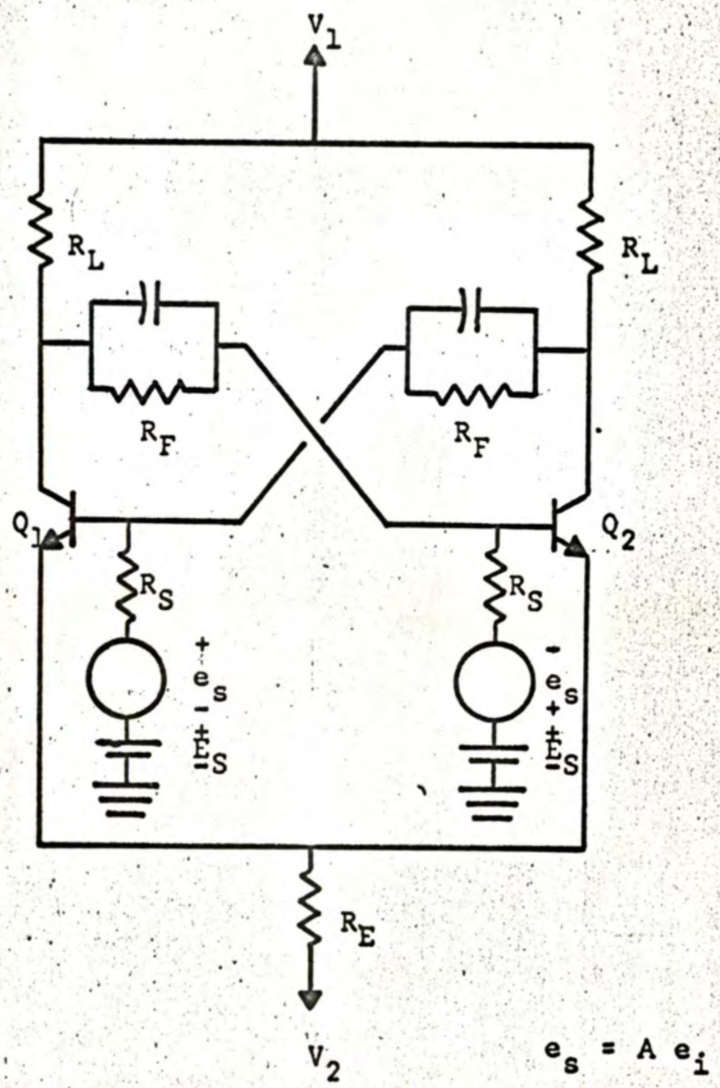


Fig. 10 Equivalent Circuit

active before Q_1, then switching will begin when Q_1 becomes active. In the first mode, the previously saturated transistor initiates switching. In the second mode, the previously cut off transistor initiates switching. The second mode is preferable because each switching threshold depends on the current gain of only one of the transistors.

Assuming that switching is initiated when the cut off transistor becomes active, the transfer characteristic is given by (Cf. fig. 11):

$$e_1 = \frac{[(V_1-V_2-V_{BE})R_S + (E_S-V_{BE}-V_2)(R_L+R_F)][(\beta_1-1)R_L-R_F]R_S}{A\{[2(\beta_1+1)R_E+(\beta_1+1)R_L+R_F]R_S+2(\beta_1+1)R_E(R_L+R_F)\}(R_L+R_F)}$$

$$e_2 = -\frac{[(V_1-V_2-V_{BE})R_S + (E_S-V_{BE}-V_2)(R_L+R_F)][(\beta_2-1)R_L-R_F]R_S}{A\{[2(\beta_2+1)R_E+(\beta_2+1)R_L+R_F]R_S+2(\beta_2+1)R_E(R_L+R_F)\}(R_L+R_F)}$$

$$e_3 = \frac{V_1R_E + V_2R_L}{R_E + R_L}$$

$$e_4 = V_1$$

The transfer characteristic depends on the current gains of the transistors. Thus, temperature instability is introduced through the current gains.

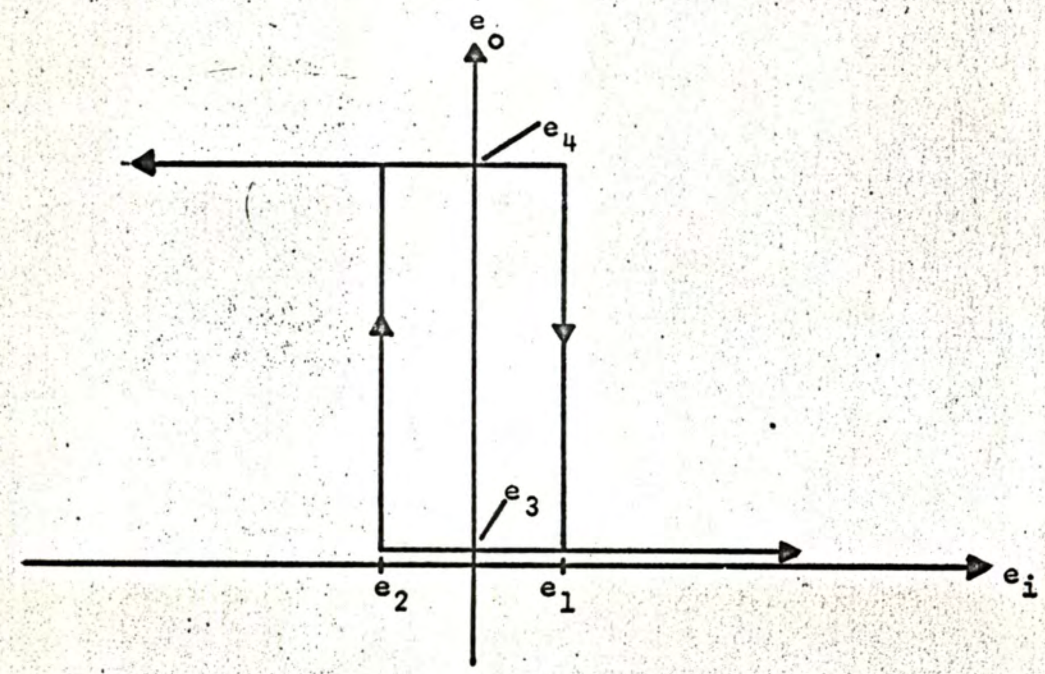


Fig. 11 Transfer Characteristic

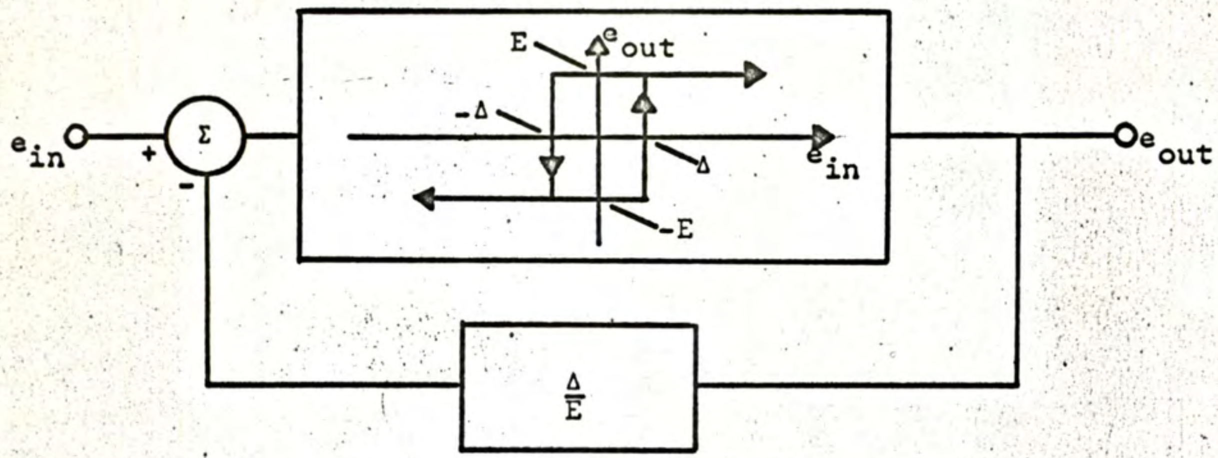
However, the gain of the differential amplifier decreases the hysteresis width and fluctuations by the same amount. So, the differential amplifier does not magnify the fluctuations relative to the hysteresis width. If the hysteresis is stable without the differential amplifier, it will be stable with the differential amplifier assuming, of course, that the differential amplifier is stable.

It is difficult to obtain a wide bandwidth with the differentially driven flip-flop and simultaneously maintain d.c. stability. In order to realize a better bandwidth, it is necessary to use higher quiescent currents; but, higher currents result in greater d.c. drift.

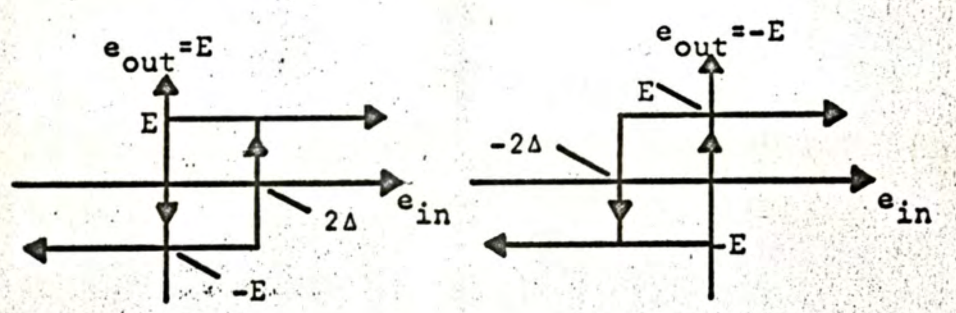
1.42 Feedback Control of the Hysteresis

Negative feedback reduces the hysteresis of a symmetric bistable device; positive feedback increases the hysteresis. Clearly, a threshold detector with fixed, stable hysteresis can have a variable effective hysteresis when feedback is used.

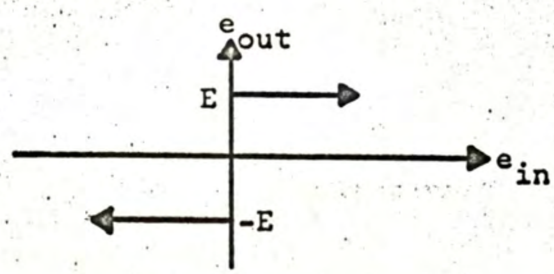
With sufficient negative feedback, a symmetric threshold detector can become a zero crossing detector. The effect of the feedback in this case is pictured in fig. 12. The feedback voltage shifts the fixed transfer characteristic along the input voltage axis. In the case of zero hysteresis, the threshold voltage required to cause the device to switch from the existing state to



Block Diagram of Zero Crossing Detector



Shifted Transfer Characteristics



Effective Transfer Characteristic

Fig. 12 Creating a Zero Crossing Detector with Negative Feedback the other state is always shifted so that it is positioned at zero input voltage.

The stability of a zero crossing detector, operating with negative feedback, depends, critically, on the stability of the hysteresis which exists initially without feedback. Negative feedback reduces the effective hysteresis; but, feedback has no effect on the fluctuations of the hysteresis. Thus, a small percent fluctuation of the hysteresis existing without feedback can cause drastic percent changes in the hysteresis of the zero crossing detector.

The problem of realizing a zero crossing detector by the feedback method really involves the design of a stable threshold detector. If a stable threshold detector can be constructed, then, when negative feedback is added, the smaller effective hysteresis will be stable. Some devices utilizing feedback to control their hysteresis will be discussed in chapter 2.

CHAPTER II

Feedback Control of Hysteresis

2.1 Introduction

Feedback control of hysteresis has been introduced in chapter 1. A number of devices working on the feedback principle have been constructed in the laboratory. Two devices will be considered here: a complementary latching flip-flop circuit, and a complementary latching tunnel diode circuit.

2.2 A Complementary Latching Flip-Flop Circuit

The schematic diagram of a practical complementary latching flip-flop circuit is shown in fig. 13. This device is designed to have a stable hysteresis which is adjustable from .1 volt to 5 volts. Although the circuit is not intended for use as a zero crossing detector, the hysteresis can be as small as 1 mv. However, the potentiometer settings become very critical at this level because this particular circuit is designed for higher levels. Operation of this or similar circuits at mv. levels is not recommended unless the ambient temperature is fairly constant because the hysteresis depends, partially, on the V_{BE}'s and β's of the transistors.

The emitter follower driver transistors are necessary in order to obtain enough drive for the output transistors. The device is capable of switching up to 1 amp. at 200 kc/s. or lower.

The

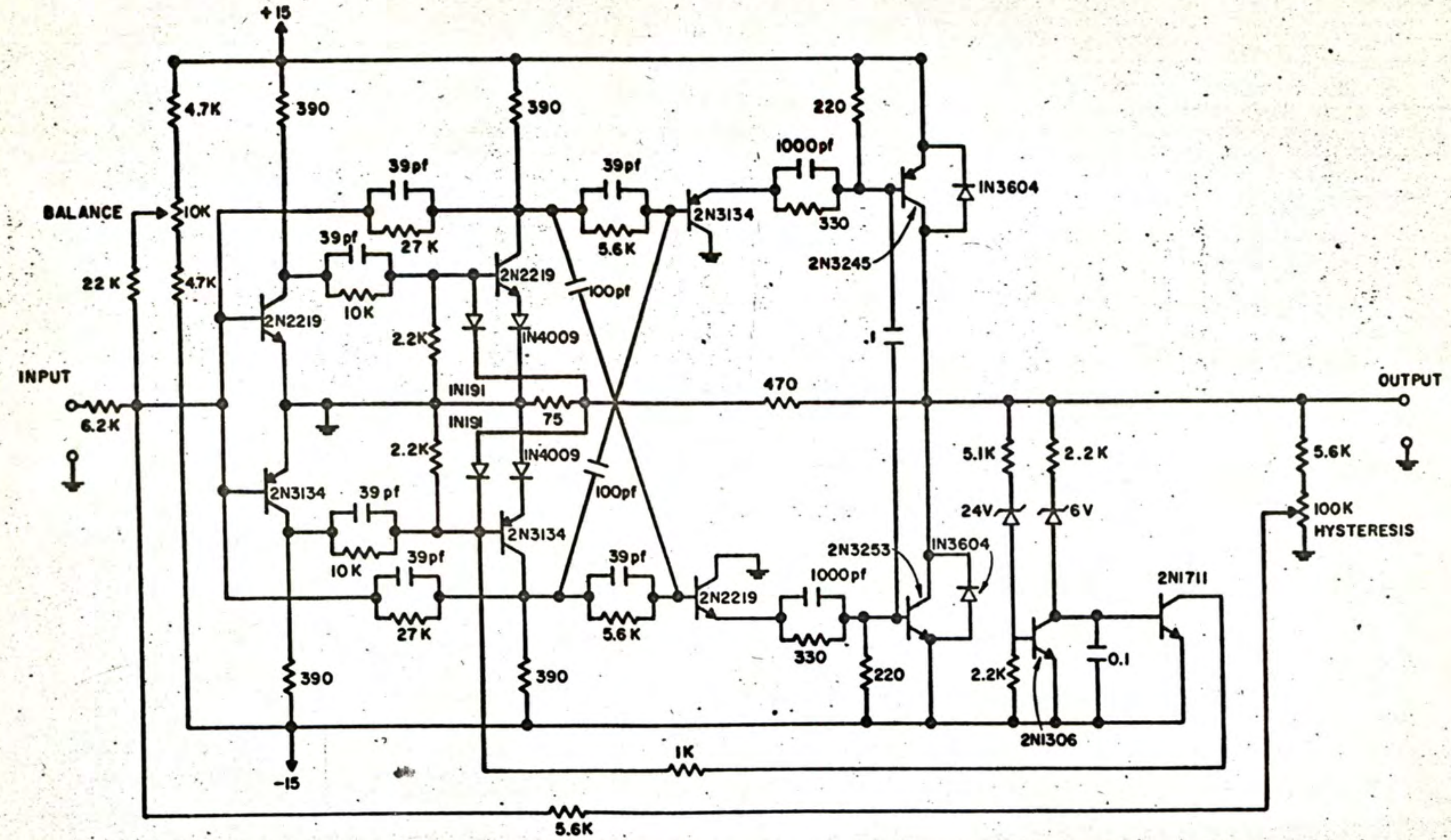


Fig. 13 An Experimental Complementary Latching Flip-Flop Circuit

The operation of the complementary latching flip-flop is indicated in the block diagram of fig. 14.

The output circuit has only one input on at a time; the other input is off. The output state determines which input is on. For example, when the output is positive, I_1 is on and I_2 is off. When the output is negative, I_2 is on and I_1 is off.

The output circuit serves two purposes. It selects two of the four thresholds present in the input circuits. This results in improved stability and reliability because otherwise the flip-flops would rely depend on a.c. coupling to achieve, in effect, two states. Furthermore, switching can be initiated at the output only by turning off the transistor which is initially on. Thus, the output transistors cannot be on at the same time.

The starting circuit is necessary to eliminate a state in which all the transistors are off. Theoretically, the starting circuit is only necessary when power is initially applied to the circuit. The starting circuit pulls the main circuit out of the "off" mode. Thereafter, the a.c. coupling in the output circuit should prevent any recurrence of the "off" mode. However, in practice, the a.c. coupling may fail occasionally causing the main circuit to switch to the "off" mode. The starting circuit is designed to detect when such a failure occurs and to automatically re-start the circuit.

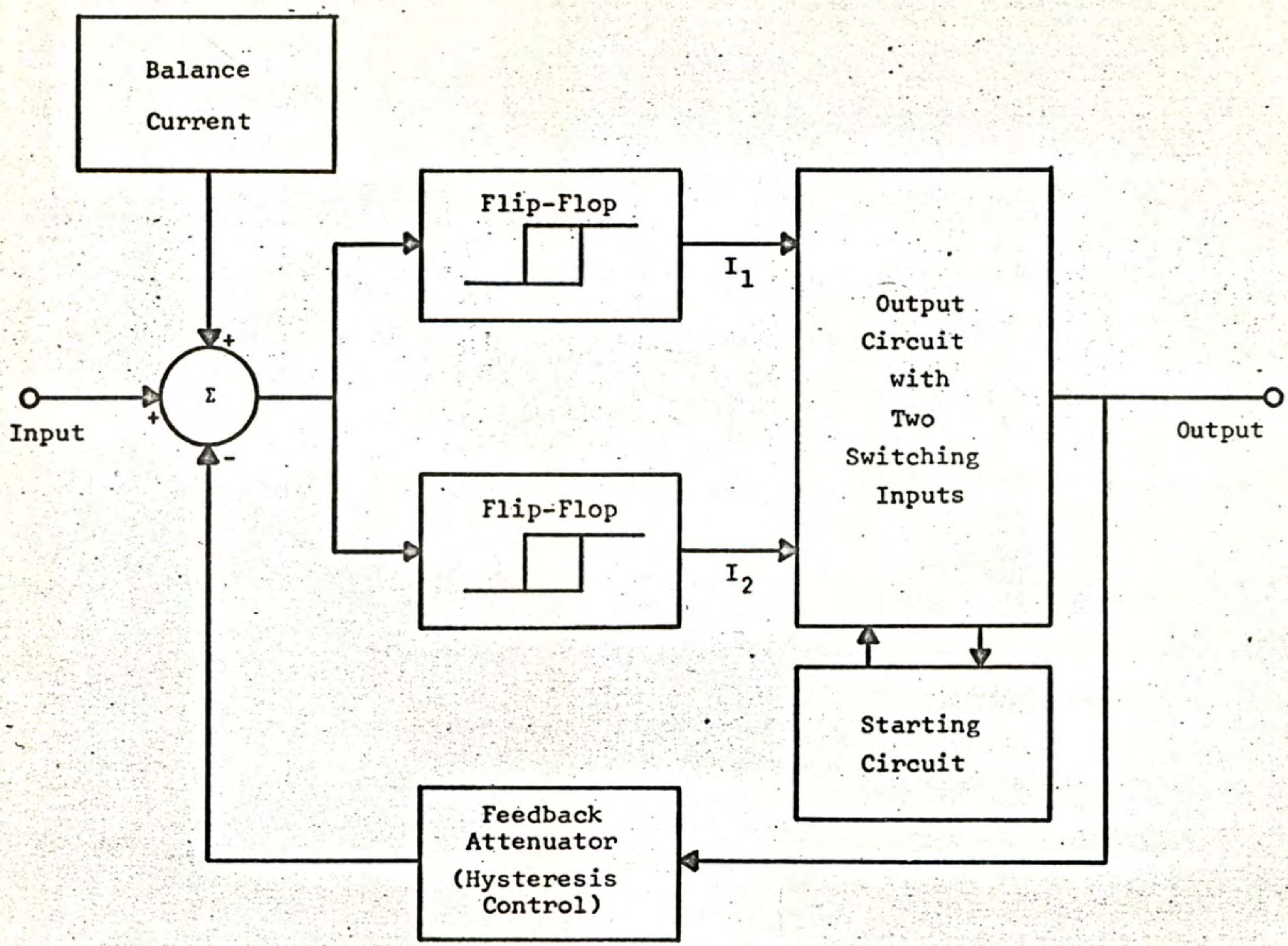


Fig. 14 Block Diagram of the Complementary Latching Flip-Flop Circuit

Referring to fig. 15, the switching of the main circuit normally occurs in the following sequence. Assume that the input signal is large, positive, and decreasing. Initially Q_1, Q_4, and Q_8 are saturated. Q_2, Q_3, Q_5, Q_7, and D_1 are off. Q_6 and D_2 are on. As the input signal decreases:

(1) Q_1 becomes active and eventually turns off. This does not cause the other transistors to switch because Q_3 is held off by the feedback from the output.

(2) Q_2 becomes active.

(3) The magnitude of the collector voltage of Q_2 decreases.

(4) Because the magnitude of the voltage at the collector of Q_2 is decreasing, the magnitude of the base current of Q_4 decreases.

(5) Q_4 becomes active.

(6) Regenerative switching occurs between Q_2 and Q_4.

(7) Q_2 turns on; Q_4 turns off.

(8) Q_6 turns off. Q_5 is pulled on by the a.c. coupling from the collector of Q_4.

(9) Q_8 turns off, and Q_7 turns on. The output voltages swit switches. At this point Q_5 and Q_7 are held on by the capacitors alone.

(10) The feedback from the output turns D_2 off. D_1 turns on so that Q_4 is held off.

(11) Immediately after D_2 turns off, Q_3 becomes active and then quickly saturates because of the current coming from Q_1 through R_D. (Q_1 is still off.)

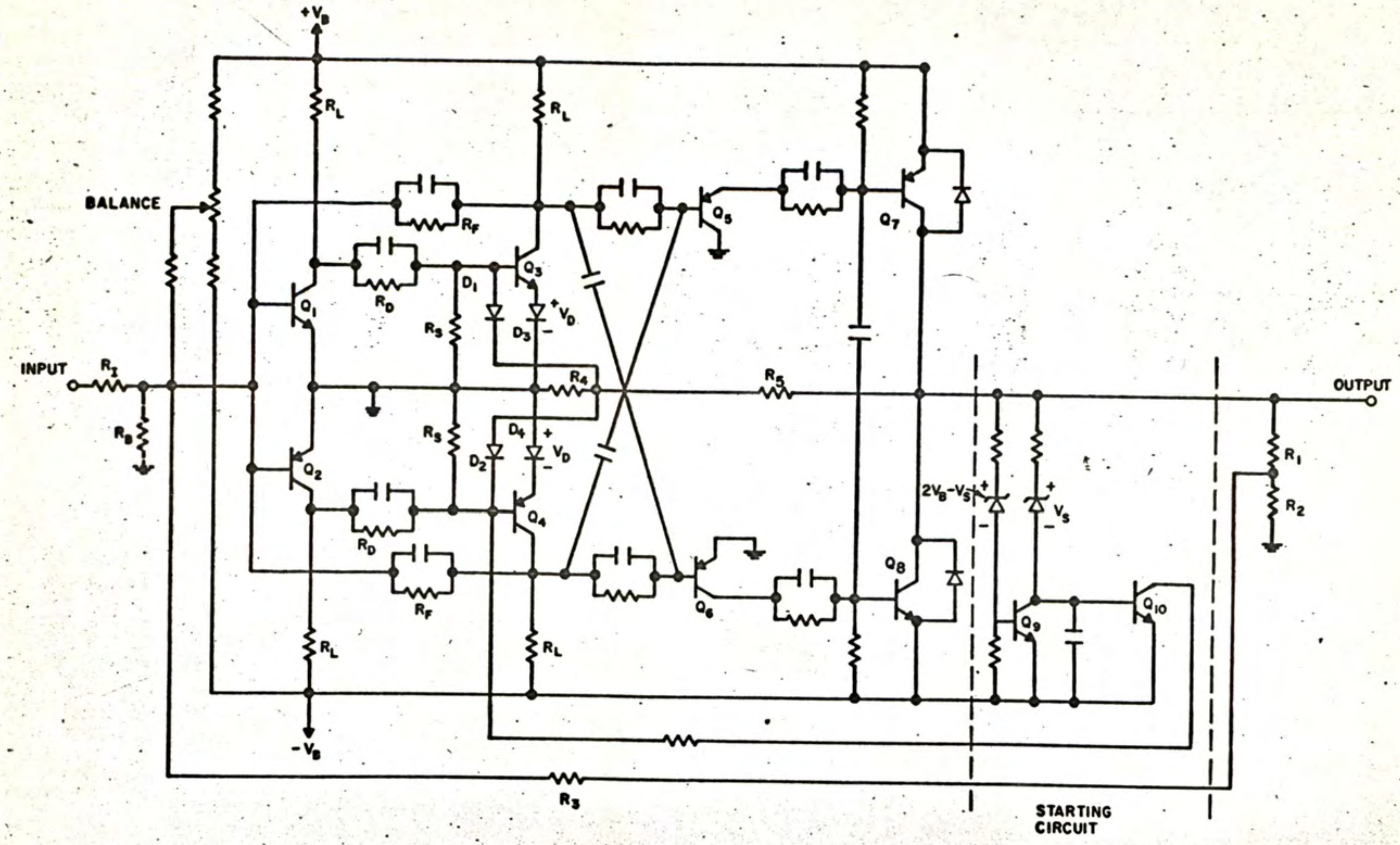


Fig. 15 A Complementary Latching Flip-Flop Circuit

(12) Q_3 holds Q_5 on.

(13) Q_5 holds Q_7 in saturation. The switching is complete. Switching in the other direction follows by symmetry.

The starting circuit affects the main circuit only when the main circuit is in the undesirable "off" mode; i.e., when all the transistors in the output circuit are off simultaneously. When the main circuit is in the off mode and when the input signals are small, Q_1 through Q_8 are off, and D_1 and D_2 are on. (Cf. fig. 15 for notation.) Starting occurs in the following manner:

(1) Q_{10} becomes active and eventually saturates. ($V_S < V_B$)

(2) Q_4 is pulled on by Q_{10}.

(3) Q_4 turns Q_6 on.

(4) Q_5 drives Q_7 on and into saturation so that the output switches to the positive state. The output is now in an allowed state.

(5) Q_{10} becomes active and then turns off. The starting circuit no longer affects the main circuit.

If the input signal is large positive or negative, then the main circuit may start without the starting circuit depending on the size of R_D and R_S relative to the feedback resistors R_4 and R_5. If the main circuit does not start automatically, then the starting circuit will start it although the switching sequence is not the same as it is with small input signals.

When the output is in the positive state, Q_9 is on. Q_9 shunts the base of Q_{10}; so that Q_{10} is off. When the output is in the negative state, Q_9 and Q_{10} are both off. Thus, the starting circuit does not affect the main circuit when the output is in either the positive or the negative state.

The diodes, D_3 and D_4, are used to lift Q_3 and Q_4 slightly above ground potential. These diodes inconjunction with D_1 and D_2 give short circuit protection at the output when the resistors are properly chosen. If the output is shorted to ground, then Q_3 and Q_4 are both held off by the diodes, D_1 and D_2. The diodes at the output protect the circuit from inductive loads.

The static hysteresis of the complementary latching flip-flop is calculated with the aid of figs. 15 & 16. The resistive balance circuit has been replaced by a resistor, R_B. The switching thresholds occur when Q_3 or Q_4 is at the breakpoint between the off and the active state. If the output is initially in the positive state, switching will occur when:

$$\frac{V_B - e_c}{R_L} - \beta_1 i_{b1} - \frac{e_c - V_{BE} - V_D}{R_D} = 0$$

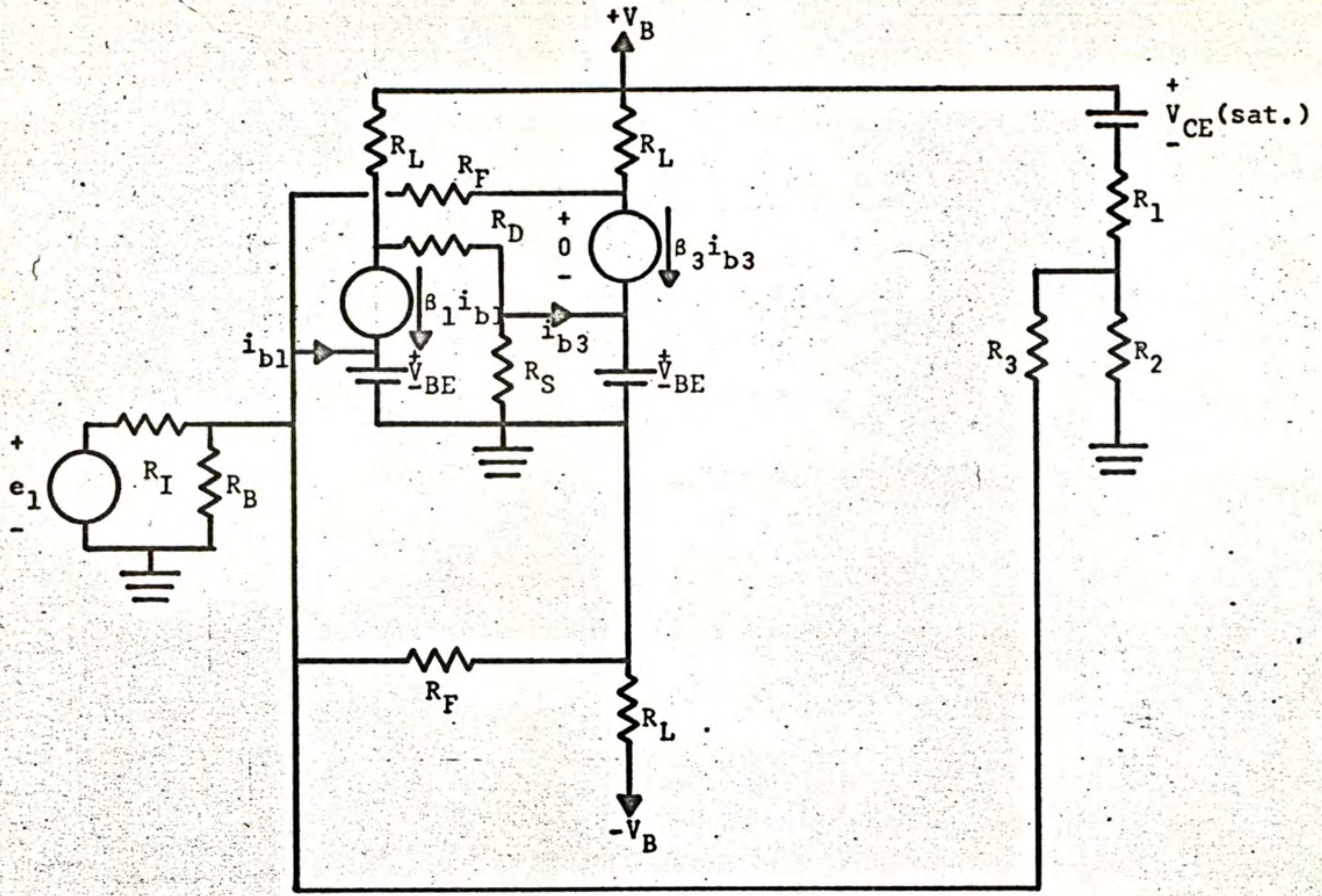


Fig. 16 Equivalent Circuit When Negative Switching Is Initiated

$$i_{b1} = \frac{V_D}{R_F} + \frac{V_{BE} - V_B}{R_F + R_L} - \frac{V_{BE}}{R_B} + \frac{e_1 - V_{BE}}{R_I}$$

$$+ (V_B - V_{CE}(sat.) - V_{BE})\frac{R_2}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$\beta_3 i_{b3} = \frac{V_B - V_{BE} - V_D}{R_L}$$

$$i_{b3} = \frac{e_c - V_{BE} - V_D}{R_D} - \frac{V_{BE} + V_D}{R_S}$$

Solving for e_1:

$$e_1 = V_B R_I \{\frac{[(\beta_3 - 1)R_L - R_D]}{\beta_1 \beta_3 R_L^2} + \frac{1}{R_L + R_F} - \frac{R_2}{R_1R_2 + R_1R_3 + R_2R_3}\}$$

$$-(V_{BE} + V_D)R_I\{\frac{[(\beta_3 - 1)R_L - R_D]}{\beta_1 \beta_3 R_L^2} + \frac{R_L + R_D}{\beta_1 R_L R_S}\}$$

$$- \frac{V_D R_I}{R_F} + V_{BE}\{1 - \frac{R_I}{R_F + R_L} + \frac{R_I}{R_B}\}$$

$$+ (V_{CE}(sat.) - V_{BE})\frac{R_2 R_I}{R_1R_2 + R_1R_3 + R_2R_3}$$

By symmetry,

$$e_2 = -V_B R_I \left\{ \frac{[(\beta_4-1)R_L-R_D]}{\beta_2\beta_4 R_L^2} + \frac{1}{R_L+R_F} - \frac{R_2}{R_1R_2 + R_1R_3 + R_2R_3} \right\}$$

$$+ (V_{BE}+V_D)R_I \left\{ \frac{[(\beta_4-1)R_L-R_D]}{\beta_2\beta_4 R_L^2} + \frac{R_L + R_D}{\beta_2 R_L R_S} \right\}$$

$$+ \frac{V_D R_I}{R_F} - V_{BE}\left\{1 - \frac{R_I}{R_F + R_L} + \frac{R_I}{R_B}\right\}$$

$$- (V_{CE}(\text{sat.}) - V_{BE}) \frac{R_2 R_I}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$e_3 = -e_4 = V_B - V_{CE}(\text{sat.})$$

The transfer characteristic is illustrated in fig. 17. The hysteresis is temperature dependent because V_{BE}, V_{CE}(sat.), V_D, and β_1 through β_4 all depend on temperature. If the unit is operated at a fixed ambient temperature, then the threshold is very stable. In fact, if the temperature is stable, a complementary latching flip-flop circuit can be designed to operate at millivolt levels.

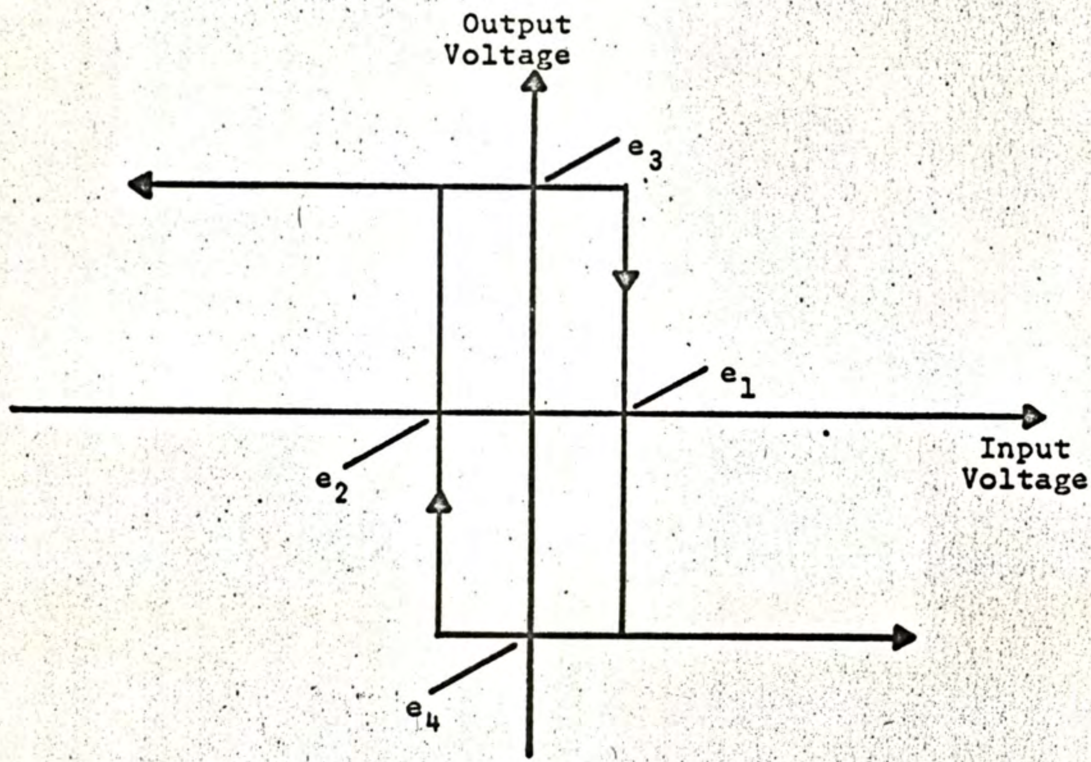


Fig. 17 Transfer Characteristic

2.3 A Tunnel Diode Stabilized Threshold Detector

A practical tunnel diode stabilized complementary latching circuit is shown in fig. 18. The hysteresis width can be adjusted from 5 volts to less than 1 millivolt by varying the negative feedback. The transfer characteristic of the device is quite stable for temperature fluctuations normally encountered in the laboratory. This device is an extremely versatile threshold detector because of the wide range of hysteresis widths. However, it is, primarily, intended for use as a zero crossing detector. Pictures of the transfer characteristic of the tunnel diode circuit are included in fig. 19. The device operates at frequencies below 500 kc/s.

The tunnel diode circuit operates on the same principles as the flip-flop device of section 2.2. However, the tunnel diode device has superior stability and speed.

The output circuit of the device is similar to the one in the flip-flop circuit. But, it serves a much more important purpose in this device. Each of the bistable tunnel diode circuits has one very stable threshold which depends on the peak current of the diode. The other threshold which depends on the valley current is not so stable. Because of the output circuit, only the stable threshold causes

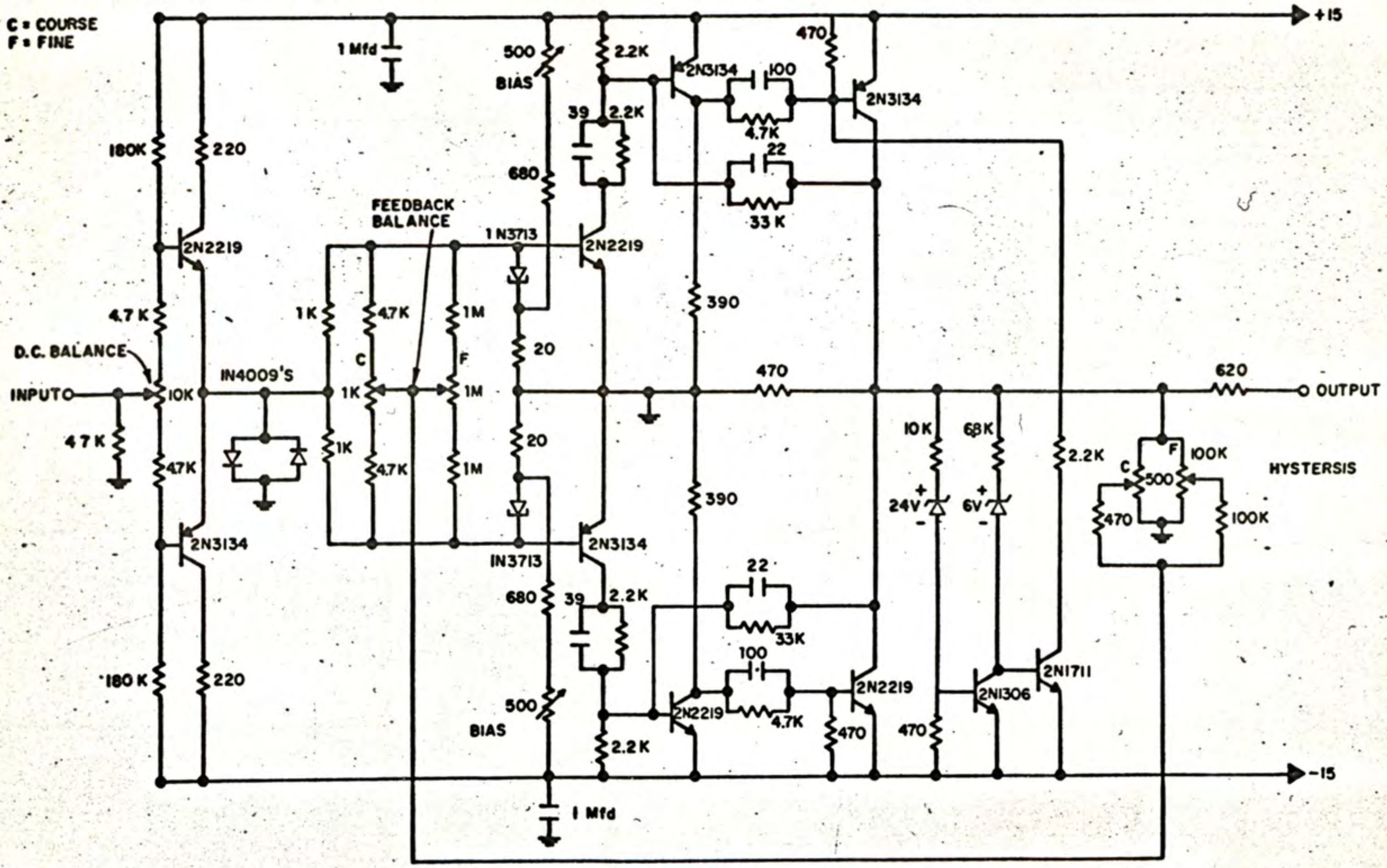
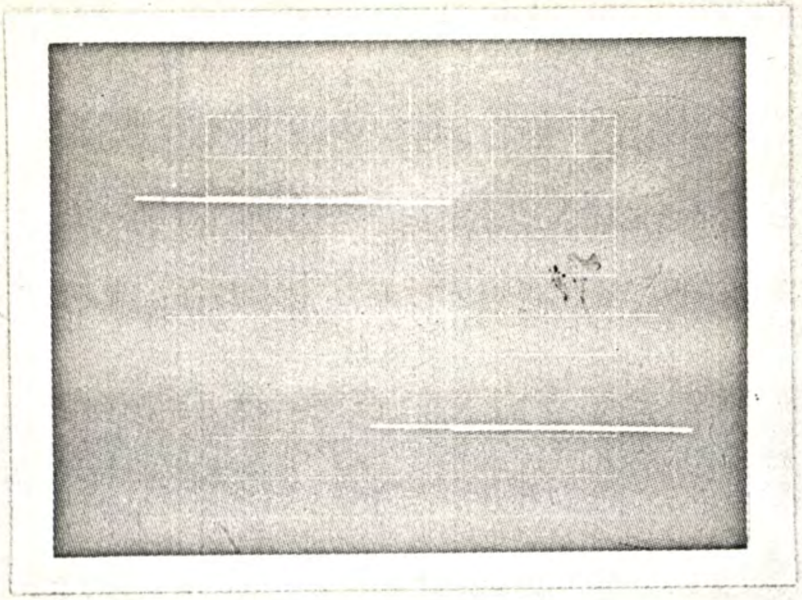
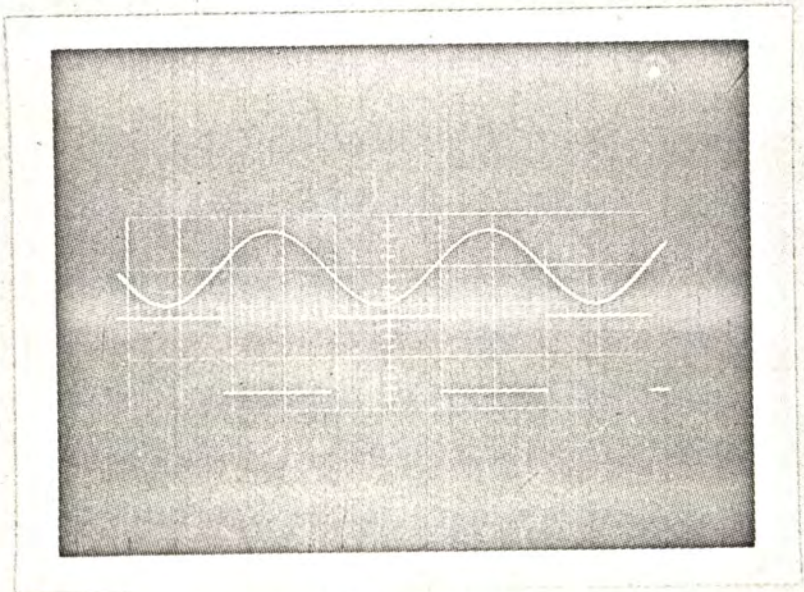


Fig. 18 An Experimental Tunnel Diode Stabilized Threshold Detector

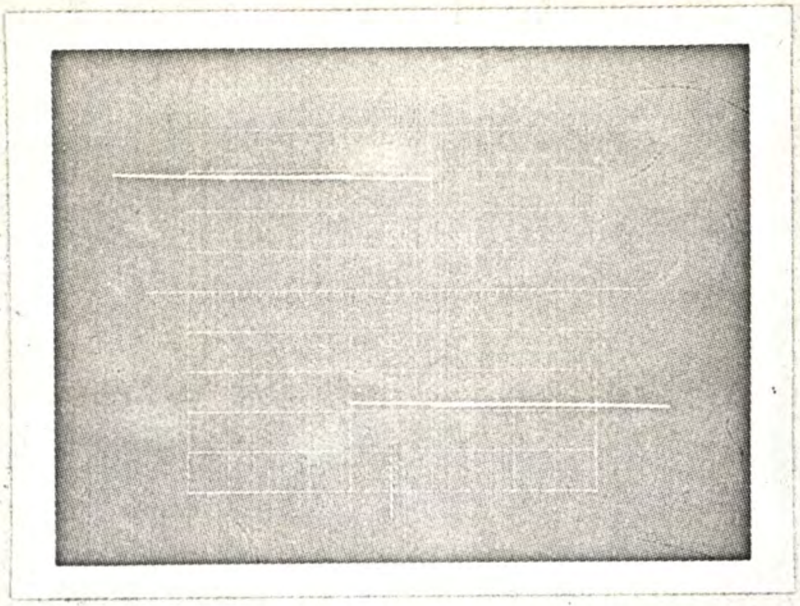


Vertical: Output 5 v./div.
Horizontal: Input .5 v./div.

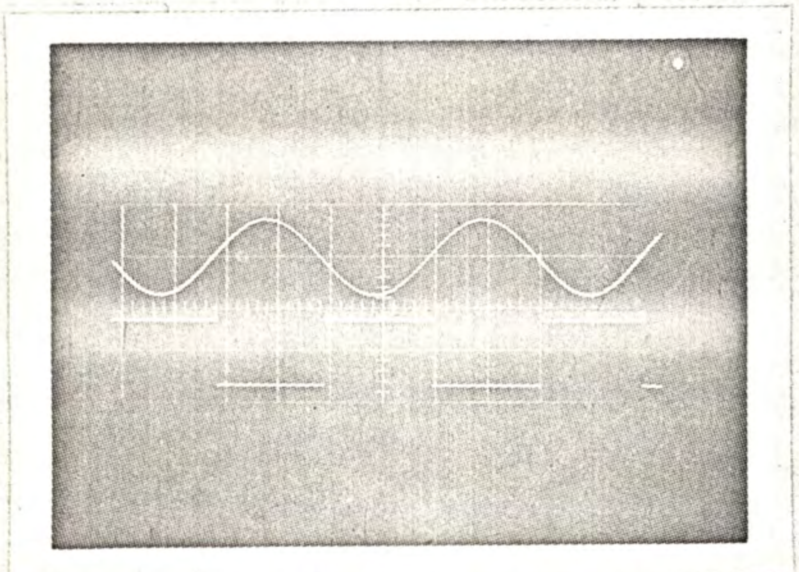


Upper Vertical: Input 5 v./div.
Lower Vertical: Output 10 v./div.
Horizontal: Time .5 ms./div.

Fig. 19(a) Experimental Transfer Characteristic
1 Volt Hysteresis

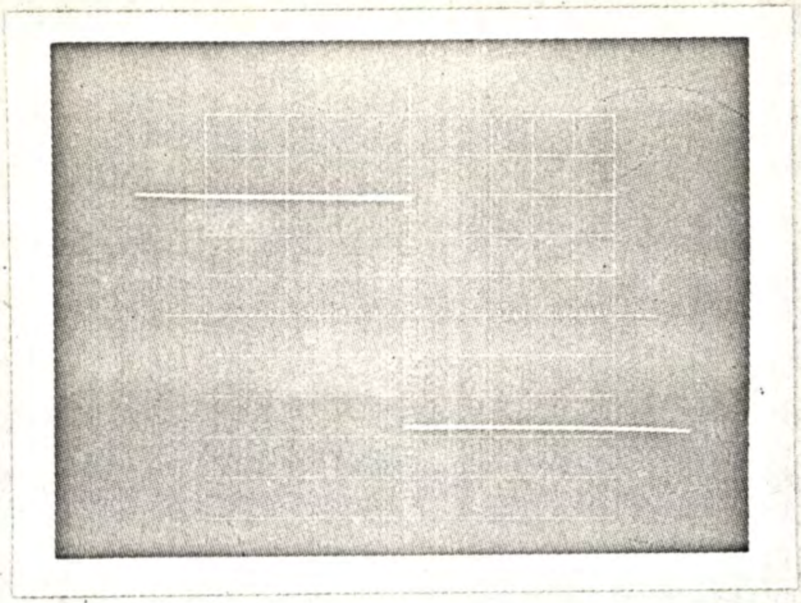


Vertical: Output 5 v./div.
Horizontal: Input 5 mv./div.

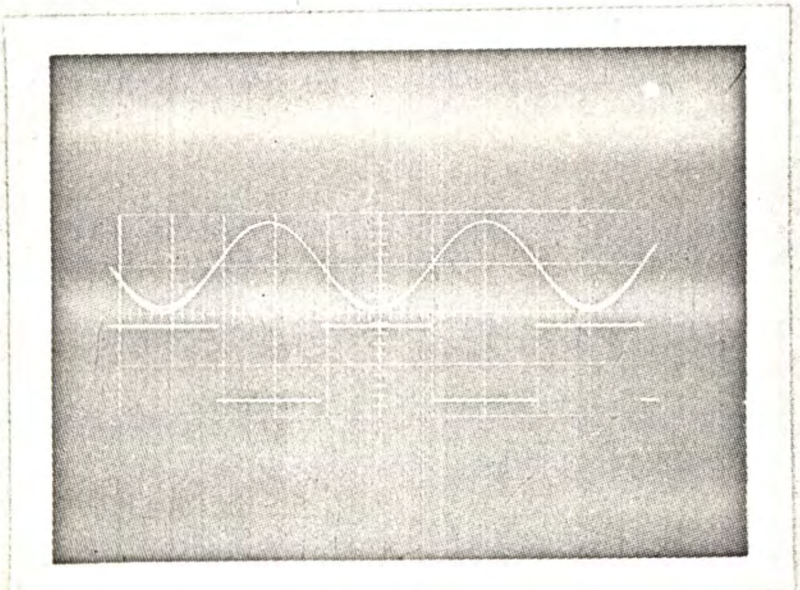


Upper Vertical: Input 50 mv./div.
Lower Vertical: Output 10 v./div.
Horizontal: Time .5 ms./div.

Fig. 19(b) Experimental Transfer Characteristic
10 Millivolts Hysteresis

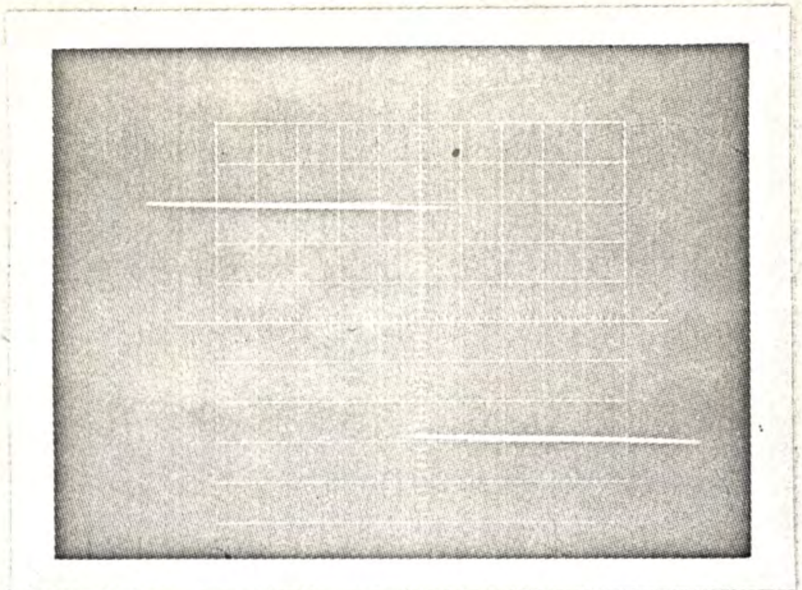


Vertical: Output 5 v./div.
Horizontal: Input 500 μv./div.

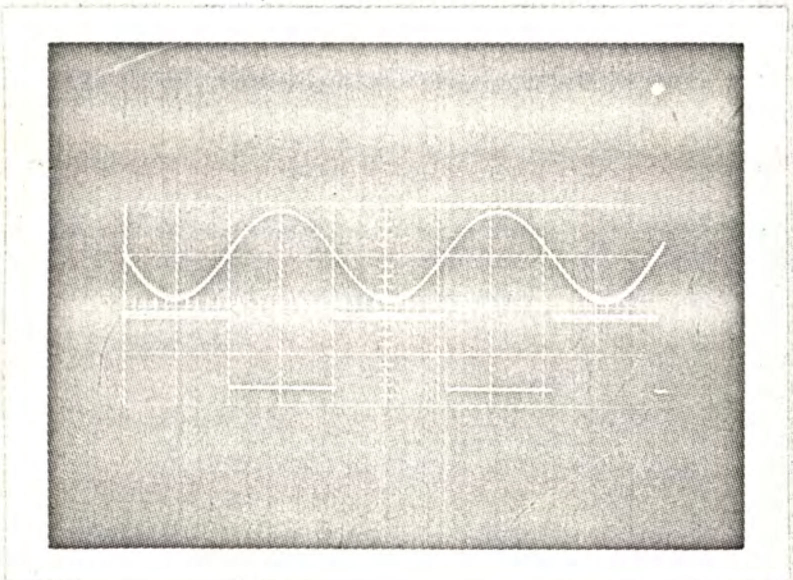


Upper Vertical: Input 500 μv./div.
Lower Vertical: Output 10 v./div.
Horizontal: Time .5 ms./div.

Fig. 19(c) Experimental Transfer Characteristic
Zero Crossing Detection



Vertical: Output 5 v./div.
Horizontal: Input 50 μv./div.



Upper Vertical: Input 50 μv./div.
Lower Vertical: Output 10 v./div.
Horizontal: Time .5 ms./div.

Fig. 19(d) Experimental Transfer Characteristic
Zero Crossing Detection

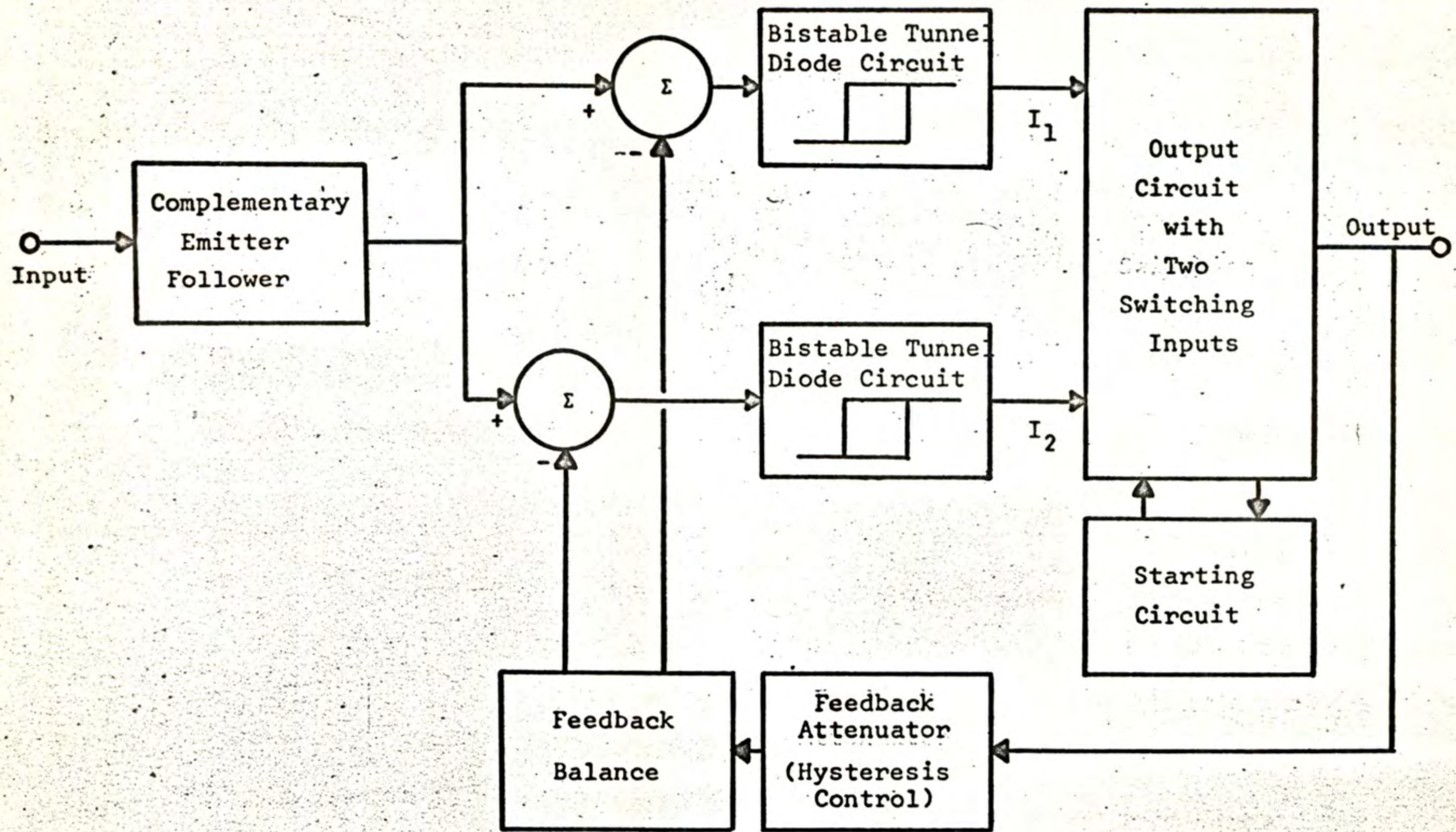


Fig. 20 Block Diagram of Tunnel Diode Stabilized Threshold Detector

switching at the output. The other unstable threshold is ignored. The output circuit also prevents both output transistors from coming on simultaneously.

The starting circuit is necessary to eliminate a state for which both output transistors are off simultaneously. The starting circuit acts only when such a state occurs; otherwise, it does not affect the main circuit.

The complementary emitter follower is used to obtain a high input impedance. The low output impedance of the emitter follower also improves the stability of the threshold detector by preventing interaction of the bistable tunnel diode circuits.

It may be helpful to review the static operation of a basic bistable tunnel diode circuit. Such a device together with a transistor amplifier is pictured in fig. 21(a). An equivalent circuit is included in fig. 21(b). The effect of the transistor is modeled as a diode. The base leakage current is neglected. It must be small compared with the peak current of the tunnel diode if the threshold depending on the peak current is to be stable. Silicon transistors which have leakage currents in the order of nanoamperes are used; so, the leakage current is certainly negligible in comparison with

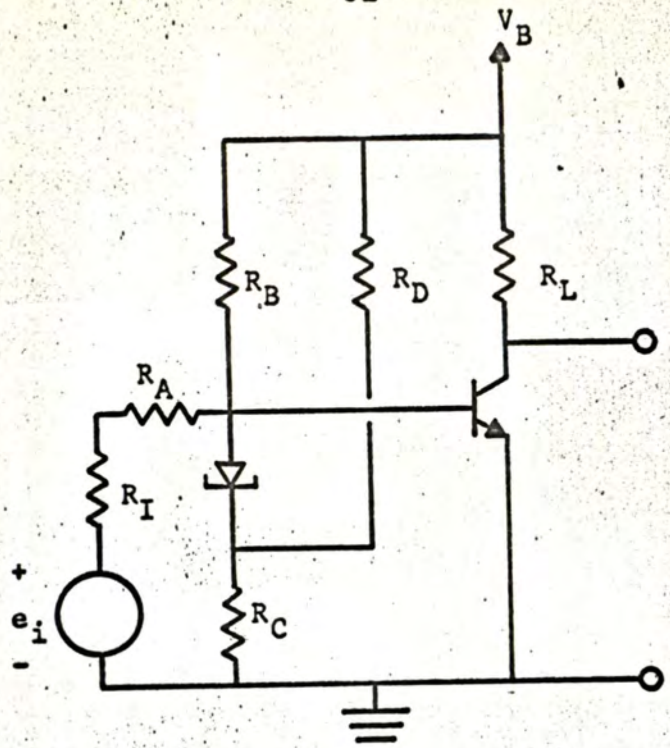


Fig. 21(a) Basic Bistable Tunnel Diode Circuit with Transistor Output

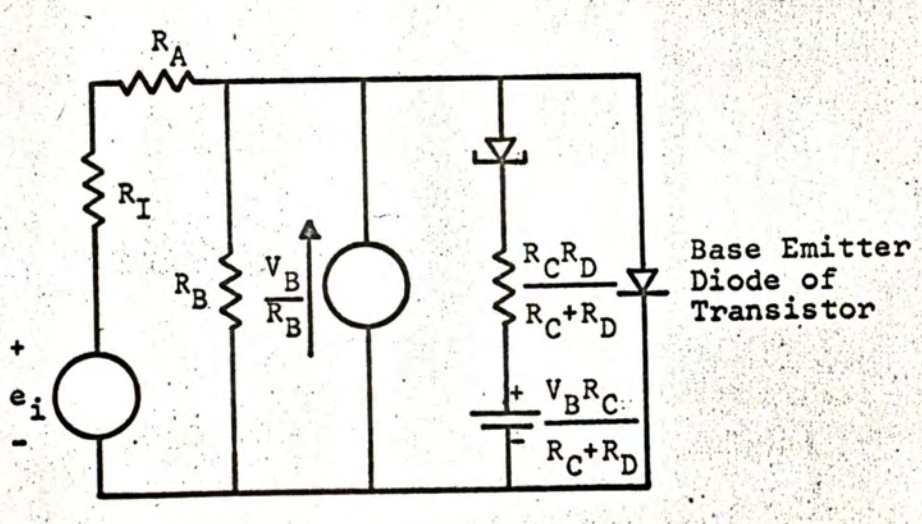


Fig. 21(b) Equivalent Circuit

the tunnel diode peak current of 1 milliampere. The R_D, R_C network is used to raise the tunnel diode about 300 millivolts above ground potential. This is necessary when a germanium tunnel diode is used in conjunction with a silicon transistor. Then the transistor is off when the tunnel diode is in the low voltage state and on when the tunnel diode is in the high voltage state. This bias voltage can be eliminated or, at least, reduced if silicon or gallium arsenide tunnel diodes are used instead of the germanium tunnel diodes.

The bias voltage is critical to the operation of the circuit. The voltage must be low enough so that when the tunnel diode is in the low voltage state, the transistor base current is negligible. For operation at millivolt levels, the base current must be 10^{-3} of the tunnel diode peak current. On the other hand, the bias voltage must be high enough so that when the tunnel diode is in the high voltage state, the base current is high enough to turn the transistor on. Preferably, the base current should be high enough to saturate the transistor. The use of silicon or gallium arsenide tunnel diodes is recommended since, then, the bias is not so critical and may even be unnecessary. The circuit of fig. 18 uses germanium tunnel diodes. The bias current and voltage must be changed if other tunnel diodes are employed.

Fig. 22(a) through (i) are self explanatory. The tunnel diode, transistor, R_B, R_C, and R_D have a nonlinear i-v characteristic which has a negative resistance region (Fig. 22(e)). The load line depending on e_i, R_I, and R_A is superimposed on the nonlinear characteristic (Fig. 22(g), (h), and (i)). e_i shifts the load line along the voltage axis. When $e_i = 0$, there are two stable states. When e_i is greater than some positive voltage, only one stable state, the high voltage state, exists. When e_i is less than some negative voltage, only the low voltage state is stable. If the circuit is initially in the low voltage state, and if e_i becomes so positive that the load line no longer intersects the low voltage region of the curve; then the circuit will switch to the high voltage state. Switching in the other direction occurs in the same manner.

The voltage at which the bistable tunnel diode circuit switches from the low voltage state to the high voltage state is dependent on the peak current of the tunnel diode. The peak current is very stable; for a selected diode, it may change by only a few percent for a temperature change of 100 °C. Therefore, the threshold which depends on the peak current is very stable.

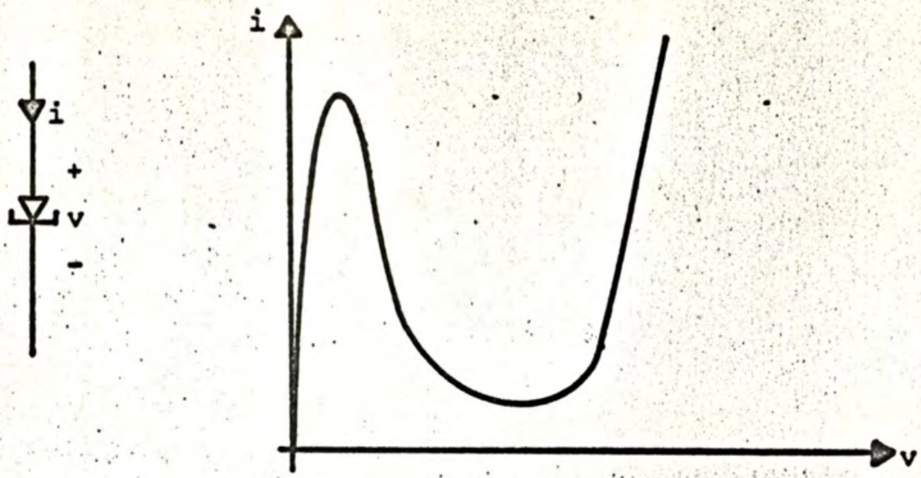


Fig. 22(a) Tunnel Diode

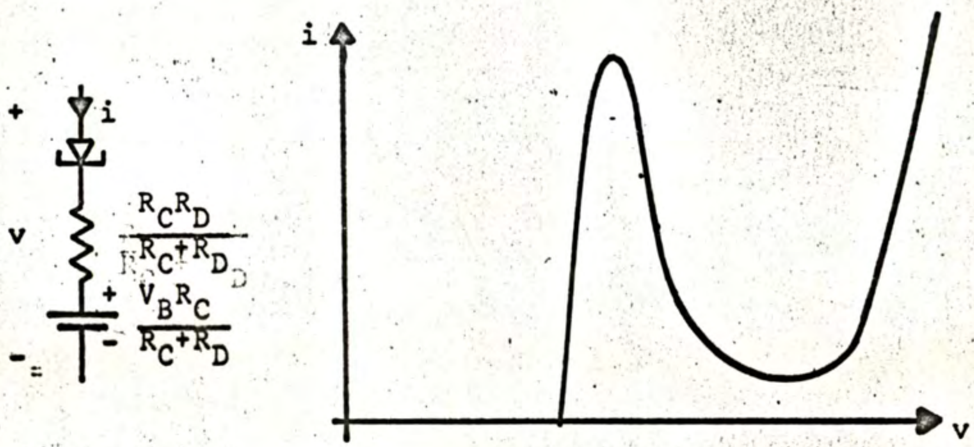


Fig. 22(b) Tunnel Diode & Voltage Bias

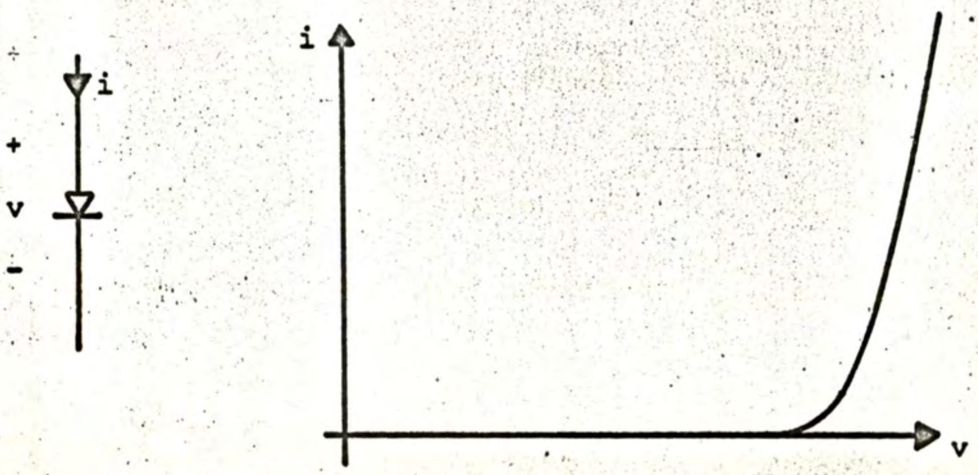


Fig. 22(c) Base-Emitter Diode of Transistor

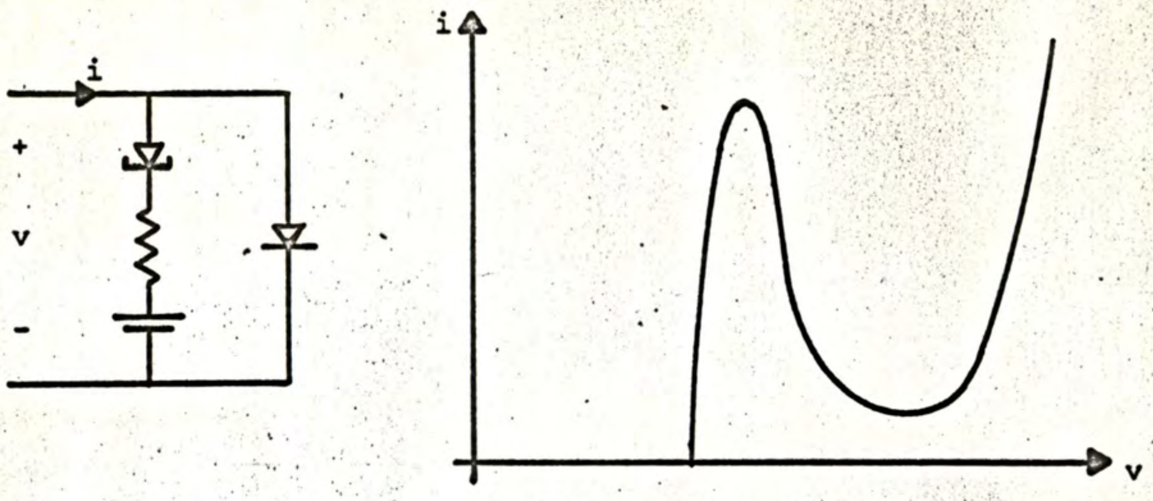


Fig. 22(d) Tunnel Diode, Voltage Bias, & Diode

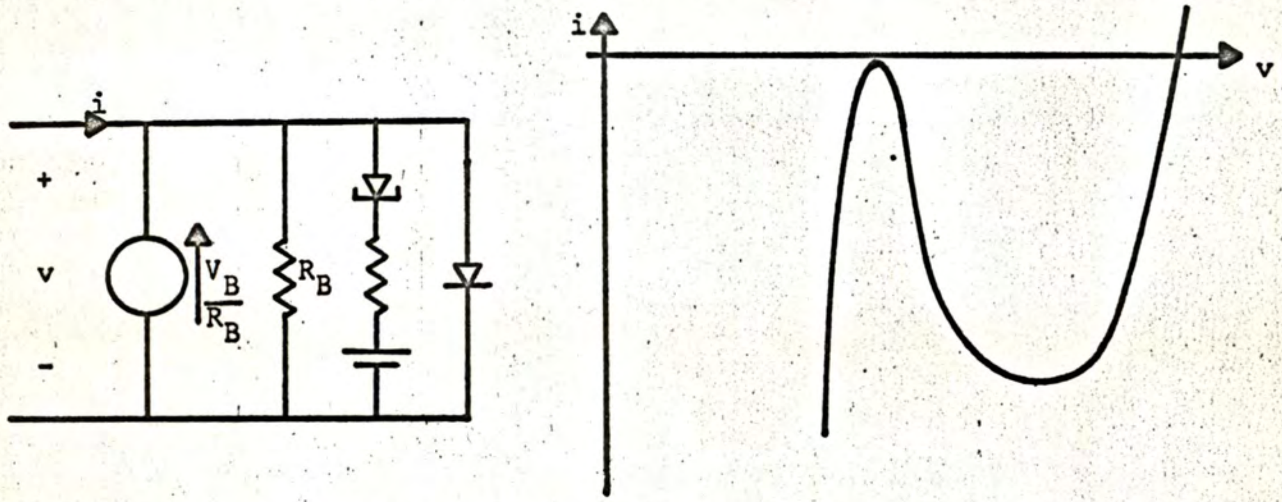


Fig. 22(e) Tunnel Diode, Voltage Bias, Diode, & Current Bias

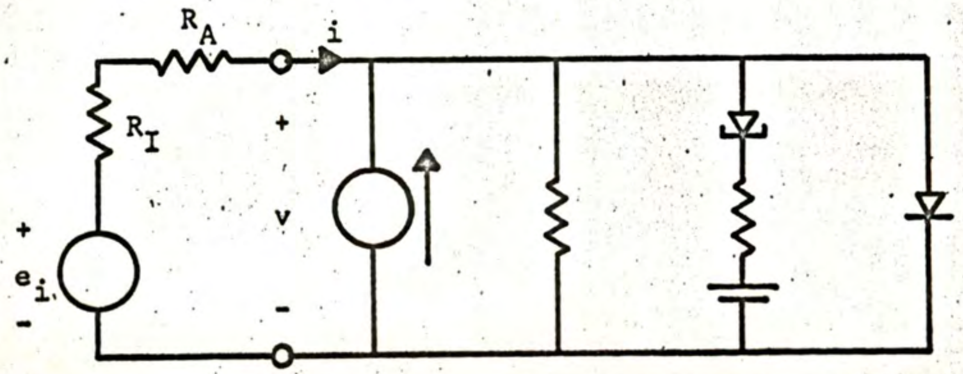


Fig. 22(f) Biased Tunnel Diode Circuit with Input Source

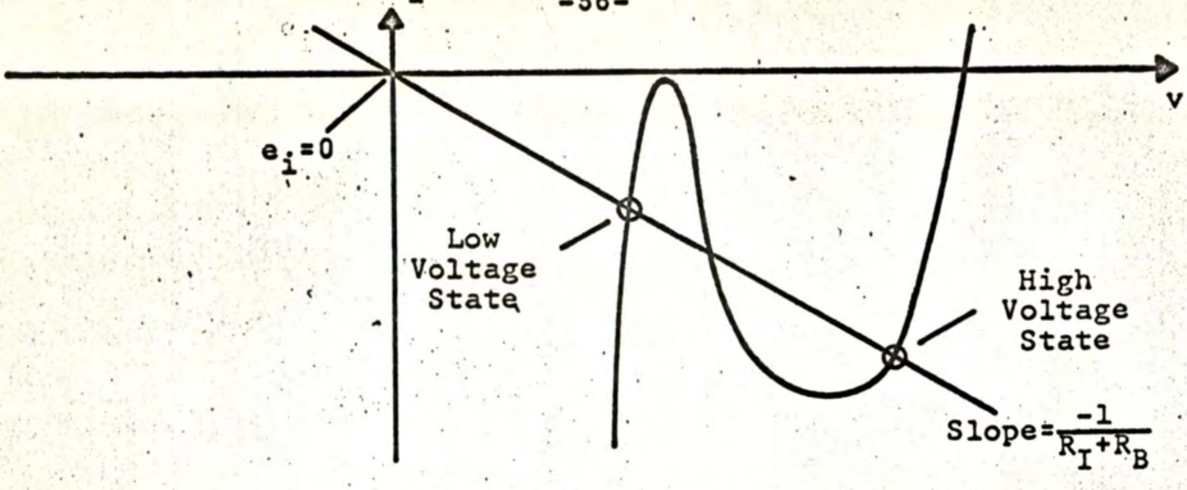


Fig. 22(g) Load Line for $e_i=0$ (Two Stable States)

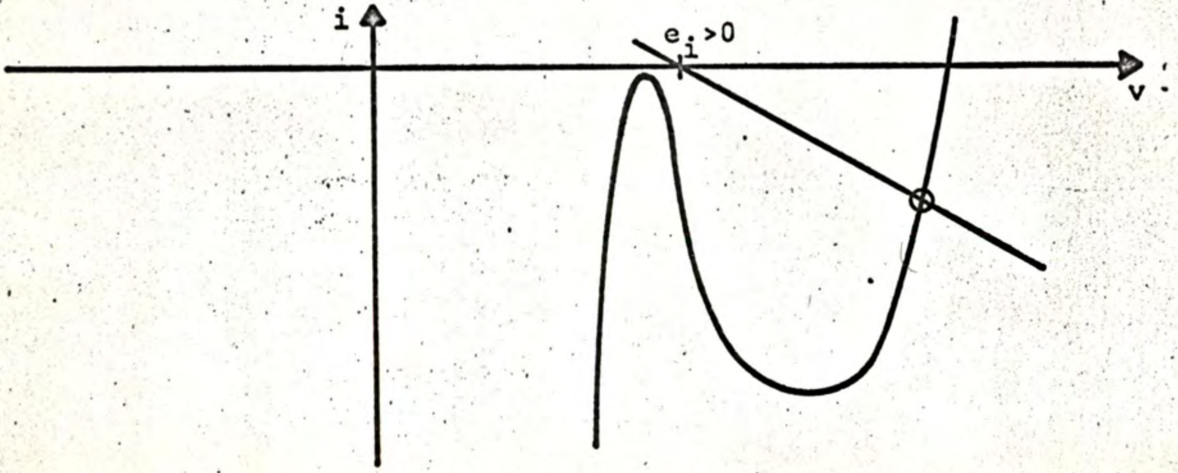


Fig. 22(h) Load Line for $e_i>0$ (Stable High Voltage State)

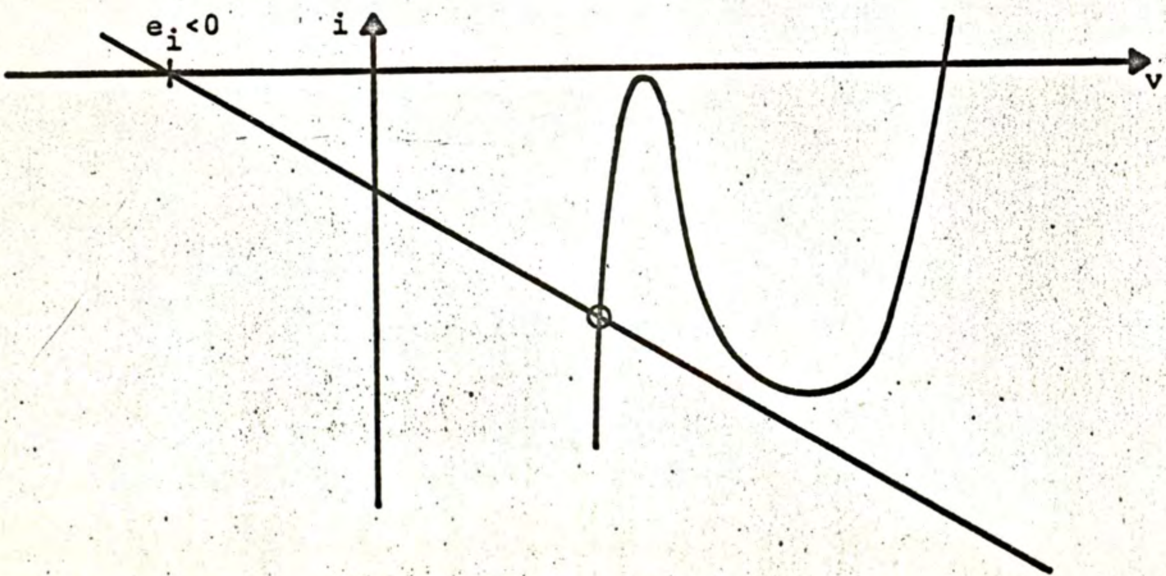


Fig. 22(i) Load Line for $e_i<0$ (Stable Low Voltage State)

The tunnel diode threshold detector is drawn in a different form in fig. 23. The emitter follower has been replaced by a voltage source, e_i, and a resistor, R_I. The clamp diodes have been eliminated since they do not affect the operation of the circuit at small signals.

Referring to fig. 23, the switching sequence is as follows: Assume that e_i is initially large, positive, and decreasing. Initially, TD_1 is in the high voltage state; TD_2 is in the low voltage state. Q_1, Q_3, and Q_6 are saturated; Q_2, Q_4, and Q_5 are off. As the signal decreases:

(1) TD_1 switches from the high voltage state to the low voltage state.

(2) Q_1 switches off. No switching is initiated in Q_3 or Q_5 because Q_3 is held in saturation by the feedback from the output.

(3) TD_2 switches to the high voltage state. This initiates switching.

(4) Q_2 switches on.

(5) Q_2 pulls Q_4 on and into saturation.

(6) Q_4 turns Q_6 off. The output voltage rises.

(7) The a.c. coupling from the output causes Q_3 to become active.

(8) The magnitude of the collector voltage of Q_3 increases causing Q_5 to become active.

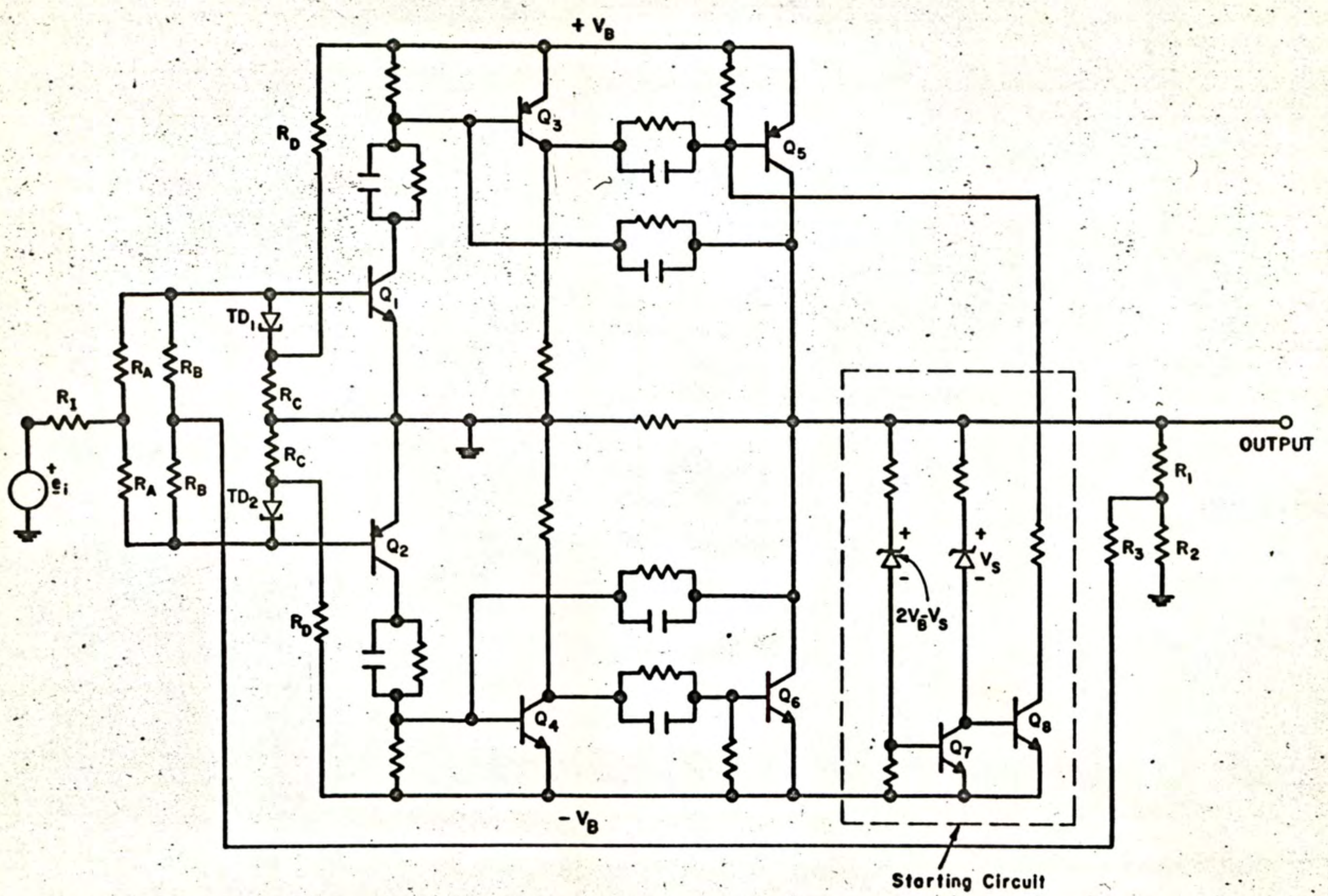


Fig. 23 Tunnel Diode Stabilized Threshold Detector

(9) Q_3 and Q_5 switch regeneratively. The output voltage reaches the positive state.
Switching in the other direction follows by symmetry.

The discussion, above, gives the impression that the switching occurs in rough steps. This is not true; switching at the output is rapid (30 ns.) and continuous.

From the discussion of the switching sequence, it is apparent that the operation of the output circuit depends, critically, on the a.c. coupling. Without the a.c. coupling, the output voltage would switch to ground potential and remain there. When the a.c. coupling capacitors are properly selected, the output switches smoothly between the positive voltage and the negative voltage.

The starting circuit affects the main circuit only when both output transistors are off simultaneously. Again, referring to fig. 23, the starting sequence is as follows. Assume that the input signal is zero. Initially, TD_1 and TD_2 are in the low voltage state. Q_1, Q_2, Q_5, Q_6, Q_7, and Q_8 are off. Q_3 and Q_4 are saturated.

(1) Q_8 becomes active and eventually saturates. ($V_S < V_B$)

(2) Q_8 pulls Q_5 on and into saturation. The output switches to the positive state.

(3) The feedback from the output causes Q_3 to become active and to eventually turn off. The circuit is now in an allowed state.

(4) Q_7 becomes active and eventually saturates.

(5) Q_7 shunts the base of Q_8 causing Q_8 to turn off. The starting circuit no longer affects the main circuit.

The hysteresis of the tunnel diode circuit is calculated with the aid of figs. 24, 25, 26, and 27. The complementary emitter follower has, again, been omitted because it has no effect on the switching and very little effect on the hysteresis. A simple piecewise linear approximation of the tunnel diode i-v characteristic can be used. The piecewise linear approximation is shown in fig. 24.

In the circuit of fig. 26,

$$V_{T1} = \frac{[V_B - V_{CE}(\text{sat.})]R_2}{R_1 + R_2}$$

$$V_{T2} = \frac{V_B R_C}{R_C + R_D}$$

$$R_{T1} = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1 + R_2}$$

$$R_{T2} = \frac{R_D R_C}{R_D + R_C}$$

A very accurate, but complicated, expression for the hysteresis can be obtained by solving the circuit of

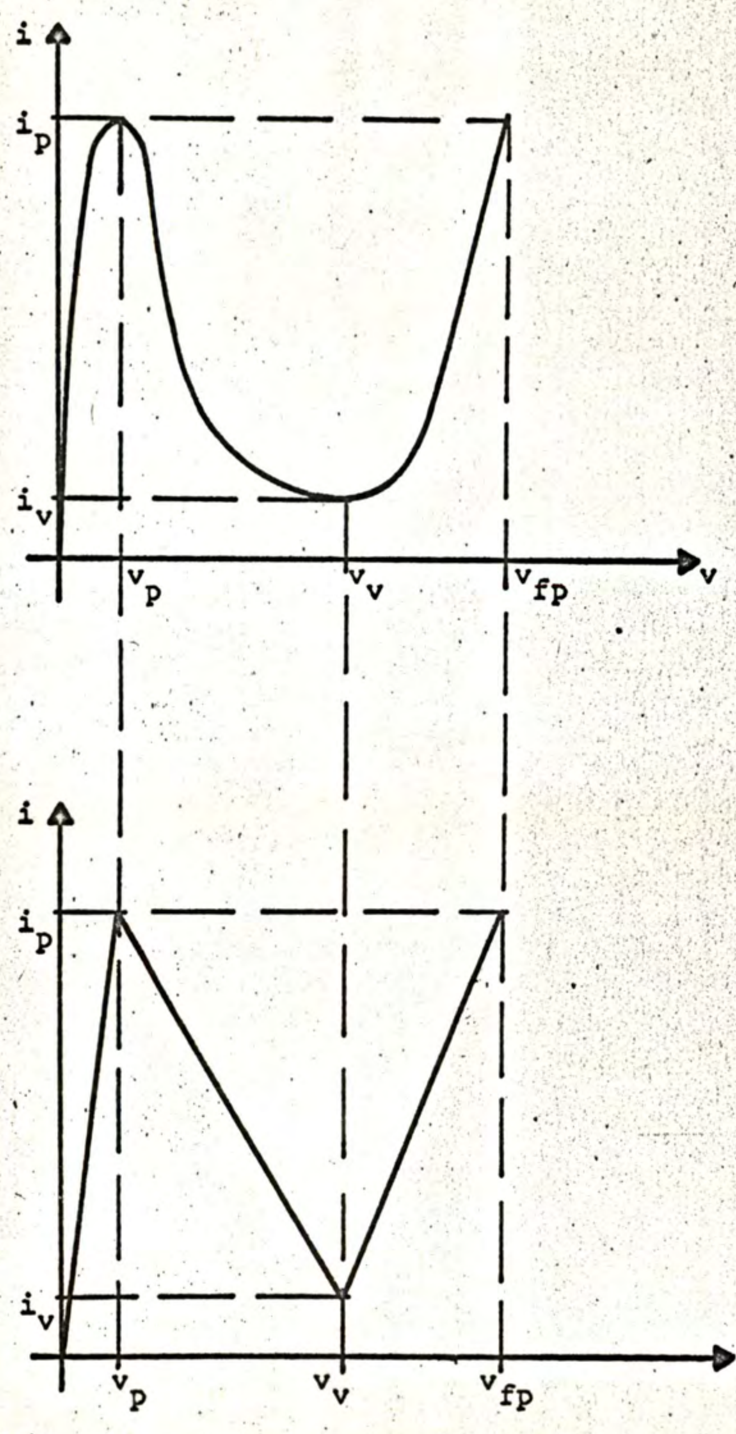


Fig. 24 Piecewise Linear Tunnel Diode V-I Characteristic

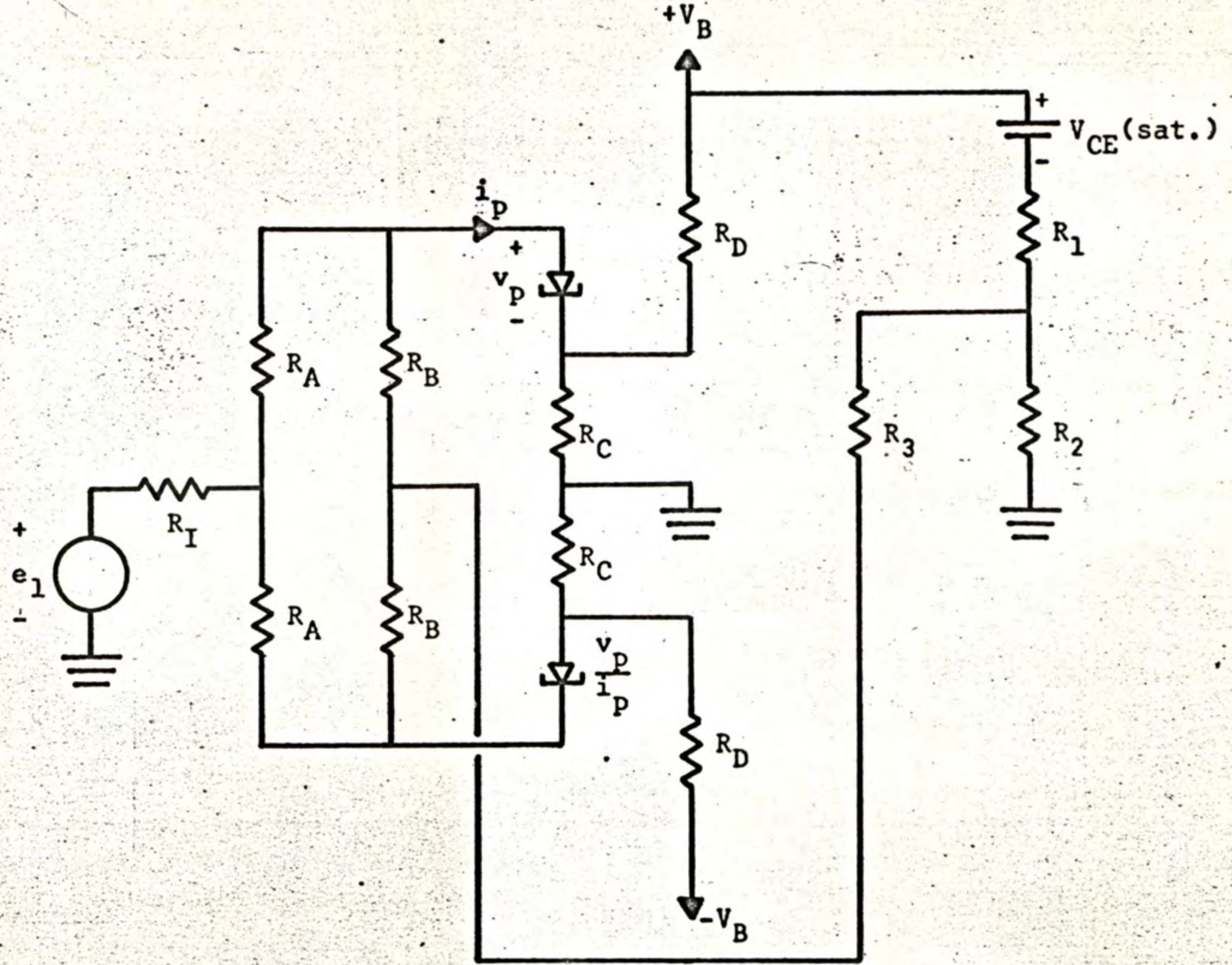


Fig. 25 Equivalent Circuit at Positive Threshold

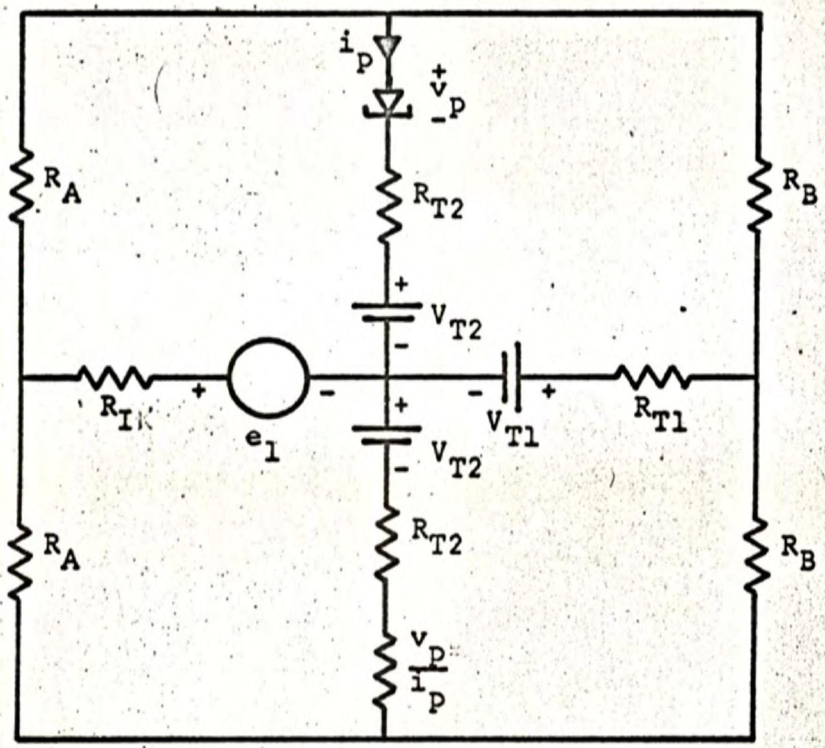


Fig. 26 Another Configuration of the Equivalent Circuit

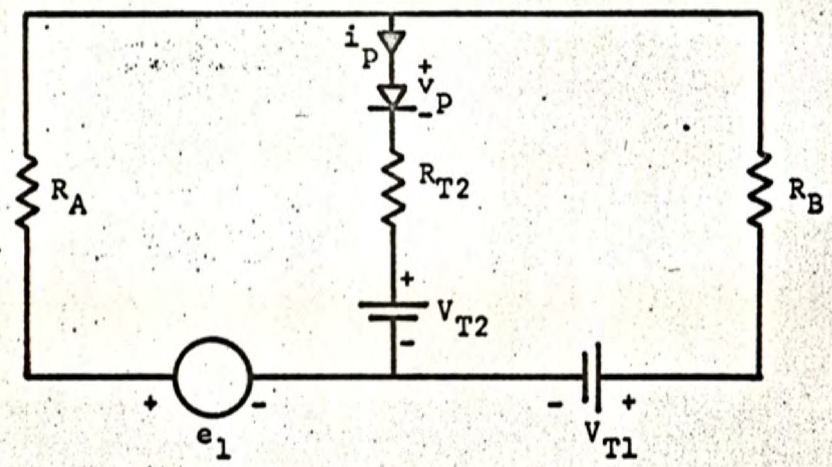


Fig. 27 Approximate Equivalent Circuit at Positive Threshold

figs. 25 & 26. However, a simpler approximate solution can be obtained from the circuit of fig. 27 if two conditions are met. If

$$R_I << R_A \quad \text{and}$$
$$R_{T1} << R_B,$$

then the coupling between the upper and lower parts of fig. 26 is weak. The effect of the lower part on the upper part can be neglected and vice versa. Then only the simple circuit of fig. 27 must be analyzed. In practice, the inequalities, above, are satisfied since it is desirable to minimize the coupling between the two tunnel diode circuits.

The approximate transfer characteristic calculated from fig. 27 is shown in fig. 28. Referring to fig. 28,

$$e_1 = - e_2 = (v_p + \frac{i_p R_D R_C + V_B R_C}{R_C + R_D}) \cdot \frac{R_A + R_B}{R_B} + i_p R_A$$
$$- [V_B - V_{CE}(\text{sat.})] \cdot \frac{R_A R_2}{R_B(R_1 + R_2)}$$

$$e_3 = - e_4 = V_B - V_{CE}(\text{sat.})$$

Although v_p, i_p, and V_{CE}(sat.) all depend on temperature, the thermal stability of the tunnel diode threshold detector is very good. The temperature dependence of v_p and i_p is extremely weak. The

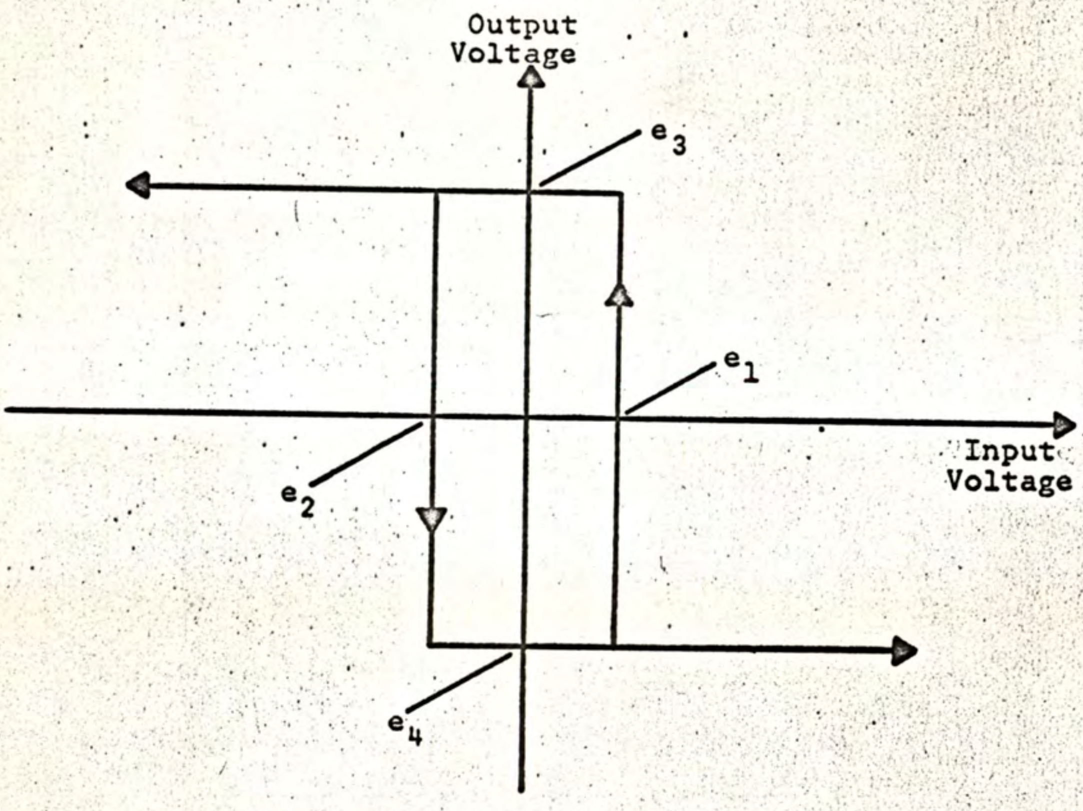


Fig. 28 Transfer Characteristic

fluctuation of these parameters with temperature is generally less than 0.1%/°C. Specifically, the peak current and voltage of a germanium tunnel diode, 1N3713, have the following temperature coefficients:

$$\left.\frac{dv_p}{dT}\right|_{T=25°C} = 65\ \mu v/°C$$

$$-1\ \mu a/°C < \left.\frac{di_p}{dT}\right|_{T=25°C} < .3\ \mu a/°C$$

typically, $$\left.\frac{di_p}{dT}\right|_{T=25°C} = -.3\ \mu a/°C.$$ (5)

Temperature variations have a greater effect on V_{CE}(sat.); however, fluctuations in V_{CE}(sat.) are not important. As long as $V_B >> V_{CE}$(sat.), fluctuations in V_{CE}(sat.) do not have a strong effect on the transfer characteristic. A millivolt variation of V_{CE}(sat.) might cause a 10 microvolt variation in the transfer characteristic.

The tunnel diode stabilized threshold detector can be adjusted to have a hysteresis of less than a millivolt. Since the series input resistors are 1000 ohms (Cf. fig. 18), the signal currents are of the order of microamperes when the input voltage is in the millivolt range. When the circuit is operating with a millivolt input signal, the tunnel diodes are switching when their current changes by a microampere out of a milliampere. In other words, the tunnel diodes are switching consistantly from the low voltage state to the high voltage state when their current changes by 0.1 %.

The tunnel diode stabilized threshold detector is a sophisticated device. It is also an expensive device. But, the experimental circuit has less than 1 millivolt of hysteresis from d.c. to 500 kc/s even though the circuit is by no means optimum. The hysteresis of the device is directly dependent on the supply voltages; so, the supplies must be regulated. Since the tunnel diodes set the switching thresholds, one would expect the device to work at frequencies well above 500 kc/s. However, the transistors limit the frequency range. Faster transistors should alleviate this problem. Some form of compensation in the feedback path might, also be useful.

The most important point is that the stable thresholds from two separate tunnel diode circuits can be selected by a special output circuit and used to form a stable threshold detector. Negative feedback can, then, be used to adjust the hysteresis.

APPENDIX

The transistor model used in the calculation of the theoretical expressions is a simplified version of the Ebers-Moll model. The simplified model is shown in fig. 29. Without such a simple model, the algebra becomes prohibitively complex. Furthermore, results calculated with more accurate models often yield complicated expressions which do not give much insight into the basic operation of the circuit.

Referring to fig.29, the diodes are assumed to be ideal switches. V_{BE} represents the voltage drop across the base to emitter diode. V_{BE} is approximated as a constant although it is in reality a logarithmic function of the emitter current. A useful equation, predicting the behavior of V_{BE}, is

$$dV_{BE} = -25\left(\frac{kT}{q}\right)\frac{dT}{T} + \left(\frac{kT}{q}\right)\frac{dI_E}{I_E}$$

where I_E is the current flowing out of the emitter. β is also assumed to be constant. Actually, β drops at low and high current levels; however, in many transistors there is a wide range of current levels for which β is constant. The β of silicon transistors usually increases with temperature.

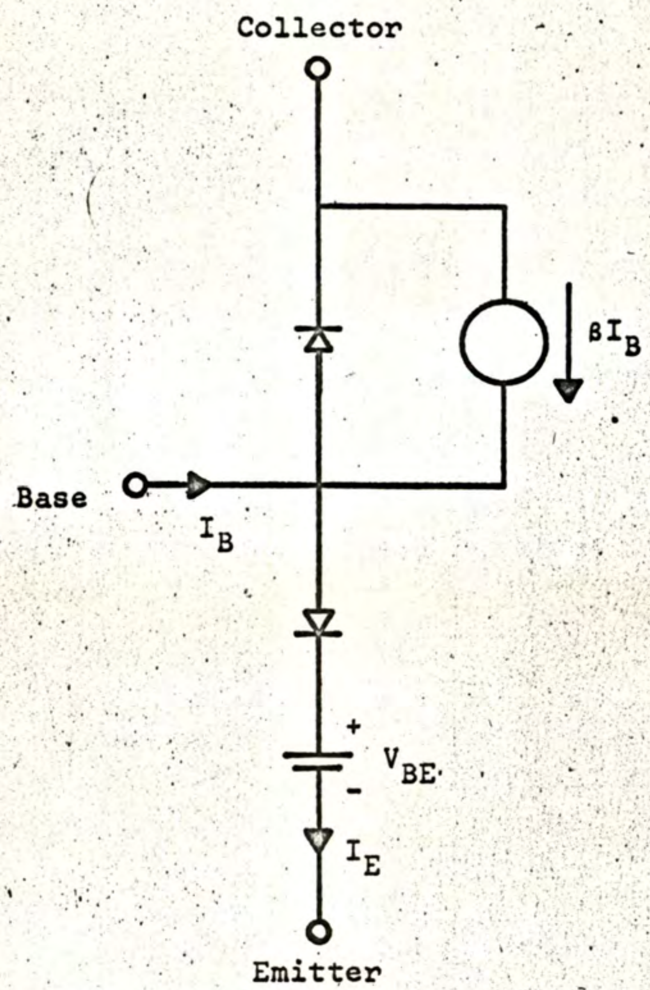


Fig. 29 A Simple Transistor Model

Generally, $1\ \%/°C < \frac{d\beta}{dT} < 5\ \%/°C.$

When the transistor is saturated, the collector to emitter voltage is approximated by a constant voltage, V_{CE}(sat.). V_{CE}(sat.) is a function of the collector current and the temperature. At moderate currents,

$$V_{CE}(\text{sat.}) \approx \frac{kT}{q}\left[\ln \frac{\beta}{\beta_r} + 2\right]$$

where β is the forward current gain as before, and β_r is the reverse current gain. β_r is the current gain when the collector and emitter are interchanged. In general, $\beta >> \beta_r$.

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A SEMICONDUCTOR REGULATED POWER SUPPLY

BY

Richard D. Gloor

August 22, 1955

A SEMICONDUCTOR REGULATED POWER SUPPLY

by

RICHARD DETMORE GLOOR

B.E.E., University of Louisville

(1951)

SUBMITTED IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September, 1955

Signature of Author ______________________________
Department of Electrical Engineering, August 22, 1955

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Thesis Supervisor

Accepted by ______________________________
Chairman, Departmental Committee on Graduate Students

A SEMICONDUCTOR REGULATED POWER SUPPLY

by

Richard Detmore Gloor

Submitted to the Department of Electrical Engineering on August 22, 1955, in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

This thesis is concerned with the design of a regulated power supply having an output voltage in the range 5 to 20 volts for application to transistorized electronic systems. Vacuum-tube regulated power supplies have been considered but discarded because of their large size and inefficiency. The solution to these problems lies in the use of semiconductor components.

The avalanche breakdown characteristics of alloyed-junction silicon diodes are investigated. These diodes are found to possess sufficiently small dynamic resistance to satisfy the requirements for use as a voltage reference element. Their temperature sensitive effects may be nullified by the use of two diodes in series whose temperature coefficients cancel. These diodes may be used to provide any desired standard voltage exhibiting either positive, negative, or zero temperature coefficient.

Characteristic curves are plotted and small-signal parameters measured for two power transistors, the Minneapolis-Honeywell Regulator Company types H-2 and H-4. Variations of the small-signal parameters with emitter current and frequency are measured. These transistors are found to be suitable for series regulating elements and comparison amplifiers. Silicon power diodes are tested for use as rectifiers in the power supply. Their low voltage drop and high rectification efficiency make them ideally suited for this application.

The evolution of the final vacuum-tubeless power supply circuit is presented in the form of a design-study. A logical development from the first circuit, a transistor analog of an elementary vacuum-tube regulator, to the final semiconductor regulated power supply is shown. Analyses are made from the standpoint of design, circuit limitations, and prediction of performance.

A six-volt power supply has been designed, constructed, and tested. Its maximum current rating is 700 milliamperes. Its stabilization factor is 0.003 and its internal impedance 0.4 ohm. These measured values of performance parameters agree well with the predicted results. Complete specifications are given in the conclusions.

Thesis Supervisor: A. B. Van Rennes
Title: Assistant Professor of Electrical Engineering

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In particular, the author is deeply indebted to his wife without whose encouragement and patience this work could not have been accomplished.

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CHAPTER I

INTRODUCTION

1.1 Objectives

The purpose of this thesis investigation is the development of a power supply to furnish voltages in the range 5 to 20 volts with an output power of up to five watts under conditions of varying load and input voltage. Semiconductor rectifier, reference, and regulator elements will be utilized.

1.2 Background

With the present trend toward transistorizing many large electronics systems, the need is felt for a power supply whose size does not represent a large percentage of the total size of the system. This power supply must furnish the low voltages necessary for transistor circuits at a current dictated by the size of the system to be powered. To insure reliable operation of the system, it must be supplied with a voltage which remains constant in spite of variations in line voltage or load.

Vacuum-tube regulated power supplies currently in use for this purpose are large and inefficient. The inefficiency results from the large voltage drop across the series regulator and the power loss in the voltage-dividing resistor used to obtain the low voltage. Physical size is large because of high values of series inductance and shunt capacitance utilized to provide low ripple voltage at the high current drawn.

A solution to the inefficiency and size problems is found with semiconductor devices. This study proposes to use a power transistor as a series regulator element to achieve efficiency with the small voltage drop involved. A semiconductor diode operated in its avalanche breakdown region will be used as the voltage reference element. Smaller capacitance will be used at points of lower current drain to reduce the output ripple voltage.

Investigations of the application of this type power supply to telephone systems have been made by the Bell Telephone Laboratories[1]. Preliminary studies on simple circuit configurations to determine feasiblity of such a device have been made by the Servomechanisms Laboratory[2] and the Lincoln Laboratory of the Massachusetts Institute of Technology.

1.3 Scope

In Chapter II a discussion is given of voltage regulators, their classification, make-up, and performance characteristics. The results of an investigation of semiconductor diodes and power transistors are given in Chapter III. Chapter IV describes the design study of an all-semiconductor power supply. Analyses are given to substantiate the experimental findings in the evolution of the final circuit. The design, construction, and testing of a six-volt, 700 milliampere power supply is detailed in Chapter V. Conclusions, including performance characteristics of the constructed power supply and suggestions for further work, are given in Chapter VI.

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1. Superscripts refer to similarly numbered items in the bibliography at the end of this report.

CHAPTER II

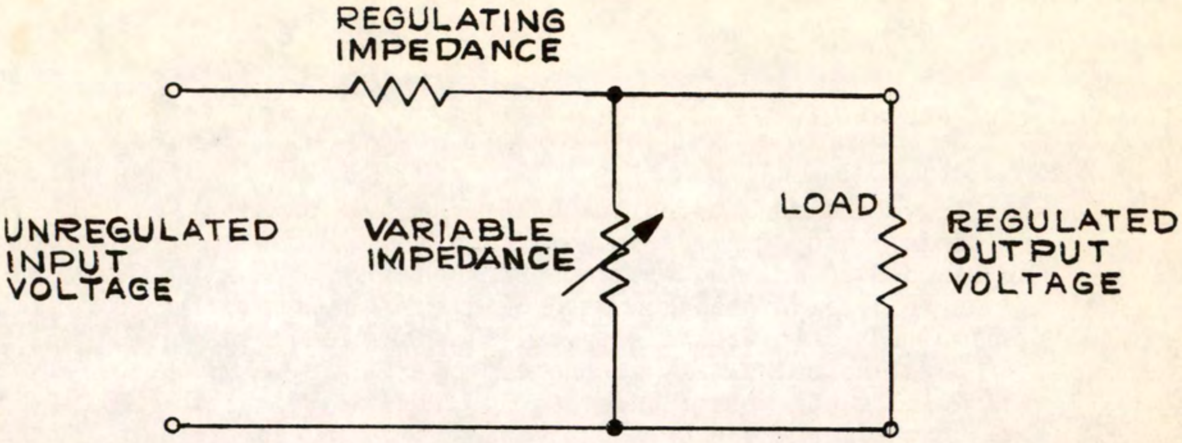
VOLTAGE REGULATORS

2.1 Classification

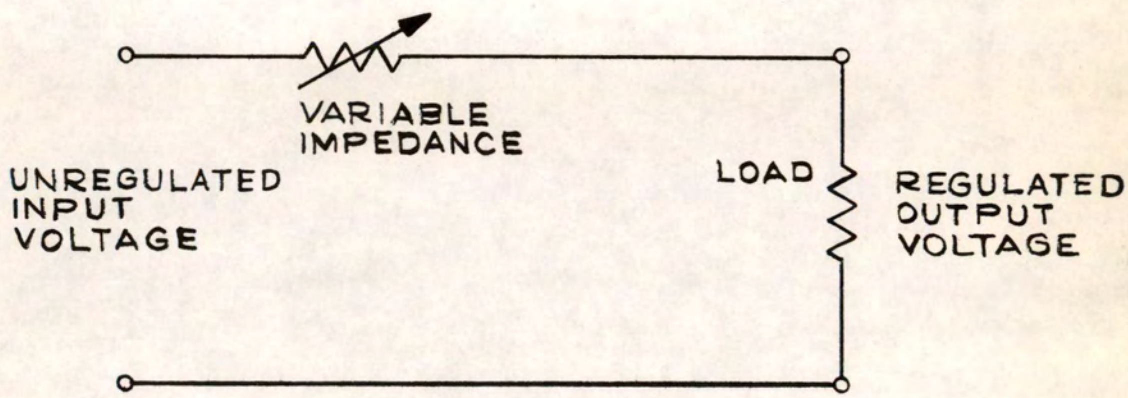
Voltage regulators may be classified as "simple" or "degenerative". Simple regulators combine linear elements with nonlinear elements to achieve low output impedance. Degenerative regulators utilize a negative feedback loop to provide an even lower impedance.

An arbitrary sub-classification is made by Benson[3] between "shunt" and "series" regulators. A shunt regulator is a variable-current device, connected in parallel with the load (see Fig. 2-1a). Both the load current and regulating-element current are drawn through a common impedance from the source of power. An ideal shunt regulating element maintains a constant voltage drop over a certain range of current. Shunt regulators are inefficient because of the wasted shunt current and because of the voltage drop and power loss in the regulating resistance.

Series regulators use a variable impedance in series with the load current as shown in Fig. 2-1b. This impedance is controlled by the load and/or line voltage to provide a constant output voltage in spite of load variations and line voltage fluctuations. Series regulators are more efficient than shunt regulators in that there is no wasted shunt current. However, when vacuum tubes are used as a series regulator, the resulting device has low efficiency, sometimes less than 50 per cent. This low efficiency is caused by the large voltage drop necessary to operate the vacuum tube as the variable series impedance.



(a)
SHUNT



(b)
SERIES

FIG. 2-1
SIMPLE VOLTAGE REGULATORS

A-63395

2.2 Basic Elements

The basic elements of a degenerative voltage regulator are: a reference element (a voltage source), a sampling circuit which provides some function of the regulator output, a comparison circuit to produce an error signal representing the deviation of the output from the desired output, and a control or regulating element.[4] Figure 2-2 shows a block diagram of a series degenerative regulator.

2.21 Reference Element

Voltage sources that are nearly constant with time are used as reference elements for regulators. The most important characteristics are constancy of voltage with aging, change of temperature, current drain, vibration, and change of position. Cold cathode gaseous discharge tubes have been successfully used in the past and more recently certain semiconductor junction diodes have been utilized as voltage standards. Since the voltage of these devices is proportional to both temperature and current, these variations constitute a source of systematic error in the reference potential. The amount of such error contributed by temperature variations may be thought of as attributable to a temperature coefficient of the standard potential. In general, it is desirable to minimize internal errors in a reference element but there are occasions where a known internal error can be used to compensate for a known error external to the standard.

2.22 Sampling Circuit

The sampling circuit provides a fraction of the regulator output for comparison with the potential of the reference element in order to derive an error signal for operation of the control element. In many cases

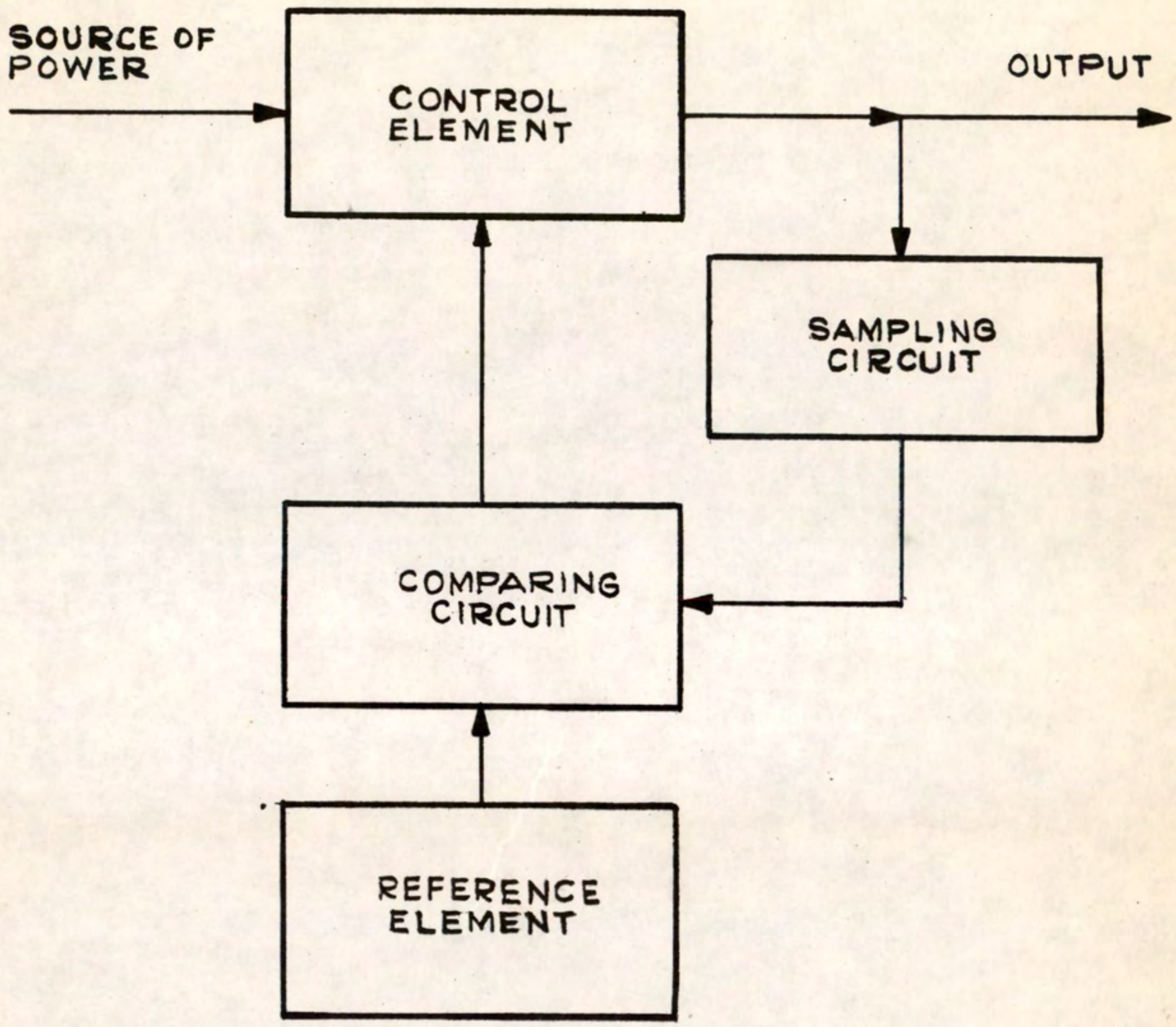


FIG. 2-2
SERIES DEGENERATIVE REGULATOR

A-63396

sampling circuits cause a loss of gain around the degenerative loop which adversely affects the performance of the regulator. The simplest form of sampling circuit is a potentiometer placed across the output from which a fraction of the output voltage may be obtained.

2.23 <u>Comparison Circuit</u>

The comparison circuit produces a signal that is a measure of the magnitude and sense of the difference between the potentials from the sampling circuit and the reference element.

2.24 <u>Control Element</u>

The element that modifies the input voltage in order to obtain the desired output voltage is called the control or regulating element. This modification of the variation of the input voltage to obtain the desired output voltage may be realized in a series regulator by a variable impedance.

2.3 <u>Performance Characteristics</u>

Factors of importance in specifying the performance of regulators include voltage, current, and power-handling capacity; output ripple; range of inputs and outputs; long-time stability; short-time or dynamic stability; and frequency characteristics. Two important characteristics which are commonly used to describe regulators are their load regulation and line regulation.

2.31 <u>Load Regulation</u>

Load regulation is defined as the ratio of the change in output voltage to a change in load current while the input voltage is held constant. Since the dimensions are those of resistance, the term is often referred to as the output, source, or internal impedance. The internal impedance is defined as:

$$R_o = - \frac{\partial E_o}{\partial I_o} \Bigg|_{E_i = \text{constant}} \tag{2-1}$$

where E_o = output voltage†

I_o = output or load current

E_i = input voltage to the regulator.

The negative sign takes account of the fact that an increase in output current normally produces a decrease in output voltage which is imagined to be produced by the voltage drop in a positive internal resistance. Internal resistance is consistently defined in the literature by Elmore and Sands[5], Gilvarry and Rutland[6], Hill[7], Hogg[8], and Maddock[9]. When the steady-state regulation is linear throughout the load range, then a single value of impedance is sufficient to describe this characteristic. Frequently, however, due to changes in parameter magnitudes, the impedance is not single-valued and a graph of output potential versus current is necessary to describe the performance.

2.32 <u>Line Regulation</u>

Line regulation, the ratio of output voltage variation to input or supply voltage variation is frequently called the stabilization factor. This choice of word is unfortunate because of the confusion of the meaning of the word "stability" in the field of servomechanisms, i.e. "freedom

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† In this report, capitalized symbols, such as E_o, refer to total quantities whereas symbols in lower case, e_o, for example, refer to incremental quantities.

from oscillation". Throughout this report the following definition will be used:

$$\text{Stabilization factor,} \quad F = \left.\frac{\partial E_o}{\partial E_i}\right|_{I_o = \text{constant}} \qquad (2\text{-}2)$$

where E_o = output voltage

E_i = input voltage to the regulator

I_o = output or load current.

The above definition is used by different authors but called by various names, e.g.: regulation -- Greenwood, Holdam, and Macrae[4], smoothing factor -- Elmore and Sands[5], stabilization ratio -- Hogg[8], regulation factor -- Hill[7].

The reciprocal of the above definition is called stabilization ratio by Hunt and Hickman[10], Gilvarry and Rutland[6], and Seely,[11] while the normalized expression $\frac{\partial E_i}{\partial E_o} \cdot \frac{E_o}{E_i}$ is called stabilization factor by Elmore and Sands[5], and stabilization ratio by Maddock[9].

One notices that by the definition given in Eq. 2-2, F is actually the incremental gain of the regulator. The stabilization factor of a perfect regulator is zero.

2.4 Regulation Equation

The output voltage, E_o, of a voltage regulator can be considered to be a function of the input voltage, E_i, and the output current, I_o, thus:

$$E_o = f\,(E_i, I_o) \qquad (2\text{-}3)$$

The total derivative of the output voltage is:

$$dE_o = \frac{\partial E_o}{\partial E_i} dE_i + \frac{\partial E_o}{\partial I_o} dI_o \tag{2-4}$$

Equation 2-4 can be rewritten using the previously defined F and R_o (see Eqs. 2-1 and 2-2):

$$dE_o = FdE_i - R_o dI_o \tag{2-5}$$

Equation 2-5, also written

$$\Delta E_o = F\Delta E_i - R_o \Delta I_o \tag{2-5a}$$

and

$$e_o = Fe_i - R_o i_o \tag{2-5b}$$

is called the regulation equation.

2.5 Advantages of Semiconductor Elements

Semiconductor diodes and transistors are obvious choices of components for use in regulators because of the following advantages: efficiency, size, lack of warm-up, and reliability.

In Chapters 3 and 4, it will be shown how the new power transistors, capable of dissipating 10 to 20 watts, may be used as control elements and as amplifiers in the comparing circuit. Silicon diodes operated in the reverse direction provide convenient voltage standards for use as the reference element. Excellent discussions of the use of semiconductor devices in regulators have been given by Chase[1], Smith[12], and Hamilton[13].

CHAPTER III

INVESTIGATION OF SEMICONDUCTOR COMPONENTS

3.1 Voltage Reference Elements

The first problem to be solved in the design of a low-voltage regulated power supply is the choice of a voltage reference element. The cold-cathode gaseous discharge tube used in high-voltage supplies is not available with a voltage rating less than 75 volts. Chase[1], Hamilton[13], and Smith[12] have suggested the use of the silicon alloy junction diode as a voltage-reference standard.

3.2 Silicon Diodes

Pearson and Sawyer[14] have described the preparation and properties of a new type of silicon diode, namely, the alloyed p-n junction diode. The unique features of this unit are: (a) large rectification† ratios (as high as 10^8 at 1 volt), (b) extremely low reverse currents (of the order of 10^{-10} amperes), (c) the ability to operate at high ambient temperatures, (d) a flat "Zener" voltage characteristic over several decades of current.

3.21 Avalanche Breakdown in Silicon

The theory of p-n junctions in semiconductors as developed by Shockley[15] may be applied to alloyed silicon diodes. The current-voltage characteristics at nominal voltages of either polarity is given by $I = I_o\,(e^{-\frac{qV}{kT}} - 1)$, where I is the current density in amperes per cm^2, V is the voltage across the junction (positive for current in the forward direction and negative in the reverse), q is the electronic charge, k is Boltzmann's constant, T is the absolute temperature and I_o is a constant determined by the properties of the silicon.

† The rectification ratio is defined as the ratio of forward current at a given positive voltage to the inverse current at the same negative voltage.

The reverse characteristics show a low current of the order of 10^{-10} amperes, and then an abrupt transition to the so-called "Zener" behavior wherein the current increases at nearly constant voltage over as many as six decades (for reverse voltages above a given critical value, $I = KV^n$, with n as high as 1500). It is this property of the diode that makes it useful as a voltage reference element.

In discussing the "Zener" effect, McKay[16] calls it an avalanche breakdown and successfully applies the Townsend theories of avalanche breakdown in gases to intrinsic electric breakdown in solids.

3.22 Slope and Temperature Effects

The average slope of a voltage standard can be determined by plotting the change of terminal voltage over a given current range, and dividing the change in voltage by the change in current. This is sometimes called the a-c, differential or dynamic resistance to differentiate it from the absolute resistance computed from the total current and voltage. Therefore the slope resistance is defined:

$$r_s = \frac{\partial V_a}{\partial I_r} \qquad (3\text{-}1)$$

where V_a = avalanche voltage

I_r = reverse current through the diode.

The temperature effect can be determined by holding the current constant at some nominal value and noting the change in voltage drop at various temperatures. The temperature coefficient may then be found from

$$C_a = \frac{\partial V_a}{\partial T} \cdot \frac{100}{V_a} \quad \text{per cent}/^\circ\text{C or }^\circ\text{K} \qquad (3\text{-}2)$$

In a group of diodes tested by Smith,[12] temperature coefficients were found to range from about -0.04 to 0.10 and the value of r_s varied between 10 ohms and 1000 ohms for diodes having rated voltages between 4 and 40 volts. The lowest values of both C_a and r_s occurred in diodes having avalanche voltages of 4 to 6 volts.

3.23 Slope-Temperature Data

The author tested two 150 milliwatt diodes, type A5B.† The following data were obtained:

	C_a	r_s	V_a at 10 ma.
Diode No. 1	0.03 per cent/°C	8.8 ohms	6.00 volts
Diode No. 2	0.04 per cent/°C	8.5 ohms	6.25 volts

In the forward direction, these diodes were found to exhibit a similar effect, viz., constant voltage drop throughout a large range of current. The temperature coefficient of forward voltage is negative, and the constant voltage measured less than one volt. In this constant-voltage conducting region, the temperature coefficient may be defined as

$$C_f = \frac{\partial V_f}{\partial T} \cdot \frac{100}{V_f} \text{ per cent/}^\circ\text{C or }^\circ\text{K} \tag{3-3}$$

and the slope resistance as

$$r_f = \frac{\partial V_f}{\partial I_f} \tag{3-4}$$

where V_f = forward voltage drop

I_f = forward current through the diode.

† National Semiconductor Products

The following data were obtained in the forward direction:

	C_f	r_f	V_f at 10 ma.
Diode No. 1	-0.22 per cent/°C	4.2 ohms	0.769 volts
Diode No. 2	-0.20 per cent/°C	5.0 ohms	0.772 volts

Figure 3-1 is a plot of the volt-ampere characteristics of diode 2.

3.3 Minimizing Temperature Effects

In attempting to reduce the temperature coefficient of the voltage reference element, one can make use of the fact that the temperature coefficients in the forward and reverse directions have opposite algebraic sign. Two diodes are connected in series with either their anodes or their cathodes connected together. This is not a cascade connection but a "back-to-back" connection. If a current is created in either direction, it constitutes a forward current for one diode and a reverse current for the other. With this connection, one would expect the temperature effects to tend to cancel, the slope resistances to add, and the resultant voltage drop across the pair to be the sum of the individual voltage drops, V_a and V_f.

3.31 Slope-Temperature Data

Figure 3-2 shows the volt-ampere characteristics of the pair of diodes connected "back-to-back." The following data is for reverse current in diode 1 and forward current in diode 2.

	Predicted	Measured
$(V_a + V_f)$ at 10 ma.	6.772 volts	6.88 volts
$r_s + r_f$	13.8 ohms	13.5 ohms

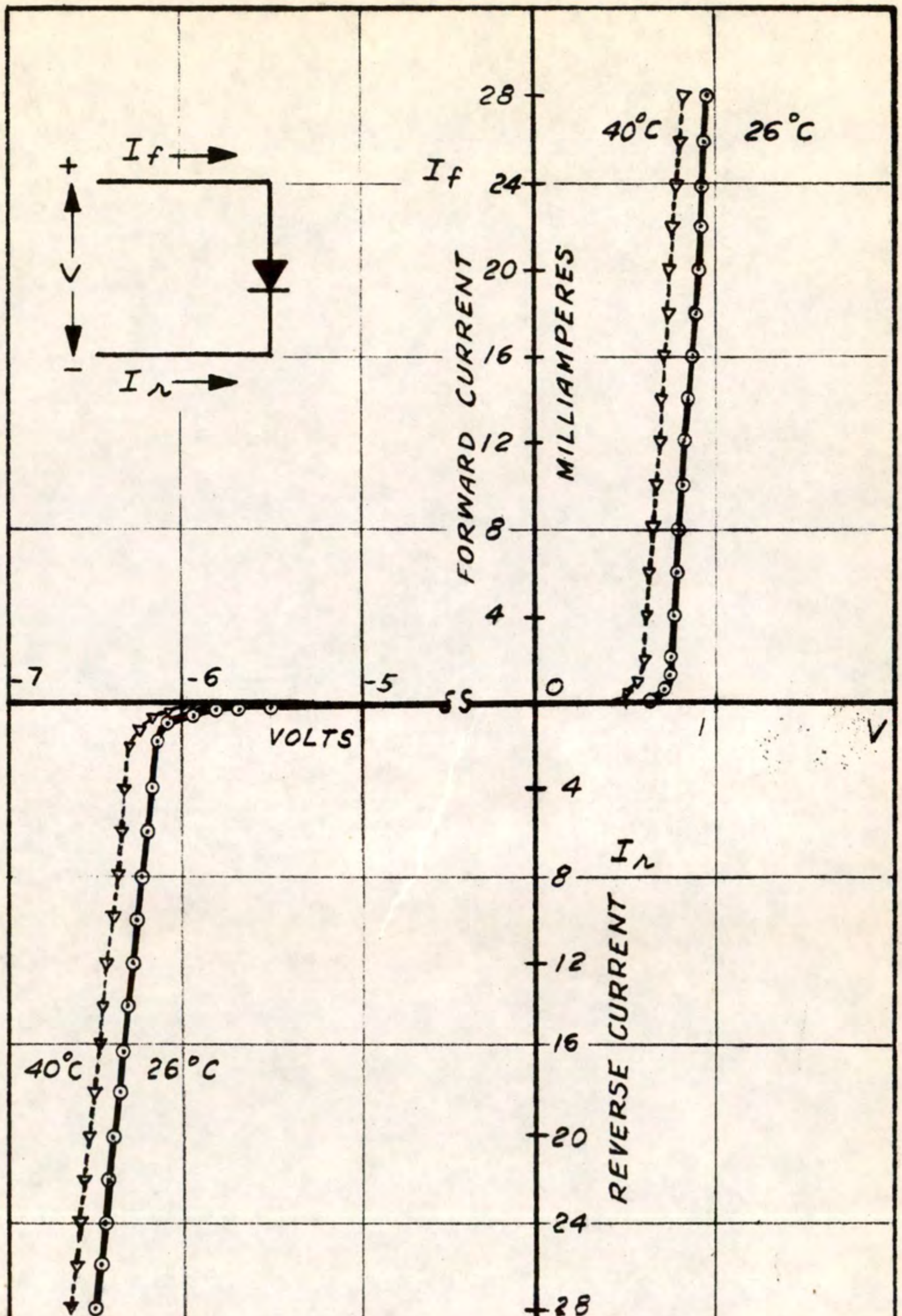


FIG. 3-1

VOLT-AMPERE CHARACTERISTICS OF ALLOYED JUNCTION SILICON DIODE

A-63397

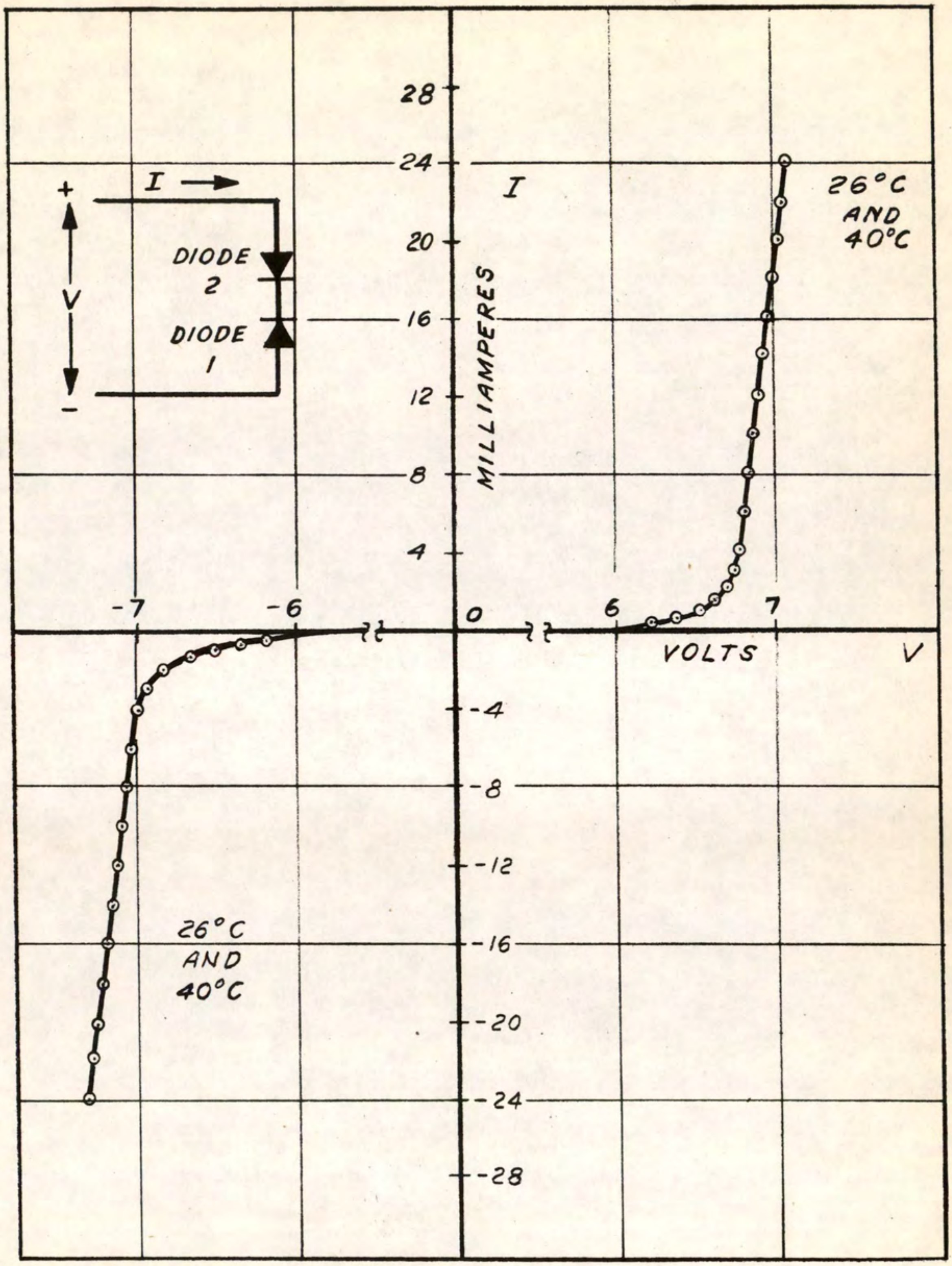


FIG. 3-2

VOLT-AMPERE CHARACTERISTICS OF TWO SERIES DIODES

A-63398

In order to check the temperature effects, the temperature coefficients are expressed in slightly different units from those defined in Eqs. 3-2 and 3-3. The units "volts per $^{\circ}C$" are used to facilitate comparison.

Temperature coefficient of diode 1	0.00179 volts/$^{\circ}C$
Temperature coefficient of diode 2	-0.00161 volts/$^{\circ}C$
Sum of the measured temperature coefficients of each diode	0.00018 volts/$^{\circ}C$
Measured temperature coefficient of the two diode combination	0.000178 volts/$^{\circ}C$

By this method it is possible to achieve a temperature coefficient lower than the value of 0.01 per cent/$^{\circ}C$ quoted for a VR 75 electron tube. Through the judicious use of several diodes connected in series, some in the forward direction and others in the reverse direction, it is possible to "tailor-make" a voltage standard having very close to a zero temperature coefficient, to a net positive temperature coefficient or to a negative temperature coefficient as desired.

It would seem more feasible to make a medium voltage standard with several low voltage diodes connected in series than with one diode. A 24-volt reference standard made up of four 6-volt diodes could have a smaller temperature coefficient and slope than a single 24-volt diode.

3.4 Additional Electrical Properties

A discussion of the electrical properties of silicon junction diodes would not be complete without some mention of the chief defects present in some units as presently made. According to Pearson and Sawyer,[14] these are chiefly two in number and might be called noise at the avalanche knee and "softness" of the reverse characteristic, or a lack of saturation of the reverse current. The noise is almost always limited to the low-current region of the avalanche characteristic, sometimes appears to be "clipped" at a fixed voltage, is generally white, and, in some units, is temperature dependent. "Softness" of the reverse characteristics means an unusual increase in the reverse current before the true avalanche voltage is reached.

3.5 Silicon Power Rectifiers

If the primary source of power for a non-vacuum-tube power supply is to be alternating voltage, a semiconductor device must be used to furnish filtered direct voltage to the regulator. One may pick from a variety of metallic rectifiers built from copper oxide, germanium, selenium, or the newcomer, silicon.

Losco[17] and Rockett[18] have described new silicon power rectifiers with junction areas of 0.05 cm^2 that pass 200 amperes/cm^2 of forward current at 1 volt. Rudenberg[19] points out that in theory, silicon rectifiers can operate at current densities 1000 times higher than those customary with copper oxide and selenium rectifiers, and with much better forward and inverse characteristics.

Before describing the electrical characteristics of silicon power rectifiers, one should take note of the advantages of germanium rectifiers. At room temperature they have good forward and inverse characteristics. However, it is difficult to manufacture single cells with peak inverse voltages exceeding 65 volts rms. Germanium power rectifiers must be considerably derated above 55°C and generally cannot operate above

75°C at any power level. In contrast, the silicon rectifiers will handle useful power levels at 125°C.

3.51 Electrical Characteristics

The electrical efficiency† of silicon rectifiers is extremely high, even at low current levels. This is due to the almost complete absence of inverse dissipation at voltages below the peak inverse voltage. At temperatures in the vicinity of 65°C, efficiency is about 98 per cent.

The inverse characteristics show an outstanding feature, viz., the increase of breakdown voltage with increased temperature. The fact that the peak inverse voltage rating is not reduced with increasing temperature is an important practical property of silicon power rectifiers.

Another important feature of a silicon power rectifier is the relatively high rectification ratio obtainable. Rectification ratios of 10^7 are attainable at room temperature. This order of magnitude is similar to the values obtained for small-area junctions. A high ratio is obtained because the forward current density at low voltages (say one volt) is increased to a greater extent by the junction area than is the leakage current. For a given power rating, the absence of appreciable inverse leakage current increases the possible output voltage in rectifier applications. Note that features (a), (b), and (c) mentioned in Section 3.2 for the 150-milliwatt diodes are also found in the larger power units.

The high current density of the silicon junction permits radical miniaturization but the small physical size achieved has drawbacks. The

---

† Rectifier efficiency is defined as 100 times the direct power output divided by the alternating power input.

internally generated heat must be conducted through the copper case of the silicon unit without exceeding the maximum junction temperature of 150°C. This fact requires that the rectifiers be mounted on a good thermal conductor, such as an aluminum chassis of reasonable area or on a well-designed fin to provide adequate convection. Proper mounting of the silicon power rectifiers is necessary if thermal limitations are not seriously to restrict the electrical dissipation permissible with these rectifiers.

3.52 <u>Experimental Data</u>

Two silicon power diodes were tested to determine their applicability as rectifiers. The larger unit, a type CK 775[†], had an average current rating of 15 amperes and a recurrent peak current rating of 50 amperes. The second diode, a type 1N347[††], had an average current rating of 1 ampere and a recurrent peak current rating of 5 amperes.

Figure 3-3 shows the volt-ampere characteristics of these two diodes.

3.6 <u>Power Transistors</u>

For a series-type regulator a control element is needed which will pass the expected load current with minimum voltage drop. Two semiconductor devices, types H-2 and H-4[†††] germanium junction transistors, have recently been introduced to the market which will fulfill these requirements. The maximum collector current ratings for these units are 800 ma. and 400 ma. respectively. With maximum collector voltage ratings of 30 volts (grounded emitter) and 60 volts (grounded base), these transistors seem ideally suited for use as control elements.

† Raytheon Manufacturing Company
†† Transitron Electronics Corporation
††† Minneapolis-Honeywell Regulator Company

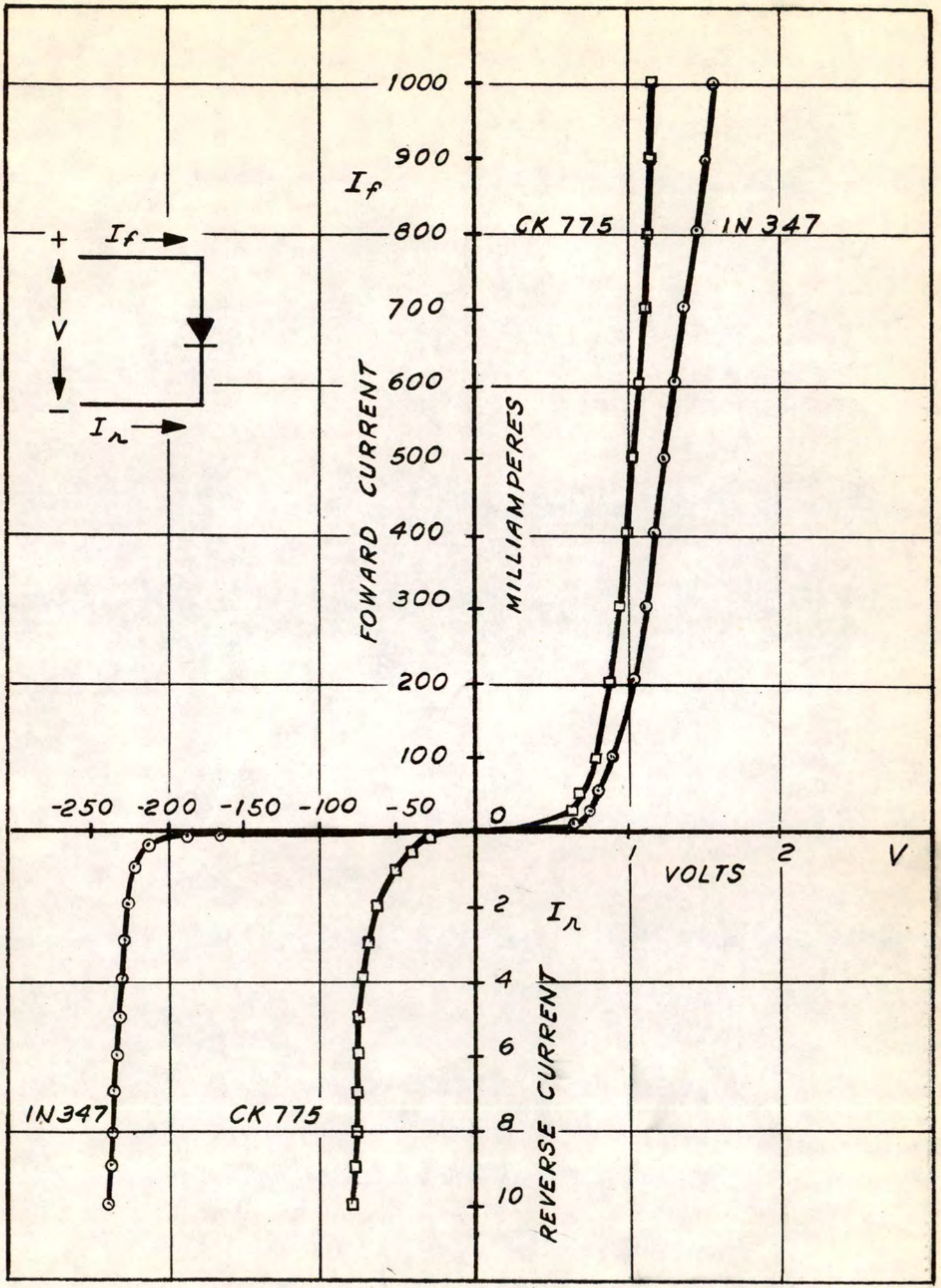


FIG. 3-3

VOLT-AMPERE CHARACTERISTICS OF SILICON POWER DIODES

A-63399

3.61 Collector Dissipation Limitation

The power-handling ability of germanium junction transistors is limited primarily by the maximum heat dissipation of the transistor, the maximum collector voltage, and the maximum collector current. The importance of heat dissipation is well known and presently is the basis for power classification of transistors. In the case of the H-2 which is rated as a 20 watt transistor, one immediately notices that the unit cannot be operated at its maximum collector voltage and maximum collector current simultaneously. Mooers[20] states that the germanium junction in the H-2 would melt in free air with 1/4 watt dissipation. Roka,[21] in describing the construction of this transistor, states that the collector is soldered directly to the copper case of the metal shell. The heat generated at the collector junction is conducted to the outside through this heat transferring copper case. To assure adequate transfer of heat from the copper case, as in the case of the power diode, the manufacturer recommends that the transistor be mounted on an external heat sink. In many cases, it is necessary that the collector remain electrically isolated from the heat sink. This isolation can be accomplished by inserting an 0.002-inch mica washer between the copper heat-conducting section of the transistor shell and the chassis or heat sink. This connection provides good electrical insulation with a minimum of thermal insulation. By applying a drop of silicone oil between the mica sheet and the metal surfaces, the temperature drop across the mica insulator is reduced to less than 2°F per watt of heat transfer.

3.62 Case Temperature Derating

Mooers[20] states that the power ratings of 20 watts for the H-2 and 5 watts for the H-4 are made on the basis that the case temperature be maintained at 70°F. If the case temperature is allowed to rise, the maximum collector dissipation must be reduced at the rate of approximately 0.15 watts per ^{o}F for the H-2 and 0.04 watts per ^{o}F for the H-4. Under no circumstances must the case temperature be allowed to exceed 250^{o}F. Figure 3-4 shows the manufacturer's limits for collector dissipation for the H-2 and H-4.

3.63 Power-Temperature Data

To measure temperature of the transistor case during succeeding experiments, an iron-constantan thermocouple was attached to the mounting stud and the temperature was monitored continuously. The transistor was mounted on a sheet of aluminum, 6" x 6" x 1/8" and air was circulated past this heat sink by means of a small fan.

Figure 3-4 is a plot of the collector dissipation versus the mounting case temperature using the heat sink just described. Since the physical packages for the H-2 and H-4 are almost identical, equal case temperatures would be expected for equal collector dissipations. This fact is borne out by the plotted points. The most important information to be obtained from this plot is that for the particular heat sink used, the maximum steady-state collector dissipation allowable for the H-2 is 11 watts and, for the H-4, 4 watts.

The proper design and construction of a heat sink for these transistors cannot be overemphasized. A slight burr on the mounting hole of the aluminum plate will prevent intimate contact with the copper case and greatly reduce the maximum allowable dissipation. An interesting discussion of the thermal and mechanical design of the power transistors is given by Fletcher.[22]

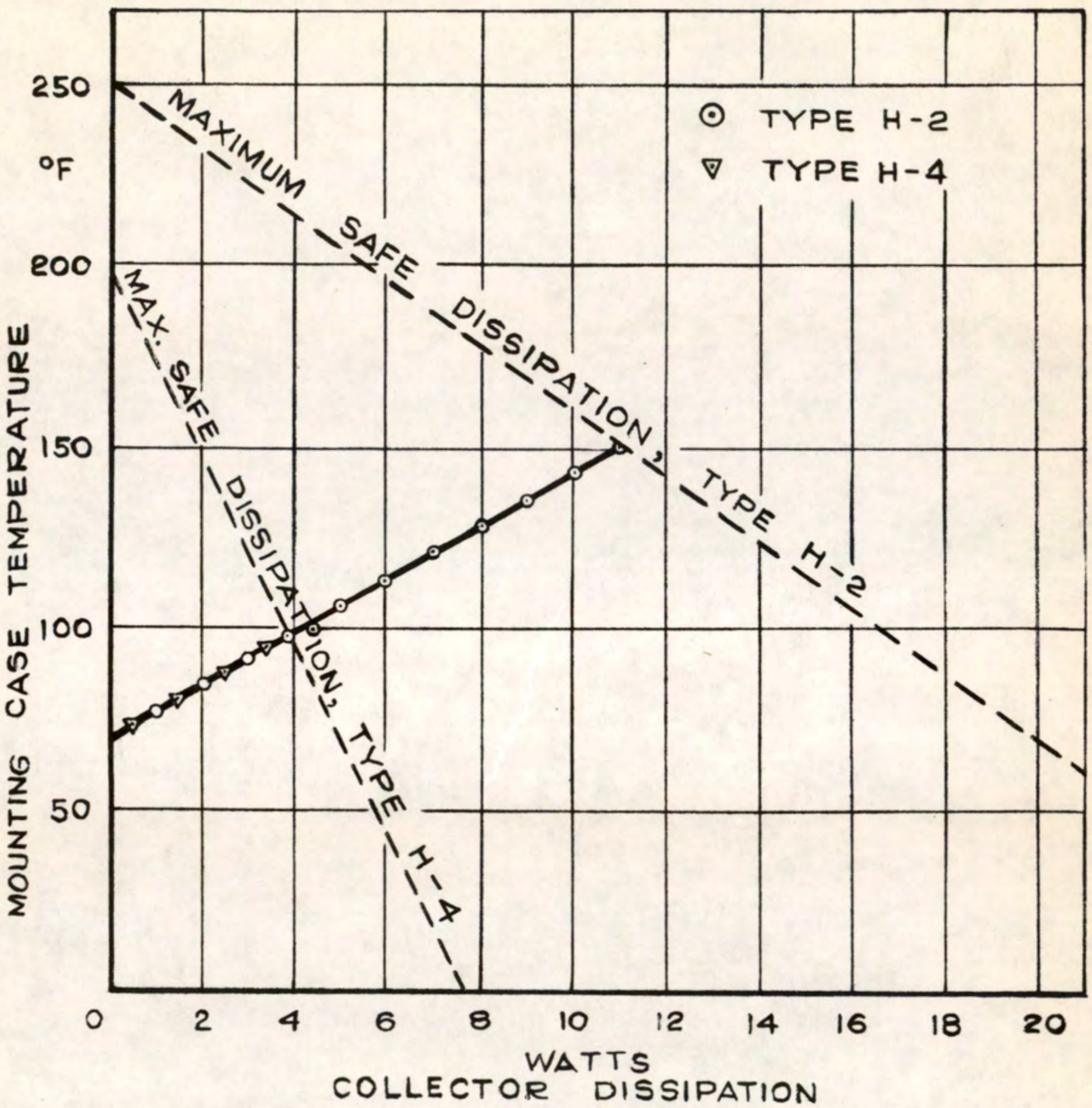


FIG. 3-4

MOUNTING CASE TEMPERATURE
vs. COLLECTOR DISSIPATION
FOR POWER TRANSISTORS

A-63400

3.64 Electrical Characteristics

When confronted by a three-terminal device, one can choose a number of volt-ampere characteristics which form a complete description of the electrical characteristics. For amplifier applications, the collector family of characteristics is usually most useful. This family is a plot of collector-to-emitter voltage versus collector current for various values of base current (for the grounded emitter circuit) or collector-to-base voltage versus collector current for various values of emitter current (for the grounded base configuration). One is immediately faced with the practical problem that commercial transistor curve plotters cannot furnish the currents required by power transistors.

In considering the point-by-point measurement of the characteristic curves, the author realized that this method of measurement most accurately duplicates the actual operating conditions of the transistor as a d-c amplifier. If pulsing techniques were used to sweep the collector voltage through some range of values, the transistor would not reach the temperatures attained in actual operation, say, as the series control element. Since semiconductor parameters are known to vary with temperature, the measurements made by pulse techniques would not be applicable to the mode of operation encountered in this report. Figures 3-5 to 3-8 show the collector families of characteristic curves for the power transistors, types H-2 and H-4.

3.65 Small-Signal Parameters

For purposes of circuit analysis, Ryder and Kircher[23] consider a transistor to be an active four-terminal network. In general, a four-terminal network can be described by any one of six pairs of two simultaneous alge-

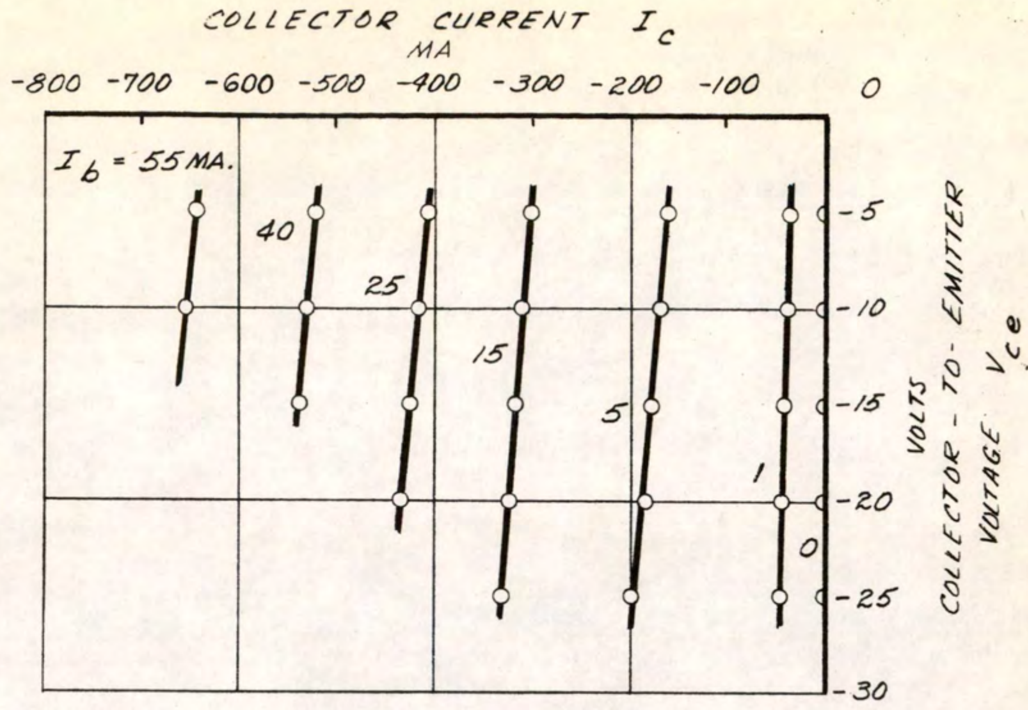


FIG. 3-5

GROUNDED EMITTER CONFIGURATION

CHARACTERISTIC CURVES FOR POWER TRANSISTOR TYPE H-2

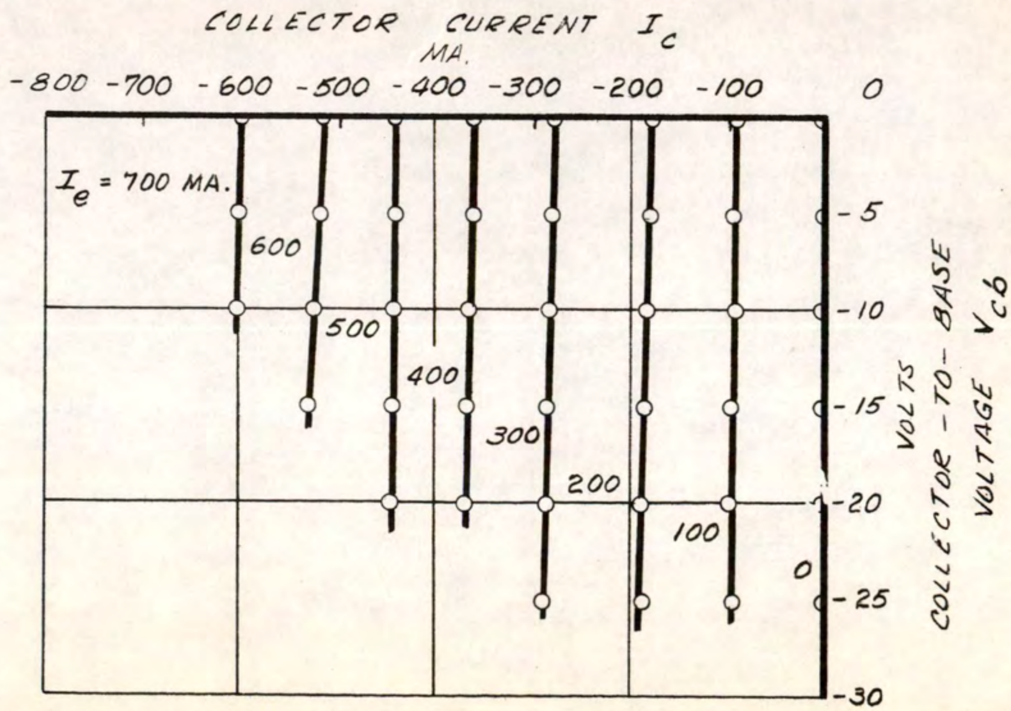


FIG. 3-6

GROUNDED BASE CONFIGURATION

A-62241

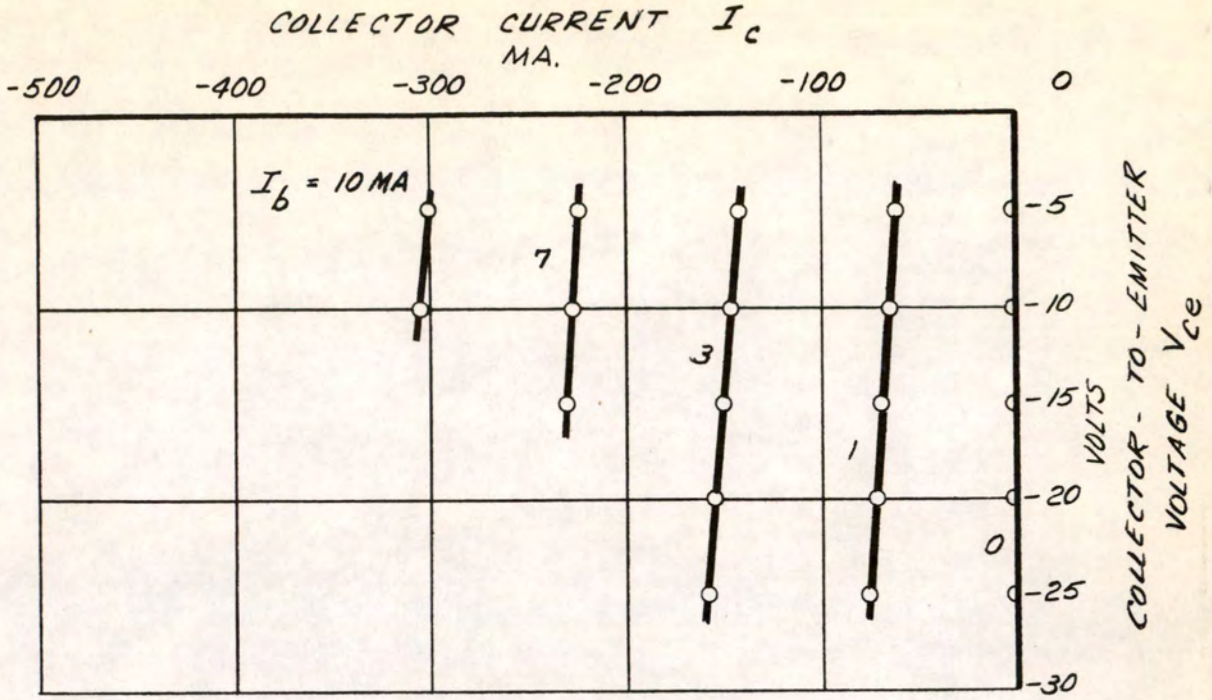


FIG. 3-7

GROUNDED EMITTER CONFIGURATION

CHARACTERISTIC CURVES FOR POWER TRANSISTOR TYPE H-4

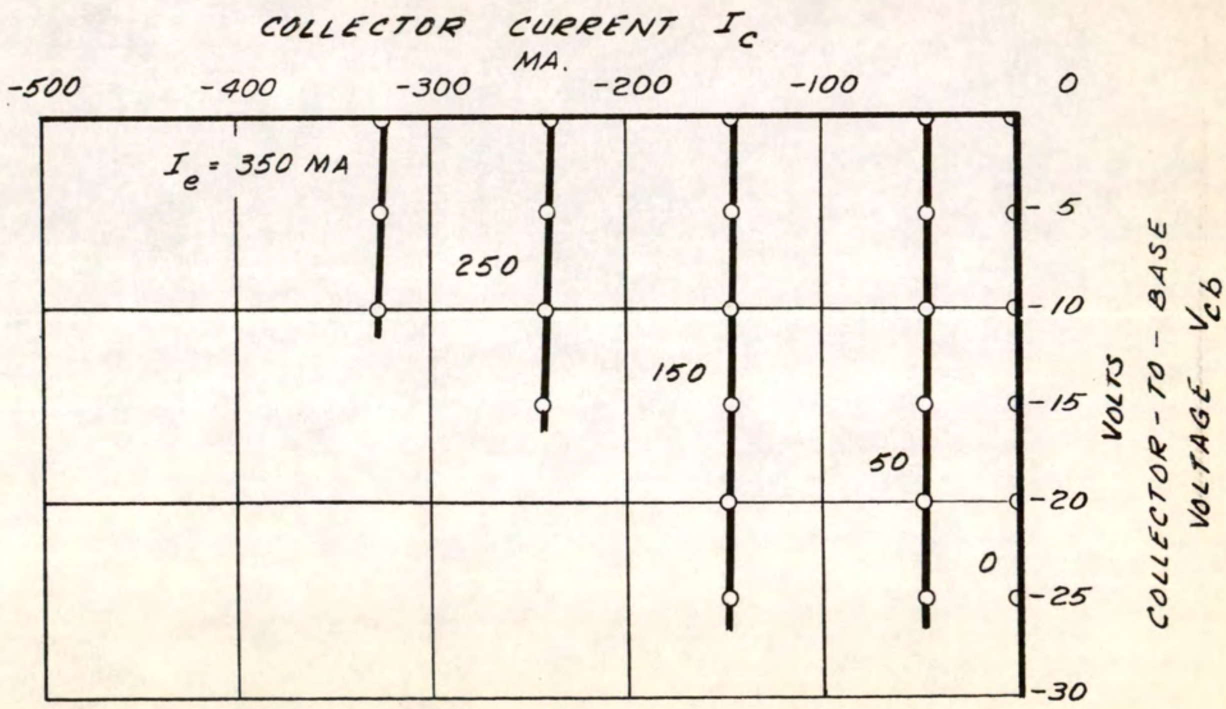


FIG. 3-8

GROUNDED BASE CONFIGURATION

A-62242

braic equations. Under small-signal conditions, the four coefficients in each pair of equations are constants, independent of alternating voltage and current (although they may be quite dependent upon the quiescent direct voltage and current that is applied). These six sets of four coefficients each constitutes a set of small-signal parameters. Note, however, that only four independent parameters actually exist. Corresponding to each set of parameters as described, several different equivalent circuits can be devised; the elements in such circuits also may be considered as parameters. Furthermore, for a transistor, which can be connected in three possible configurations, viz., grounded-base, grounded-emitter, and grounded-collector, the number of possible representations is increased threefold.

Figure 3-9 shows a low-frequency incremental equivalent T circuit applicable when the transistor is operated in the "active" or amplifier mode. This circuit is discussed by Pritchard[24] and Cooper.[25]

The a-c, incremental, or small-signal parameters are:

r_e - incremental emitter resistance

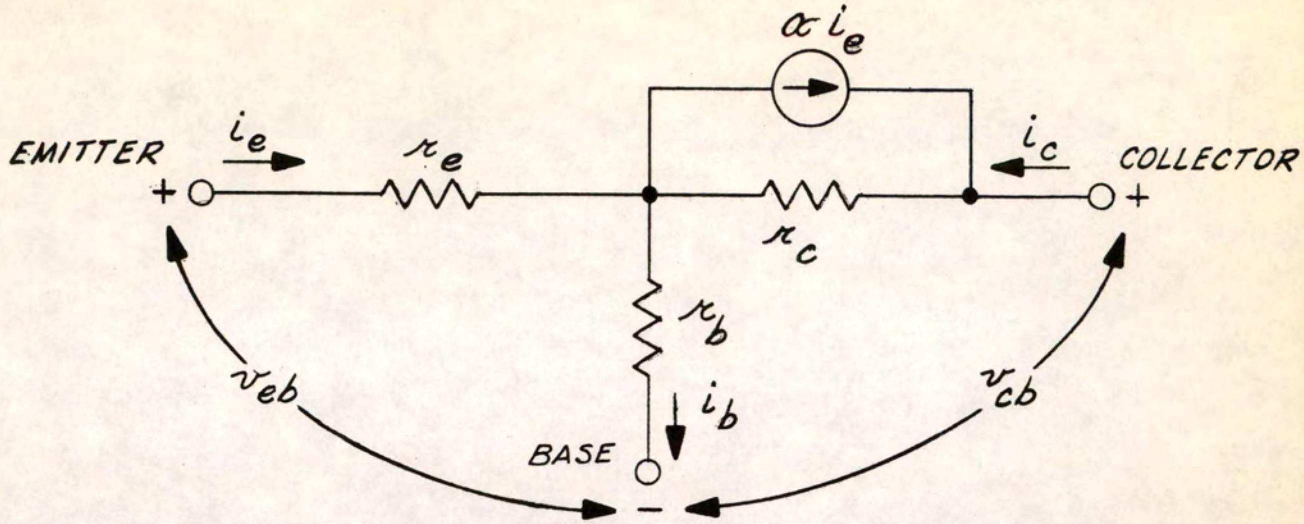
r_b - incremental base resistance

r_c - incremental collector resistance

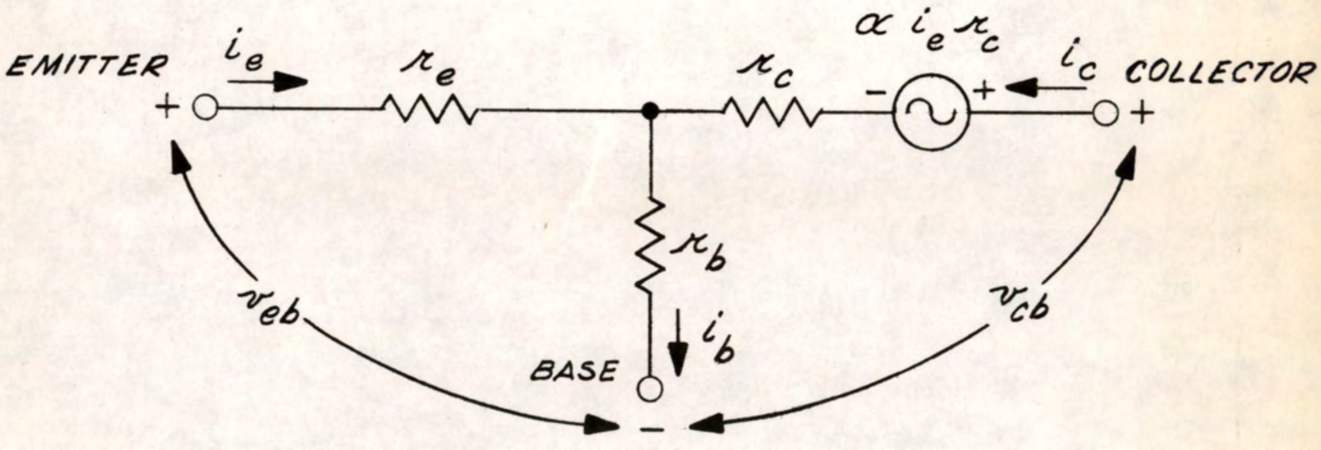
α - current amplification factor.

Methods for measuring the small signal parameters have been outlined by Lehovec,[26] Knight,[27] and Shea[28] and Nussbaum.[29] These methods entail the superposition of a small a-c signal on the biasing current and the measuring of the ratios of a-c currents and voltages. The following expressions are used to calculate the values of r_e, r_b, r_c, and α.

$$r_e = \left. \frac{v_{eb}}{i_c} \right|_{i_b = 0} \tag{3-5}$$



(a.) WITH CURRENT GENERATOR



(b.) WITH VOLTAGE GENERATOR

FIG. 3-9

TRANSISTOR INCREMENTAL EQUIVALENT CIRCUIT

A-62243

$$r_b = \left. \frac{v_{eb}}{i_c} \right|_{i_e = 0} \qquad (3\text{-}6)$$

$$\beta = \frac{\alpha}{1-\alpha} = \left. \frac{i_c}{i_b} \right|_{v_{ce} = 0} \qquad (3\text{-}7)$$

$$r_b + r_c = \left. \frac{v_{cb}}{i_c} \right|_{i_e = 0} \qquad (3\text{-}8)$$

The above methods require that the a-c signals be kept isolated from the d-c biasing supplies. This isolation requires large inductors whose impedance blocks the flow of alternating current but which must be capable of passing the large direct current necessary to bias the transistor into the proper operating region. Nussbaum[29] states that the chokes he used were specially wound. The author did not use the a-c method of measurement because of the unavailability of suitable chokes.

It is possible to vary the direct biasing currents and voltages and to measure the ratios of changes in these quantities. Equations 3-5 to 3-8 can be rewritten in terms of small changes in the direct currents and voltages. The following expressions then define the small signal parameters:

$$r_e = \left. \frac{\Delta V_{eb}}{\Delta I_c} \right|_{I_b = \text{constant}} \qquad (3\text{-}5a)$$

$$r_b = \left. \frac{\Delta V_{eb}}{\Delta I_c} \right|_{I_e = \text{constant}} \qquad (3\text{-}6a)$$

$$\beta = \frac{\alpha}{1-\alpha} = \left. \frac{\Delta I_c}{\Delta I_b} \right|_{V_{ce} = \text{constant}} \qquad (3\text{-}7a)$$

$$r_b + r_c = \left. \frac{\Delta V_{cb}}{\Delta I_c} \right|_{I_e = \text{constant}} \qquad (3\text{-}8a)$$

Figures 3-10 to 3-13 are plots of values of small-signal parameters versus emitter current.

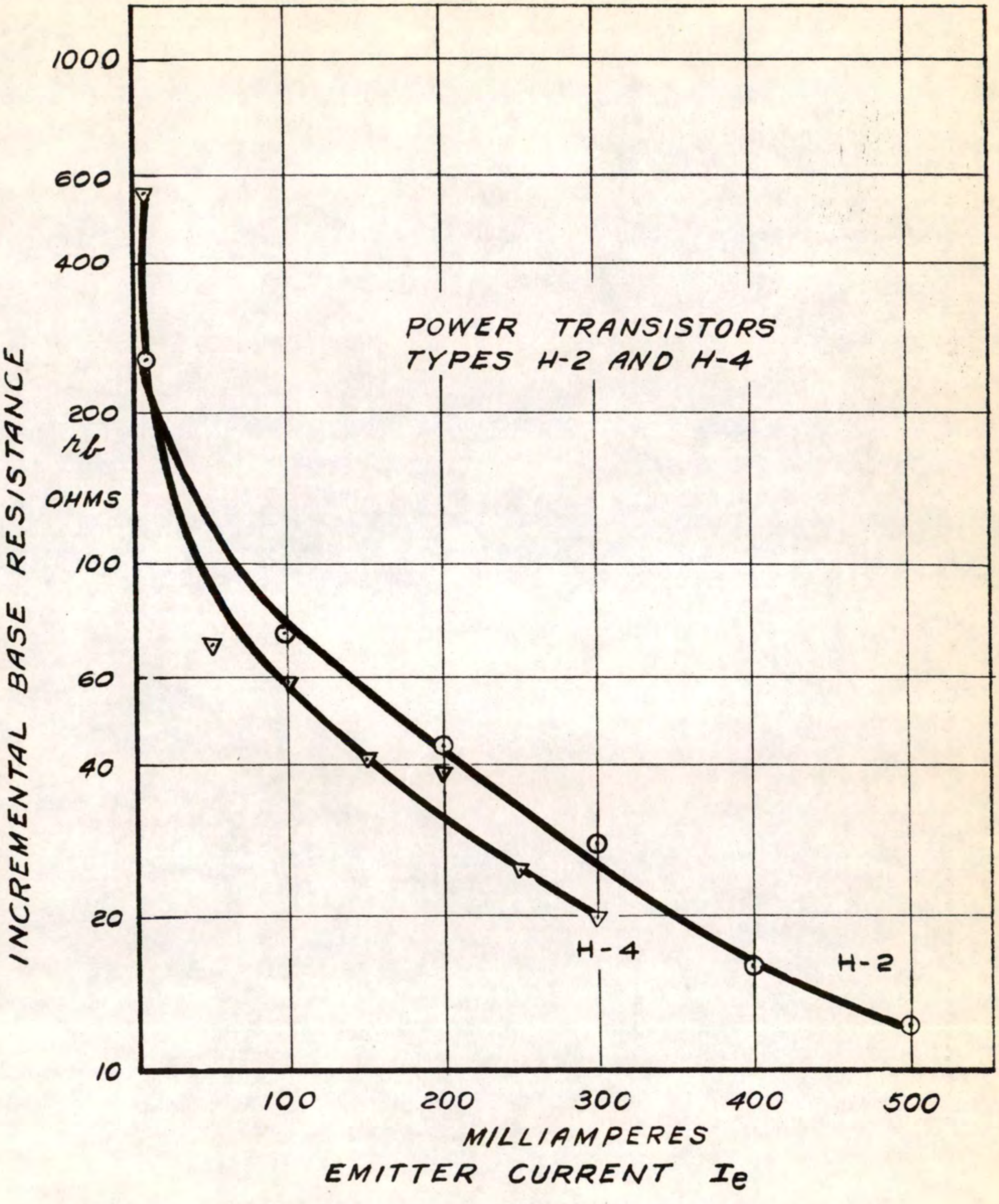


FIG. 3-10
INCREMENTAL BASE RESISTANCE
vs EMITTER CURRENT

A-62244

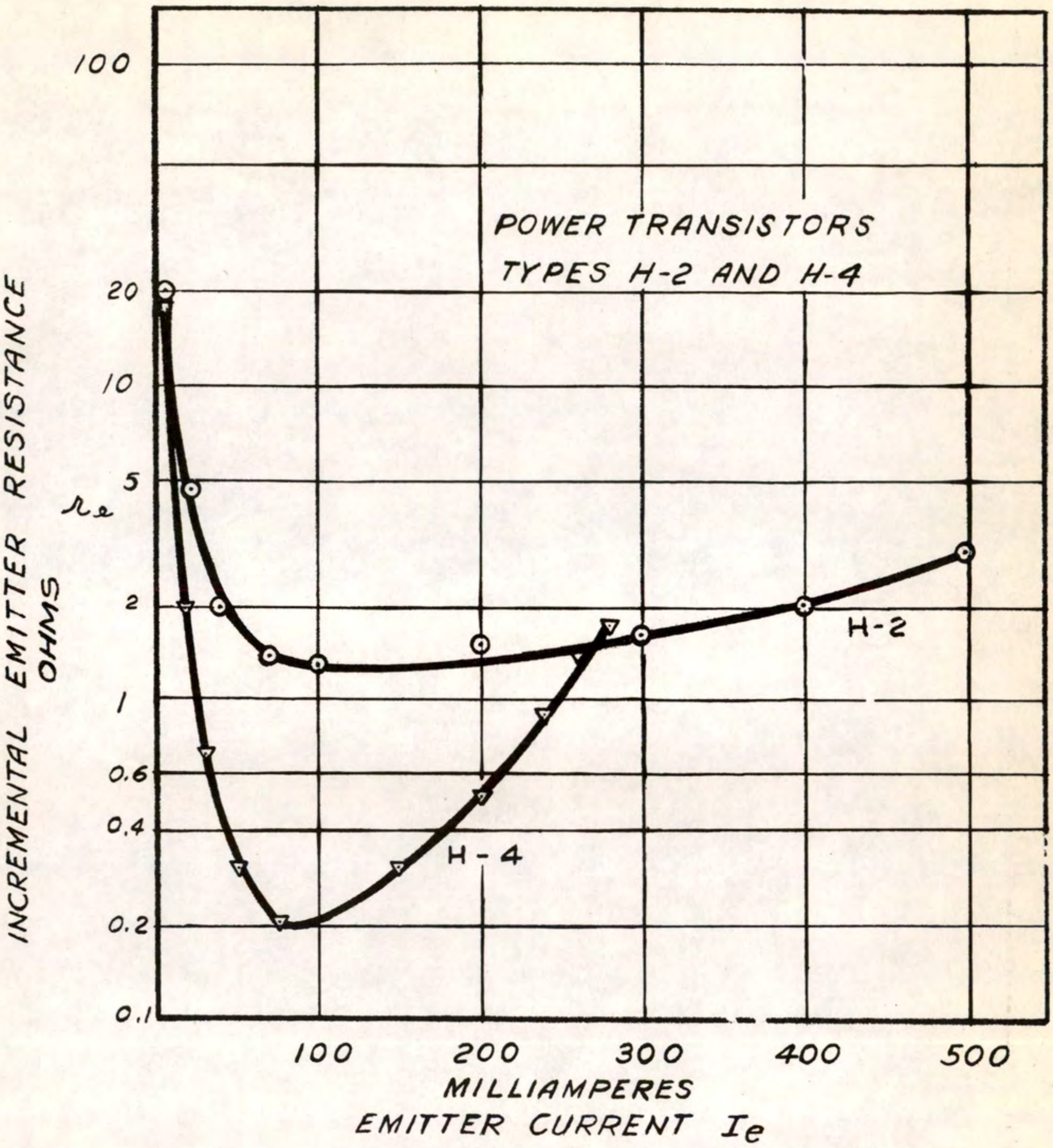


FIG. 3-11

INCREMENTAL EMITTER RESISTANCE vs EMITTER CURRENT

A-62245

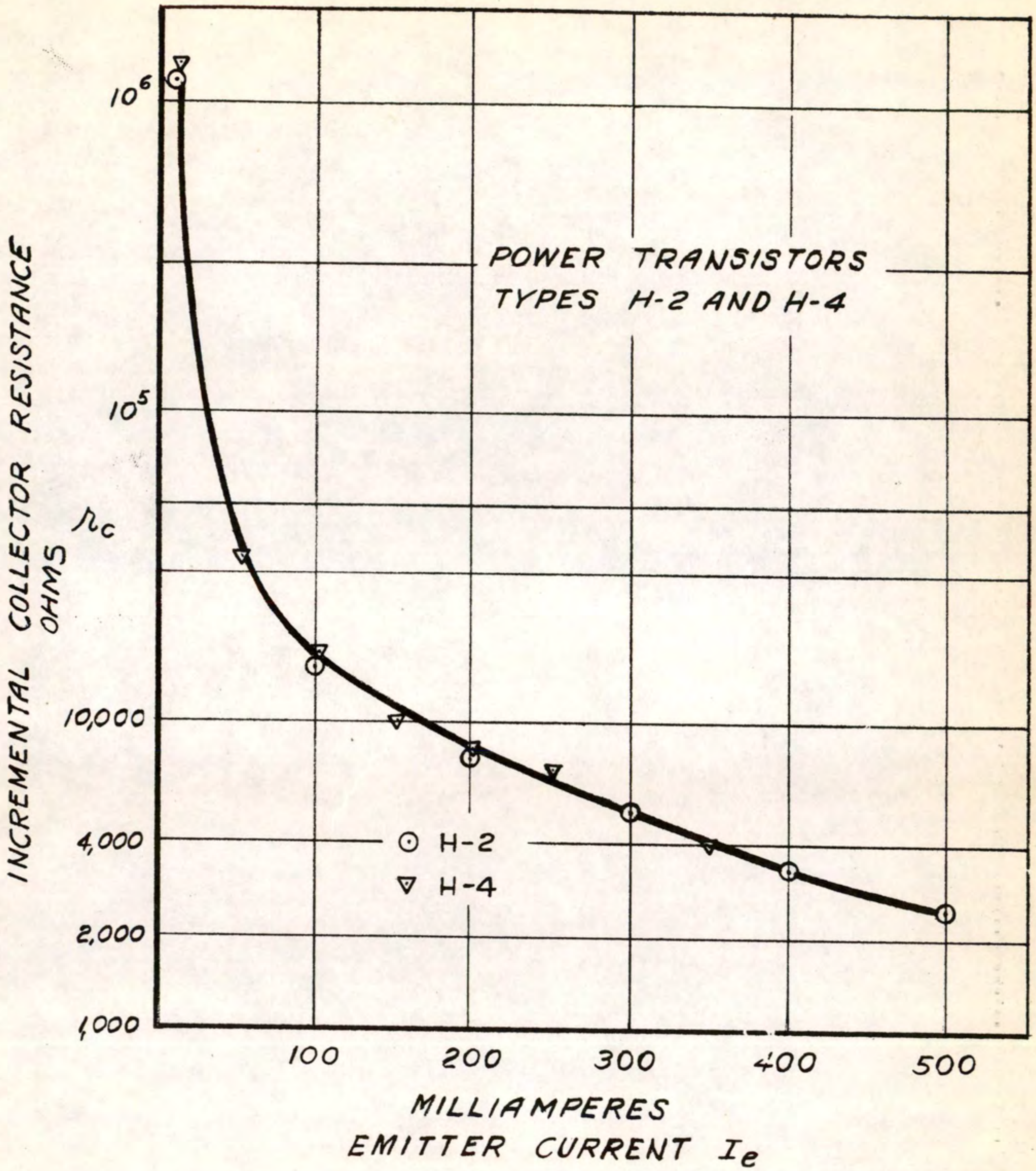


FIG. 3-12
INCREMENTAL COLLECTOR RESISTANCE
vs EMITTER CURRENT

A-62246

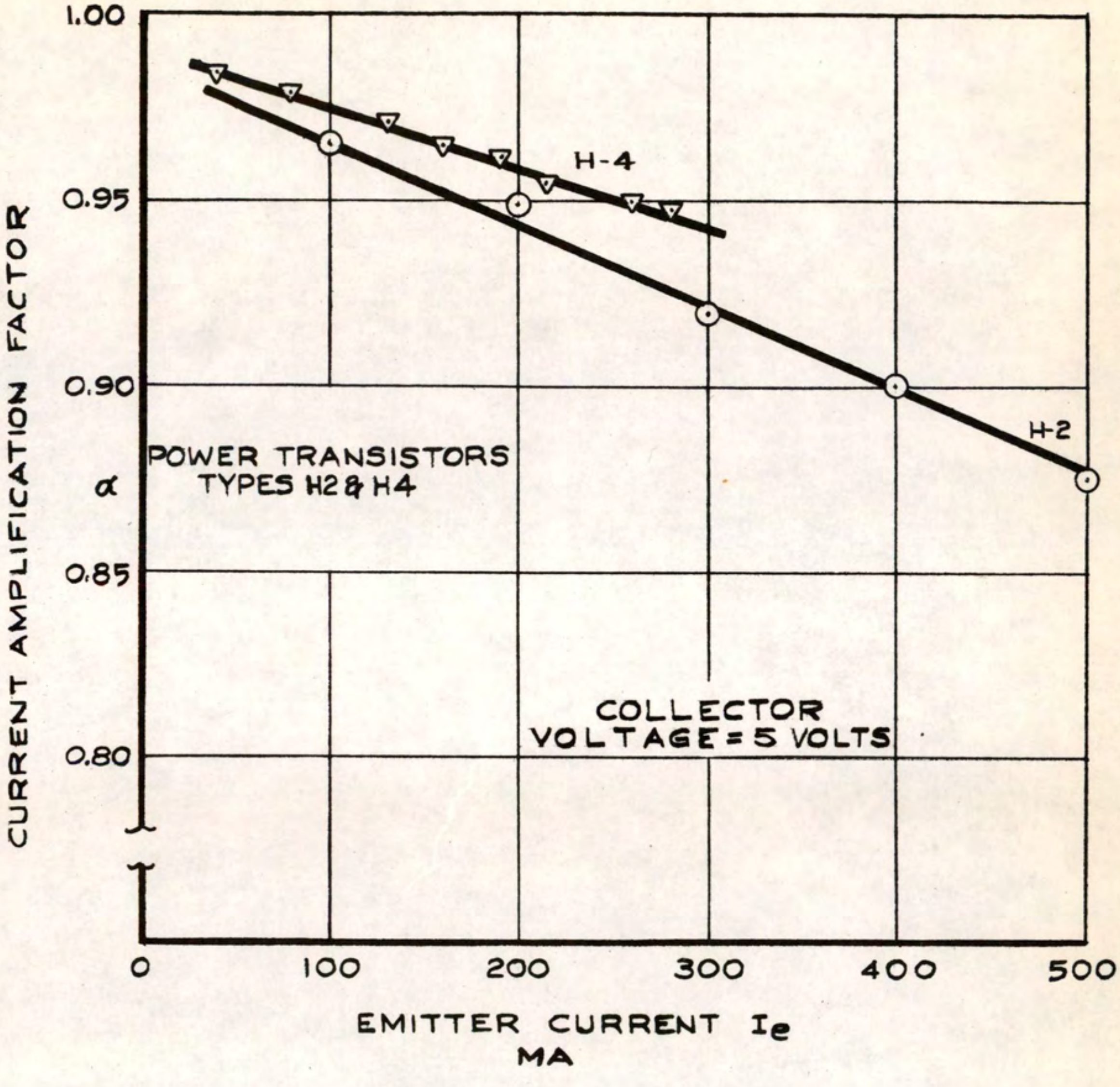


FIG. 3-13

CURRENT AMPLIFICATION FACTOR VS EMITTER CURRENT

A-62181

3.66 Variations of Small-Signal Parameters

The variation of the small signal parameters with operating point, temperature, and frequency has been discussed by many authors, Shea,[28] Nussbaum,[29] Coblenz and Owens,[30] Pritchard,[31] and Webster[32] among others. According to Nussbaum,[29] comparison of experimental data for high power transistors with the theory for low-power transistors shows that for parameters which depend upon frequency, the agreement was quite good, but that differences exist in the case of those parameters which are a function of current. This observation leads to the conclusion that further work must be done on the theory of junction power transistors.

To measure the frequency dependence of the current-amplification factor, α, the author used a commercial transistor tester.† The measurements were made with an emitter current of 1 milliampere at a collector voltage of 5 volts. Figures 3-14 and 3-15 are plots of α versus frequency. For comparison purposes, the analytical expression

$$\alpha = \frac{\alpha_o}{1 + j\frac{f}{f_o}} \tag{3-9}$$

suggested by Shea[28] has been plotted. In Eq. 3-9, α_o is the low frequency current amplification factor and f_o is the α cut-off frequency at which α is 0.707 times its low-frequency value.

3.67 Large-Signal Parameters

An equivalent circuit similar to Fig. 3-9a may be used for large-signal circuit analysis. The large-signal or d-c resistances are given by capital letters: R_e, R_b, and R_c. The d-c α and d-c β are a measure of the ratio of total output caused by a specified total input current

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† Baird Associates Transistor Test Set

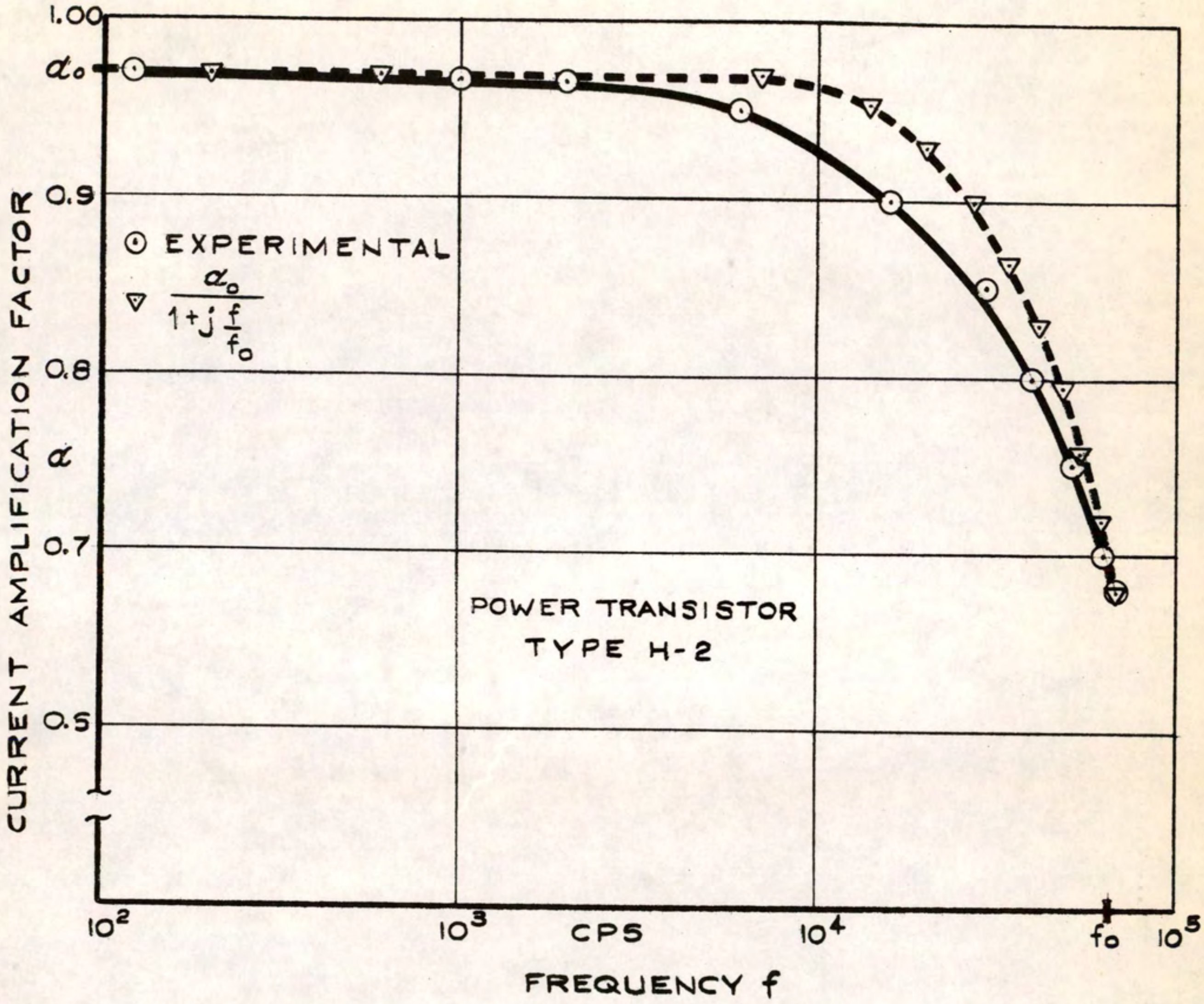

FIG. 3-14

CURRENT AMPLIFICATION FACTOR VS FREQUENCY

A-62182

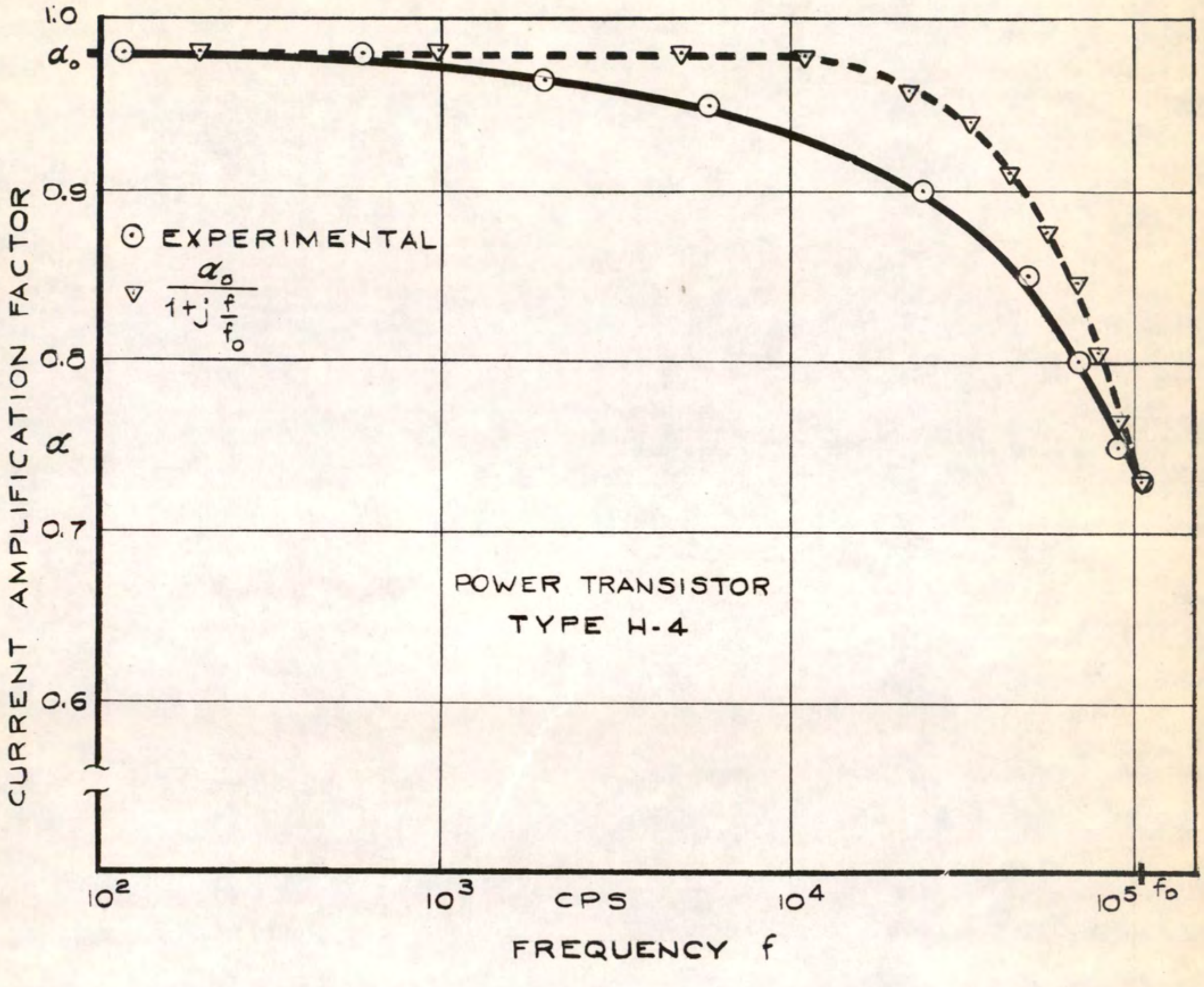

FIG. 3-15

CURRENT AMPLIFICATION FACTOR
VS FREQUENCY

A-62183

and may be defined as follows:

$$\alpha_{dc} = \frac{I_c - I_{co}}{I_e} \bigg|\, V_{cb} = \text{constant} \tag{3-10}$$

$$\beta_{dc} = \frac{I_c - I'_{co}}{I_b} \bigg|\, V_{ce} = \text{constant} \tag{3-11}$$

where I_{co} is the collector-base leakage current with the emitter open circuited, and I'_{co} is the collector-emitter leakage with the base open circuited. Mooers[33] gives the following equations relating α_{dc} and α, β_{dc} and β.

$$\alpha_{dc} = \frac{1}{I_e} \int_0^{I_e} \alpha \, dI_e \tag{3-12}$$

$$\beta_{dc} = \frac{1}{I_b} \int_0^{I_b} \beta \, dI_b \tag{3-13}$$

Recently Gray[34] has investigated the pulse-circuit characteristics of power transistors. He reports on measured values of the large-signal parameters and shows the effects of quiescent current and frequency on the d-c alpha.

3.7 Summary

The results of semiconductor research have been discussed in this chapter. Silicon diodes have been found to be useful both as voltage-reference elements and as rectifiers. Germanium power transistors exhibit good characteristics for use as control elements and comparison amplifiers. In the next chapter these devices will be combined to build a voltage-regulated power supply.

Figure 3-16 shows the semiconductor components which were investigated.

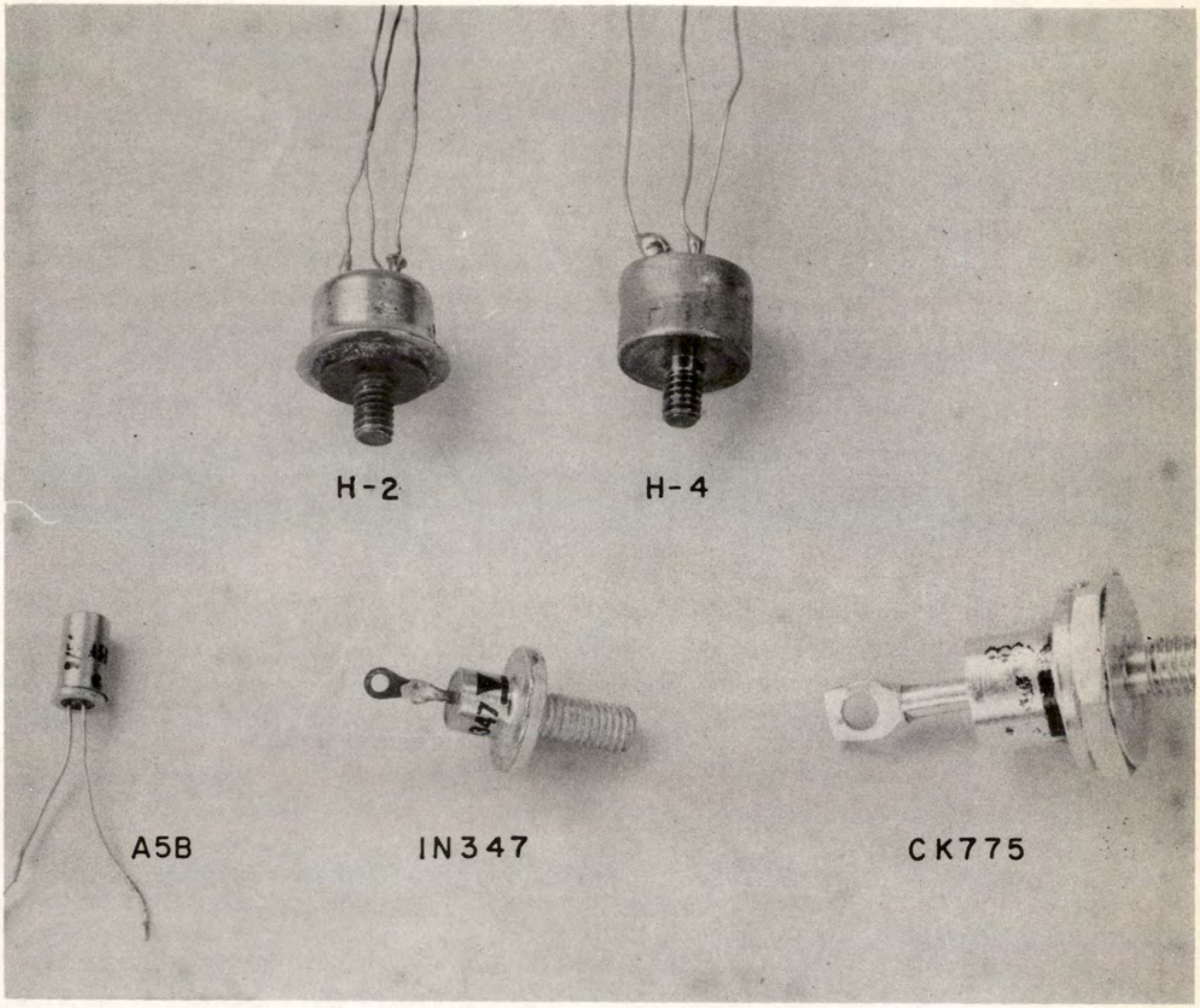

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FIG. 3-16

SEMICONDUCTOR COMPONENTS

A-62184

CHAPTER IV

POWER-SUPPLY DESIGN STUDY

4.1 Elementary Regulator

In this chapter, attention will first be given to the design of an elementary regulator; further development will yield the final semiconductor degenerative regulator. Finally, a rectifier-filter combination to supply power to the regulator will be considered.

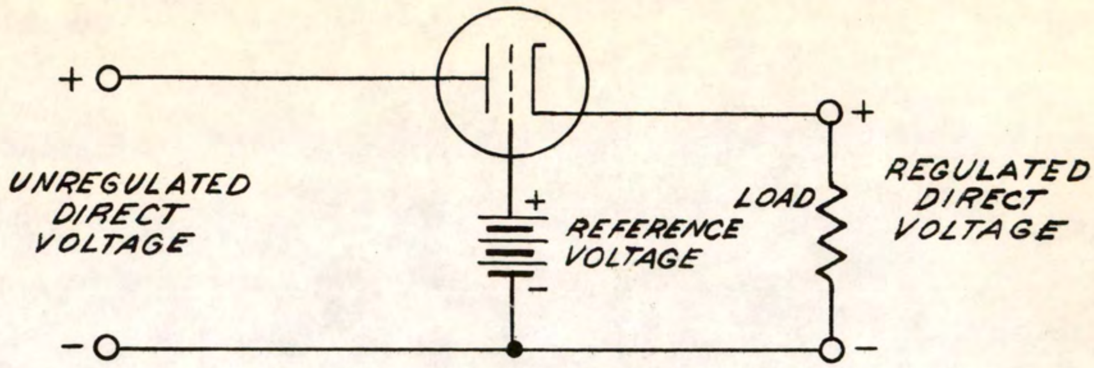
Figure 4-1(a) shows an elementary vacuum-tube regulator. Its operation is merely that of a cathode follower wherein the voltage across the load or cathode resistor is maintained slightly greater than the grid or reference voltage. Figure 4-1(b) shows a circuit suggested by Spencer.[2] It is the exact transistor analog of the vacuum-tube circuit given in Fig. 4-1(a). It is properly called an emitter follower and is similar in operation to the cathode follower. The control element is a pnp transistor and the regulated voltage must therefore have its positive terminal grounded. The maximum current which can be supplied by this circuit is limited by the maximum collector current of the transistor.

4.11 Performance Analysis

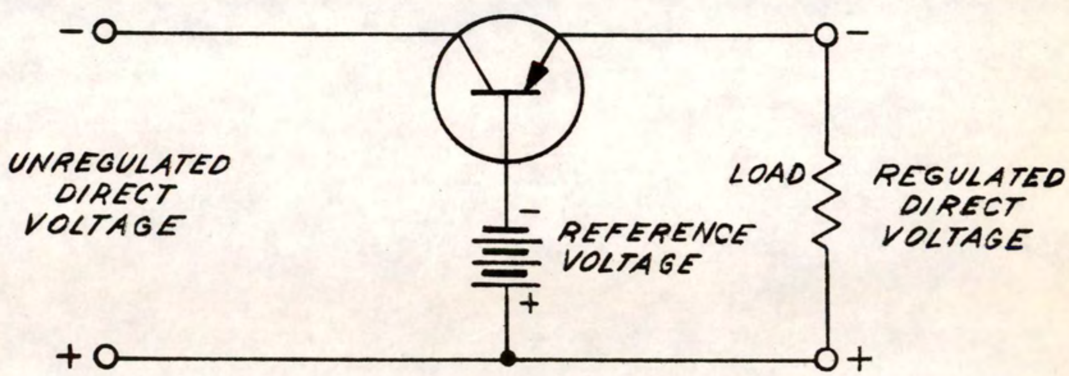
The incremental equivalent circuit for the transistor, Fig. 3-9(b), may be used to obtain analytical expressions for stabilization factor, F, and internal impedance, R_o, of the elementary transistor voltage regulator, Fig. 4-1(b). Appendix A shows these calculations. The exact expressions are:

$$F = \frac{r_b}{r_b + r_c} \tag{4-1}$$

$$R_o = \frac{r_e r_b + r_c\left[r_e + r_b(1-\alpha)\right]}{r_b + r_c} . \tag{4-2}$$



(a)
VACUUM TUBE
VOLTAGE REGULATOR



(b)
TRANSISTOR
VOLTAGE REGULATOR

FIG. 4-1
ELEMENTARY VOLTAGE REGULATORS

A-62256

From experimental data, the following inequalities

$$r_c \gg r_b > r_e \tag{4-3}$$

may be used to obtain approximations to expressions such as Eqs. 4-1 and 4-2. Different approximations may be required for different operating points of the transistor. Careful consideration should be given to the exact expression before making an approximation which is to apply throughout a wide range of operation. The approximations for Eqs. 4-1 and 4-2 are

$$F \approx \frac{r_b}{r_c} \tag{4-1a}$$

$$R_o \approx r_e + r_b\,(1-\alpha). \tag{4-2a}$$

From Eq. 4-1a, one can see that transistors with low r_b make the best regulators. Since the small-signal parameters have been shown to vary with the operating point of the transistor, the values of F and R_o can be expected to vary with different magnitudes of emitter or load current. The minimum value of R_o for a circuit using a high-alpha transistor would be r_e (from Eq. 4-2a).

4.12 Diode Reference

A silicon diode can readily be used to supply the reference voltage in the regulator circuit, Fig. 4-1b. This modification is shown in Fig. 4-2.

Consider this circuit with the load terminals open-circuited, i.e. with $I_o = 0$. Since I_{co} is small, it is neglected in all analyses of this and succeeding chapters. Neglecting I_{co}, one finds that $I_b = 0$ and $I = I_r$. One can then choose a value of I_r which operates the reference diode in the

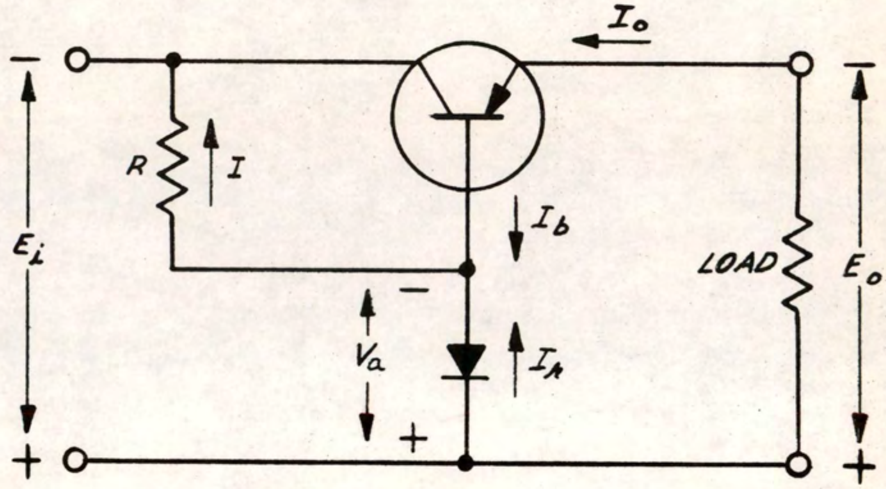


FIG. 4-2

ELEMENTARY VOLTAGE REGULATOR
WITH SILICON-DIODE
REFERENCE ELEMENT

A-62257

constant-voltage region of its reverse characteristics. The value of resistance, R, necessary to produce this current may be calculated from

$$R = \frac{E_i - V_a}{I_r} . \tag{4-4}$$

With the load reconnected and some load current, I_o, being delivered, the base current I_b, of the transistor may be calculated from the expression:

$$I_b = I_o\ (1-\alpha_{dc}) . \tag{4-5}$$

The reference current, I_r, can now be found from the node equation:

$$I_r = I - I_b = I - I_o\ (1 - \alpha_{dc}) \tag{4-6}$$

As the load resistance is decreased, I_o increases. Therefore, from Eq. 4-5, I_b increases. This increase of I_b is further enhanced by the fact that α_{dc} tends to decrease with increasing emitter current, I_o.[34]

4.13 Circuit Limitations

From the foregoing discussion, it can be seen that for some value of load current, I_r is reduced to zero. Figure 3-1 shows that the diode is no longer a voltage reference at reverse currents less than 2 milliamperes. In a circuit constructed by the author using an H-2 power transistor and a 6-volt reference diode, I_r was reduced to 2 ma. at a load current of 300 ma. This circuit thus does not permit load currents near the maximum current rating of the control element.

At first glance, a solution appears to be the selection of a small value of R to produce a large no-load value for I_r. Unfortunately the 150-milliwatt power rating of the 6-volt reference diode limits the maximum value of I_r to 25 milliamperes.

The limitations of this circuit do not justify a detailed analysis of it, but one could use Eqs. 4-1a and 4-2a as approximations of its performance. However, the base resistance, r_b, must now be increased by the parallel combination of R and dynamic resistance, r_s, of the reference diode.

4.2 Degenerative Regulator

To decrease the internal impedance and extend the range of permissible load current, a negative feedback loop can be introduced. The resulting device is a degenerative regulator. Figure 4-3 shows a degenerative vacuum-tube regulator and its transistor equivalent. Tube V_2 and its transistor counterpart Q_2 provide a stage of amplification between the sampling circuit and the regulating or control elements.

4.21 Analysis of Limitations

Consider the circuit of Fig. 4-4 under no load. Then $I_o = 0$ and $I_{b2} = I_{e1}$. The following relations among the currents can be seen:

$$I_{b1} = I_{e1}\,(1-\alpha_{dc1}) = I_{b2}(1-\alpha_{dc1}) \qquad (4\text{-}7)$$

$$I_{b2} = I_r\,(1-\alpha_{dc2}) \qquad (4\text{-}8)$$

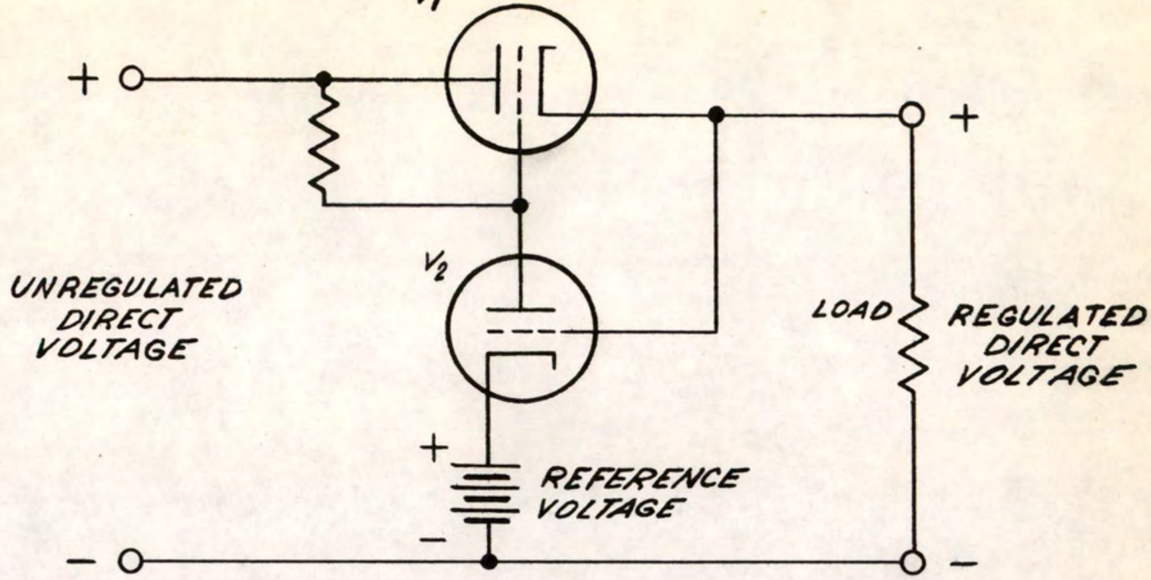
$$I_{c2} = I - I_{b1} \qquad (4\text{-}9)$$

$$I_{c2} = I - I_{b2}\,(1-\alpha_{dc1}) = I - I_r(1-\alpha_{dc1})\,(1-\alpha_{dc2}) \qquad (4\text{-}10)$$

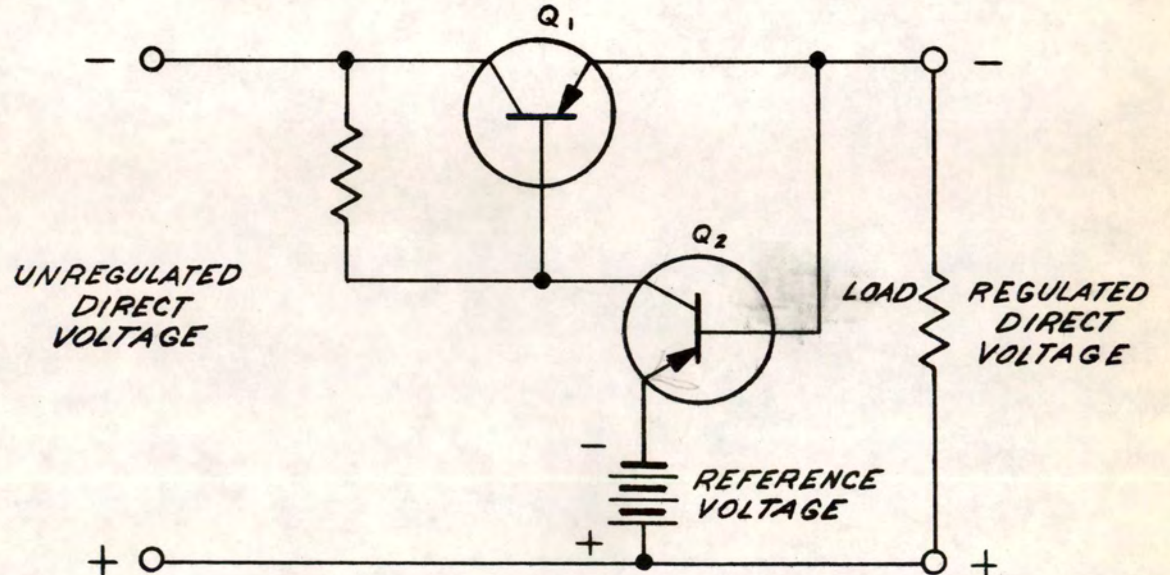
$$I_r = \frac{I_{c2}}{\alpha_{dc2}} = \frac{I - I_r(1-\alpha_{dc1})(1-\alpha_{dc2})}{\alpha_{dc2}} \qquad (4\text{-}11)$$

Solving for I_r:

$$I_r = \frac{I}{\alpha_{dc2} + (1-\alpha_{dc1})(1-\alpha_{dc2})} = \frac{I}{1 - \alpha_{dc1} + \alpha_{dc1}\,\alpha_{dc2}} \qquad (4\text{-}12)$$



(a)
VACUUM TUBE
VOLTAGE REGULATOR



(b)
TRANSISTOR
VOLTAGE REGULATOR

FIG. 4-3

DEGENERATIVE VOLTAGE REGULATORS

A-62258

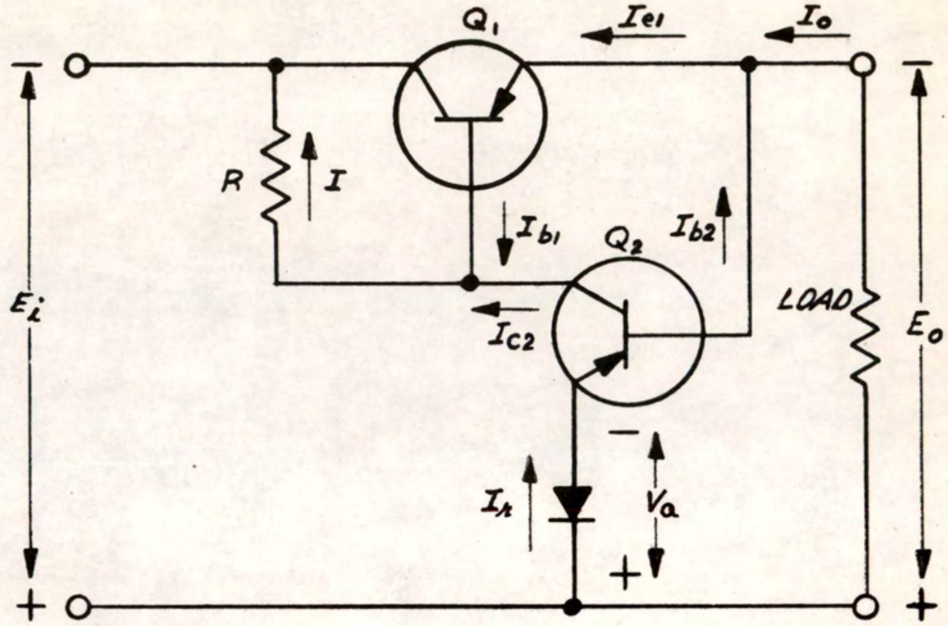


FIG. 4-4

DEGENERATIVE VOLTAGE REGULATOR
WITH SILICON-DIODE
REFERENCE ELEMENT

A-62259

Again one may choose an operating current, I_r, for the reference diode and using Eq. 4-12, determine the current, I, necessary to produce it. The value of the resistor, R, may be determined from the approximate relation:

$$R \approx \frac{E_i - V_a}{I} \quad . \qquad (4\text{-}13)$$

Under load conditions, the emitter current in transistor Q_1 is

$$I_{e1} = I_o + I_{b2} \ . \qquad (4\text{-}14)$$

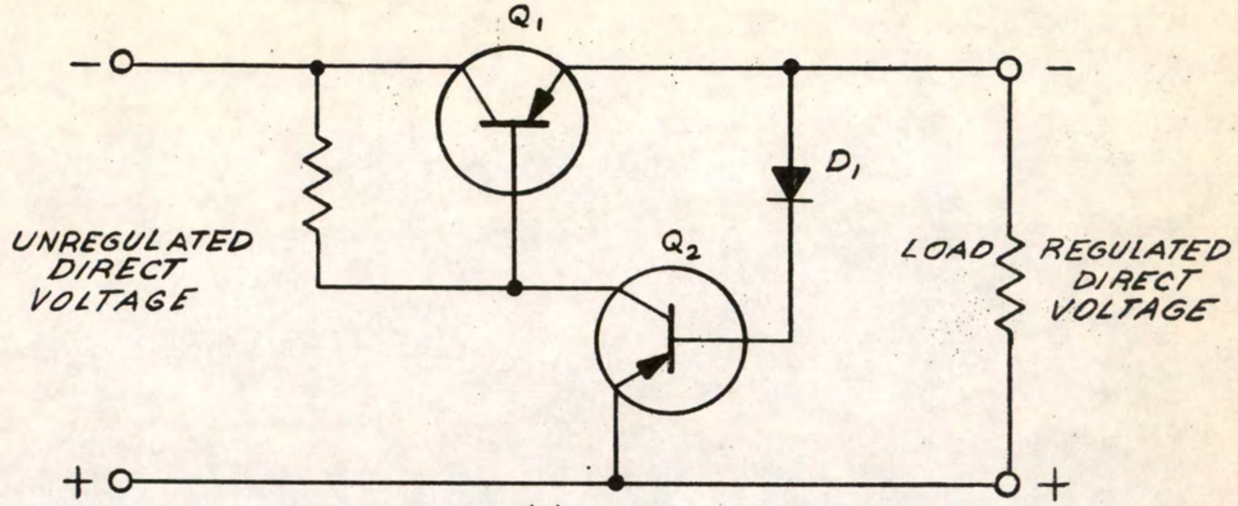
By steps similar to Eqs. 4-7 to 4-11, a new expression for the reference current is obtained.

$$I_r = \frac{I - I_o(1-\alpha_{dc1})}{\alpha_{dc2} + (1-\alpha_{dc1})(1-\alpha_{dc2})} = \frac{I - I_o(1-\alpha_{dc1})}{1 - \alpha_{dc1} + \alpha_{dc1}\,\alpha_{dc2}} \qquad (4\text{-}15)$$

The assumption that the current I remains essentially constant during load conditions is borne out by experiment. Equation 4-15 shows that the circuit of Fig. 4-4 has the same type of limitation that the simple regulator of Fig. 4-2 exhibited, i.e. the reference current is reduced during loading by a component of the load current. This fact, in addition to limiting the maximum current the circuit can furnish, adversely affects the load regulation in the useful current range. The latter effect is attributable to variations in reference current (and therefore small variations in reference voltage) caused by changes in load current.

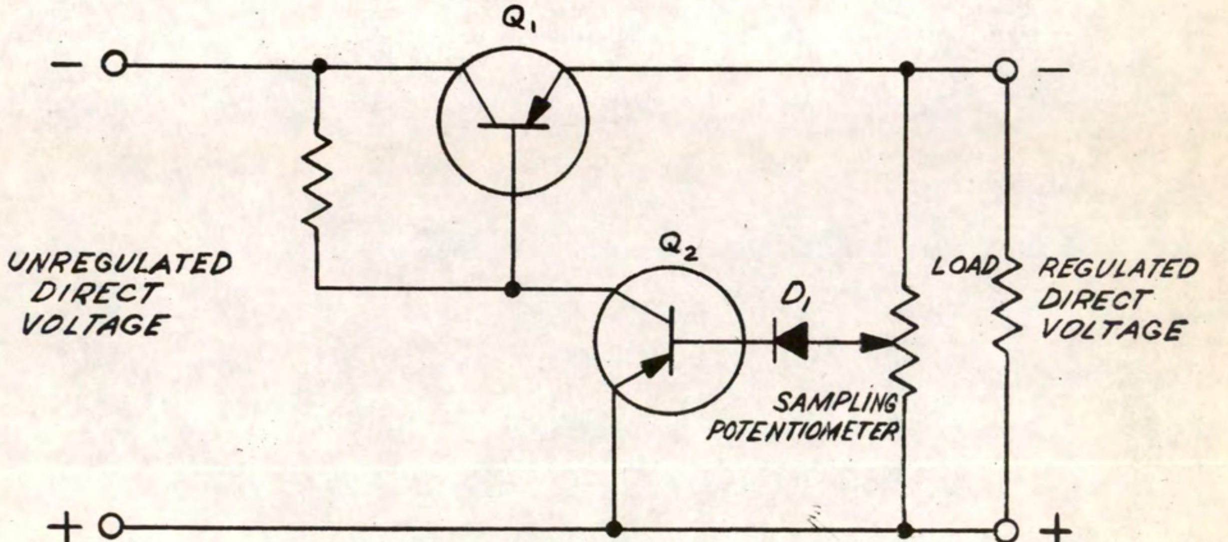
4.3 Improved Degenerative Regulator

A solution to the two problems mentioned in the last paragraph is found in the circuit shown in Fig. 4-5(a). Here, the reference diode has been moved from the emitter to the base circuit of the transistor used as a comparison amplifier. The total current, I_r, and the incremental current, i_r, in the reference element of Fig. 4-4 are now



(a)

WITHOUT VARIABLE VOLTAGE ADJUSTMENT



(b)

WITH VARIABLE VOLTAGE ADJUSTMENT

FIG. 4-5

IMPROVED DEGENERATIVE REGULATORS

A-62260

multiplied by $(1-\alpha_{dc2})$ and $(1-\alpha_2)$ respectively for the circuit of Fig. 4-5. This artifice thus can extend the maximum current of the regulator and improve the load regulation.

Figure 4-5(b) shows how a sampling potentiometer may be used to provide variable output voltage at values greater than the reference voltage. One foresees, however, a possible difficulty with either circuit of Fig. 4-5. If the value of α_{dc2} is too near unity, insufficient current will exist in the reference diode to operate it in its constant-voltage region.

4.4 Semiconductor Regulator

The introduction of several milliamperes of biasing current to insure the operation of the reference diode in its avalanche region completes the design of the semiconductor regulator. The addition of resistor R_2 to the circuit accomplishes this biasing and yields the final circuit shown in Fig. 4-6. The sampling potentiometer has been omitted because the resistance which this circuit adds to the incremental resistance of the voltage reference cannot be tolerated.

4.41 Design Analysis

Consider the circuit of Fig. 4-6 under no load and without resistor R_2. Then $I_o = I_2 = 0$. The following relation is true:

$$I_{b2} = I_r = I_{e1} \tag{4-16}$$

Under this condition, an expression similar to Eq. 4-12 can be obtained which imposes a limitation on emitter current I_{e2} as follows:

$$I_{e2} = \frac{I_1}{\alpha_{dc2} + (1-\alpha_{dc1})(1-\alpha_{dc2})} = \frac{I_1}{1 - \alpha_{dc1} + \alpha_{dc1}\alpha_{dc2}} \tag{4-17}$$

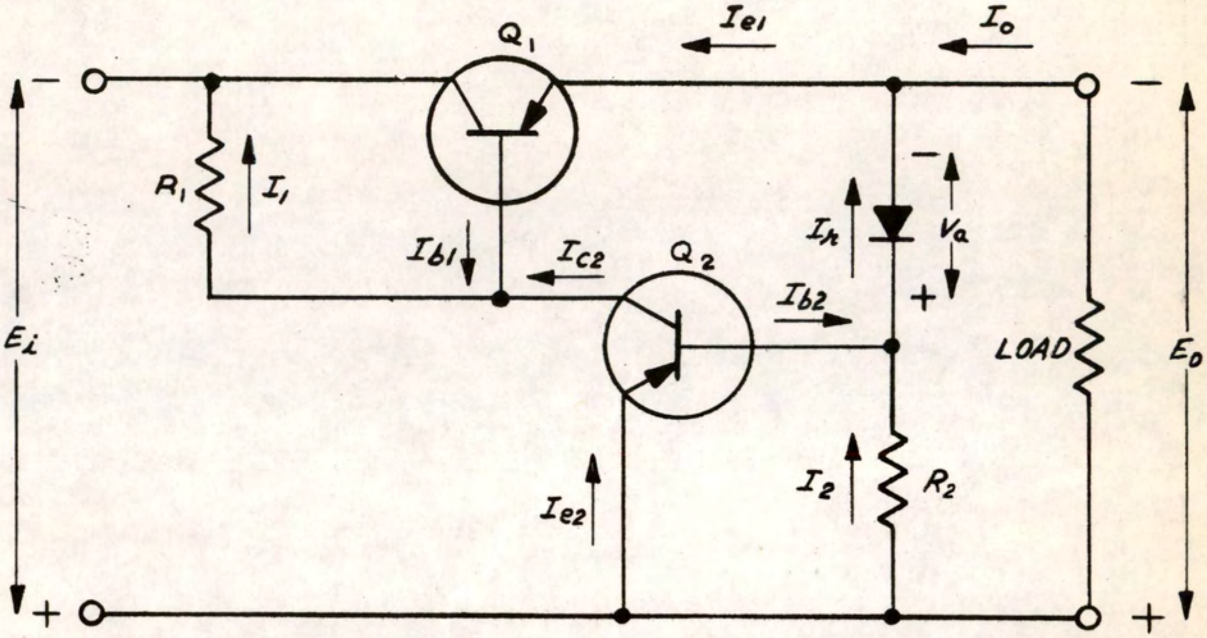


FIGURE 4-6

SEMICONDUCTOR REGULATOR

A-62261

Equation 4-17 shows that I_{e2} can never be negative at no load. A design choice for resistor R_1 may be estimated from

$$R_1 \approx \frac{E_i - V_a}{I_1} \tag{4-18}$$

since under proper operating conditions, the collector-to-ground voltage of transistor Q_2 is approximately equal to V_a. The reference current may be calculated from

$$I_r = I_{e2}(1-\alpha_{dc2}) . \tag{4-19}$$

If this value of current is not sufficiently high to operate the diode as a voltage reference, the resistor R_2 may be inserted to provide a current I_2 which will bias the diode in the proper operating region since

$$I_r = I_{b2} + I_2 . \tag{4-20}$$

To determine emitter current I_{e2} under load conditions, the emitter current of transistor Q_1 is recognized as becoming:

$$I_{e1} = I_o + I_r = I_o + I_{b2} . \tag{4-21}$$

An expression similar to Eq. 4-15 can then be obtained for I_{e2}

$$I_{e2} = \frac{I_1 - I_o(1-\alpha_{dc1})}{\alpha_{dc2} + (1-\alpha_{dc1})(1-\alpha_{dc2})} = \frac{I_1 - I_o(1-\alpha_{dc1})}{1 - \alpha_{dc1} + \alpha_{dc1}\,\alpha_{dc2}} . \tag{4-22}$$

At first glance, Eq. 4-22 would seem to indicate the presence of the same difficulty previously encountered, for at some value of load current, I_{e2} and therefore I_r is reduced to zero. Such would be the case if a 150 milliwatt transistor were used for Q_2. By using a power transistor, a value of R_1 may be chosen so that a sufficiently large value of current,

I_1, flows under no-load conditions. Then the maximum load current permitted by the collector-current rating of transistor Q_1 may be furnished without reducing I_{e2} to zero.

Care must be taken, however, that too large a value of current, I_1, does not exist under no-load conditions. Equations 4-17 and 4-19 indicate the danger of exceeding the power rating of the reference diode. If R_1 is chosen too small, the reference current, I_r, which may be satisfactory when the power supply is loaded would become excessive were the load removed.

Resistor R_2 may not be needed if α_{dc2} is sufficiently low. Its value is best determined by cut-and-try methods in the laboratory.

4.42 Performance Analysis

An incremental equivalent circuit, Fig. 3-9(b) may be used to obtain analytical expressions for the stabilization factor, F, and internal resistance, R_o, of the degenerative regulator of Fig. 4-6. Appendix B gives the results of these calculations. Since the results are lengthy, they will not be repeated here.

4.5 Rectifier Design

The design of the source of filtered direct voltage for the regulator is of considerable importance, in that a properly designed rectifier-filter combination can greatly improve the load regulation of the power supply.

An internal impedance for the rectifier-filter can be defined in a manner similar to Eq. 2-1 where the voltages and currents involved are the input and output quantities of the rectifier-filter circuit. This impedance, R_o' is defined by

$$R_o' = -\frac{\partial E_i}{\partial I_o}\bigg|_{E_s = \text{constant}} \qquad (4\text{-}23)$$

where E_i = output voltage of the rectifier
= input voltage to the regulator

I_o = output or load current

E_s = alternating voltage supplied to the rectifier.

The above definition assumes that changes in load current produce equal changes in the current drawn from the rectifier. Eq. 4-23 may be rewritten using the differential form

$$dE_i = - R_o' \, dI_o. \qquad (4\text{-}24)$$

Substitution of Eq. 4-24 into Eq. 2-5 yields

$$dE_o = - FR_o' \, dI_o - R_o \, dI_o \qquad (4\text{-}25)$$

Division of Eq. 4-25 by dI_o yields an expression which is defined as the internal impedance, R, of the entire power supply.

$$\frac{dE_o}{dI_o} = - (FR_o' + R_o) = - R. \qquad (4\text{-}26)$$

Equation 4-26 yields the useful expression

$$R = FR_o' + R_o \qquad (4\text{-}27)$$

To keep the value of R_o' small, a full-wave rectifier with capacitor filter is chosen. An RC pi filter cannot be used because the series resistance would greatly increase the internal resistance, R_o'.

Silicon power rectifiers are ideally suited for this application because of their small slope resistance. The voltage drop across the rectifier is about one volt and remains essentially constant over a large range of load currents. In the choice of a rectifier, proper consideration should be given to peak inverse voltage and peak surge current.

Given a value of maximum peak-to-peak ripple voltage, E_r, allowable at the output of the power supply, one can calculate the maximum peak-to-peak ripple voltage, E_r', allowable at the input to the regulator from the following

$$E_r' = \frac{E_r}{F} \tag{4-28}$$

If one considers the ripple voltage of a full-wave rectifier with capacitor filter to be a linear rise and fall, i.e. a "sawtooth" wave, the peak-to-peak value of this voltage is given by Seely[11] to be approximately:

$$E_r' \approx \frac{I_o}{2fC} \tag{4-29}$$

where f is the frequency of the alternating voltage supplied to the rectifier
C is the value of filter capacitance.

Equations 4-28 and 4-29 allow one to compute the value of filter capacitor necessary at full load current to maintain the output ripple below a prescribed level.

4.6 Summary

The design study of an all-semiconductor power supply, including rectifier and regulator, has been described. The components investigated and reported on in Chapter III have been found theoretically at least to fulfill the needs for this supply. The construction and testing of this system will be described in the next chapter.

CHAPTER V

EXPERIMENTAL RESULTS

5.1 Regulator Construction

The circuit shown in Fig. 4-6 was constructed using power transistor types H-2 and H-4 for Q_1 and Q_2 respectively. Silicon diode type A5B was used for the reference element. The reason for the choice of a design-center input voltage of 20 volts will become apparent in succeeding paragraphs. With E_i = 20 volts, and I_o = 0, a value of resistance for R_1 of 140 ohms produced a collector current, I_{c2}, of 100 milliamperes. The reference current, I_r, was set to 15 milliamperes by a value of 24 ohms for R_2. When a load current of 700 milliamperes was drawn from the regulator, I_{c2} was reduced to 3 milliamperes and I_r to 5 milliamperes. An output voltage of 6.40 volts was obtained at a load current of 400 milliamperes.

5.2 State Diagram

Before measuring the parameters which characterize the performance of the regulator, one must determine the limits on input voltage and output current within which the regulator will operate satisfactorily. Figure 5-1 shows a plot defining the permissible range of E_i as a function of I_o for the regulator of Fig. 4-6. Before measurements were made for the plotted points, two constraints were drawn. The first was a vertical line drawn through the value of maximum collector current for transistor Q_1 (800 milliamperes for type H-2). The second, that of collector dissipation, was determined as follows: the reference element was a six-volt diode (see Sect. 3.23: diode No. 1) and therefore the base-to-ground

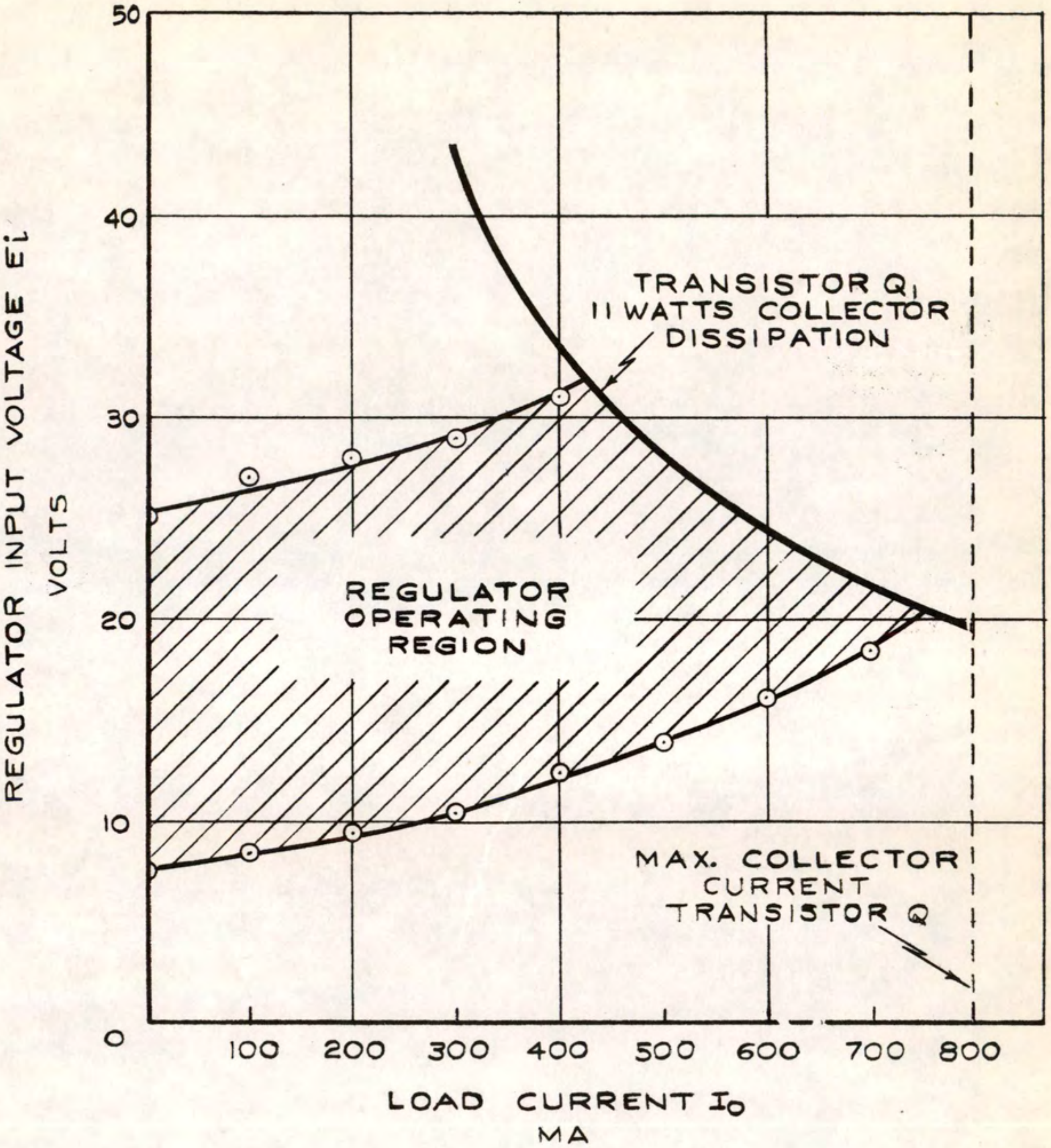


FIG. 5-1

REGULATOR STATE DIAGRAM

A-63531

voltage of transistor Q_1 was approximately six volts. With the conservative approximation that $I_{c1} \approx I_o$, one may determine collector dissipation, P, in transistor Q_1 by the equation

$$P = (E_i - 6)I_o \ . \tag{5-1}$$

Figure 3-4 indicates the maximum collector dissipation for the type H-2 transistor to be 11 watts. The second constraint on Fig. 5-1 is thus the hyperbola obtained by plotting Eq. 5-1 with P equal to 11 watts. The collector-to-base voltage of transistor Q_2 is approximately equal to the reference voltage, 6 volts. Since its maximum collector current, I_{c2}, is 100 milliamperes, there is no constraint imposed by the collector dissipation in this transistor.

The experimental data on Fig. 5-1 are plotted in the form of a so-called "state" diagram. The upper curve represents a measured current in the reference diode of 20 milliamperes, or 120 milliwatts of power dissipation. The lower curve is for 5 milliamperes of reference current, I_r. The author considered this current a conservative minimum to insure the operation of the diode in the linear avalanche region. The shaded area of Fig. 5-1 therefore represents the area of operating points or states in which the regulator gives satisfactory operation. Note that the contours do not represent the locus of points for which regulation ceases entirely.

The choice of design-center input voltage of 20 volts is obvious from Fig. 5-1. Only at this voltage may a load current of 700 milliamperes be drawn and still permit regulation. A vertical line drawn through I_o = 700 milliamperes intersects the collector dissipation constraint and the lower state curve with an intercept spacing of 3 volts. This value represents the maximum allowable sum of peak-to-peak ripple voltage and changes in E_i due to varying line voltage at the input to the regulator.

5.3 Regulation Curves

Within the limits of regulation established from the state diagram, tests may be made on the regulator without danger of exceeding maximum ratings of the components.

5.31 Predicted Performance

By means of Eqs. B-7 and B-8, one can predict the values of stabilization factor and internal impedance which will be obtained from tests on the circuit of Fig. 4-6. The calculated values are:

$F = 0.0188$

$R_o = 0.33$ ohm

5.32 Performance Data

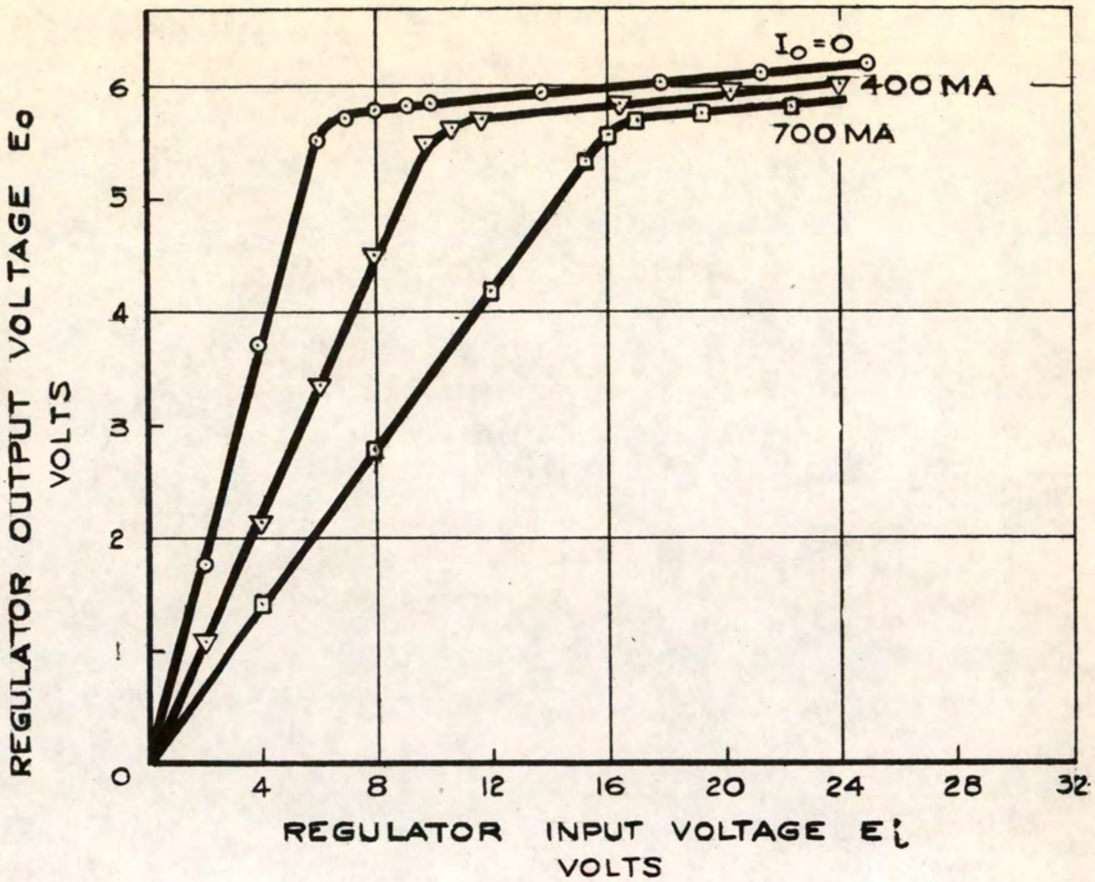
Figure 5-2(a) shows a family of curves plotting output voltage, E_o, versus input voltage, E_i, for the circuit of Fig. 4-6 with load current as parameter. In plotting the curves for various values of load current, care was taken not to operate the regulator outside the upper limit of regulation shown in Fig. 5-1. The slope of the curves in the constant voltage region is the stabilization factor, F (see Eq. 2-2).

Figure 5-2(b) shows the plot of output voltage, E_o, versus output current, I_o, for an input voltage of 20 volts. The slope of the curve is the internal impedance, R_o (see Eq. 2-1).

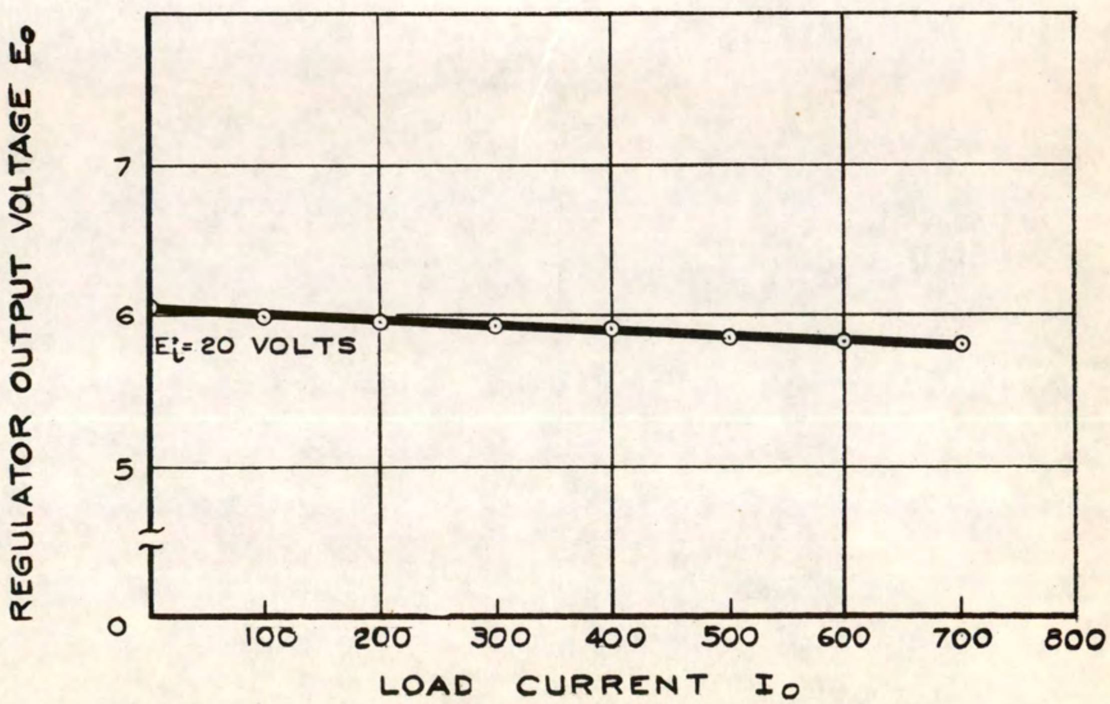
The measured values of these parameters are:

$F = 0.02$

$R_o = 0.3$ ohm



(a.)



(b.)

FIG. 5-2

REGULATION CURVES

A-63532

5.4 Circuit Improvement

With the intention of possibly improving the stabilization factor, some thought was given to the collector supply voltage for the comparison amplifier, transistor Q_2. If the top of resistor R_1 were connected to the emitter of transistor Q_1, the stabilization factor might be improved because of the regulated supply voltage for transistor Q_2. When this connection was made, the circuit of Fig. 4-6 failed to produce an output voltage. This difficulty was due to insufficient input voltage for this method of operation.

As an alternative source of regulated power for the collector of transistor Q_2, a dry battery might be used. Figure 5-3 shows circuits designed to test the improvement, if any, of such an arrangement. By the use of the circuits of Fig. 5-3 a comparison may be made both by analysis and experiment. A dry battery was used for the reference element to minimize the dynamic resistance of the standard voltage.

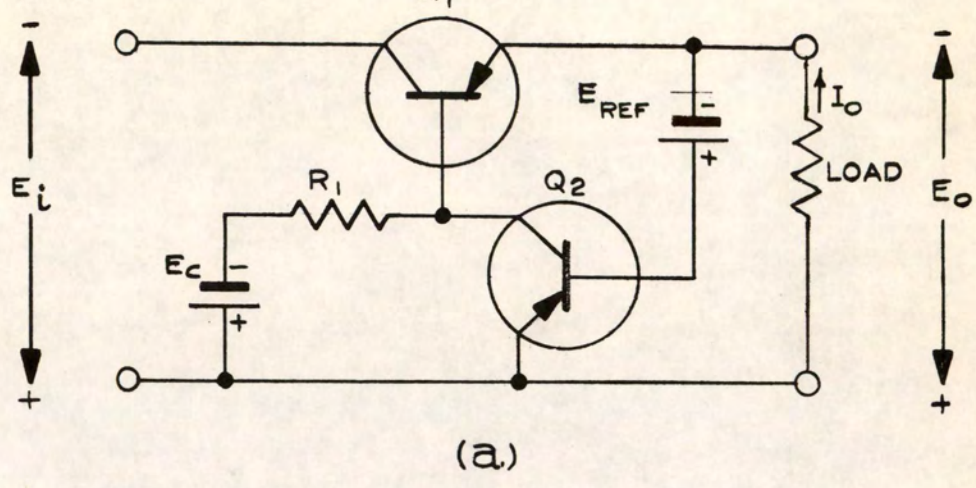
5.41 Improvement Analysis

Appendix C contains calculations based on the circuit of Fig. 5-3(a). Equation C-6 is the exact expression for the stabilization factor. Equation 4-3 enables one to make the following approximation:

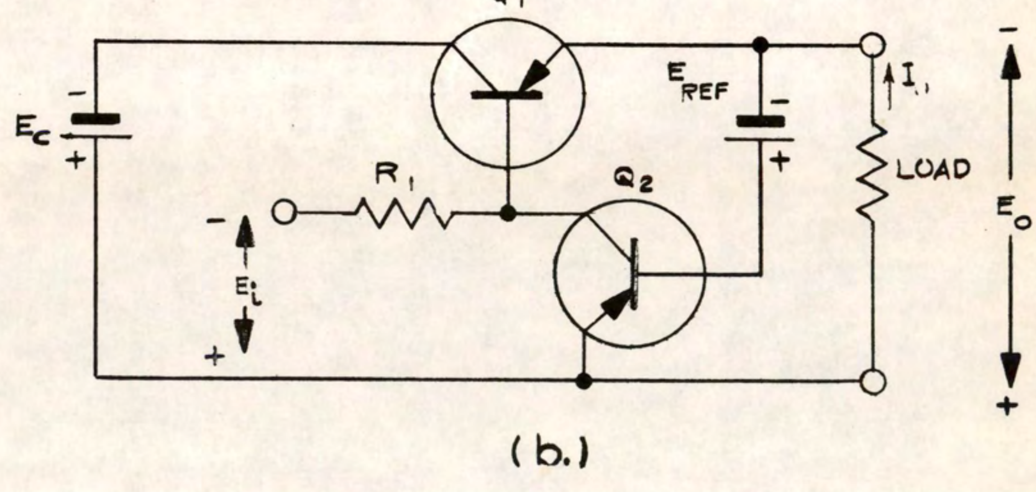
$$F \approx \frac{r_{b2}(1-\alpha_2)}{r_{c1}} \tag{5-2}$$

Appendix D shows the calculations based on the circuit of Fig. 5-3(b). Equation D-6 gives the results for the stabilization factor. Again, Eq. 4-3 may be used to yield the approximation:

$$F \approx \frac{r_{b2}(1-\alpha_2)}{R_1} \tag{5-3}$$



(a.)



(b.)

FIG. 5-3

EXPERIMENTAL CIRCUITS

A-63533

By dividing Eq. 5-3 by Eq. 5-2, one can predict that the stabilization factor of the circuit, Fig. 5-3(a), should be better than that of the circuit of Fig. 5-3(b) by a factor of $\frac{R_1}{r_{cl}}$.

5.42 Experimental Data

The circuits shown in Fig. 5-3 were constructed using the following values:

$E_c = 22.5$ volts $\qquad R_1 = 200$ ohms

$E_{ref} = 6$ volts $\qquad I_o = 200$ milliamperes

The experimental values of stabilization factor obtained were:

$F_a = 0.0003$ Circuit Fig. 5-3(a)

$F_b = 0.015$ Circuit Fig. 5-3(b)

The ratio of these two measured values is

$$\frac{F_a}{F_b} = \frac{0.0003}{0.015} = \frac{1}{50} \qquad (5\text{-}4)$$

The theoretical ratio is:

$$\frac{F_a}{F_b} \approx \frac{R_1}{r_{cl}} = \frac{200}{8000} = \frac{1}{40} \qquad (5\text{-}5)$$

5.43 Improvement Conclusions

In some applications of regulated power supplies it may be feasible to use dry batteries under low current drain to attain lower stabilization factor. The measured value of F for the circuit of Fig. 5-3(a) is two orders of magnitude better than that for the circuit of Fig. 4-6. A circuit similar to Fig. 5-3(a) has been suggested by Zimmermann.[35]

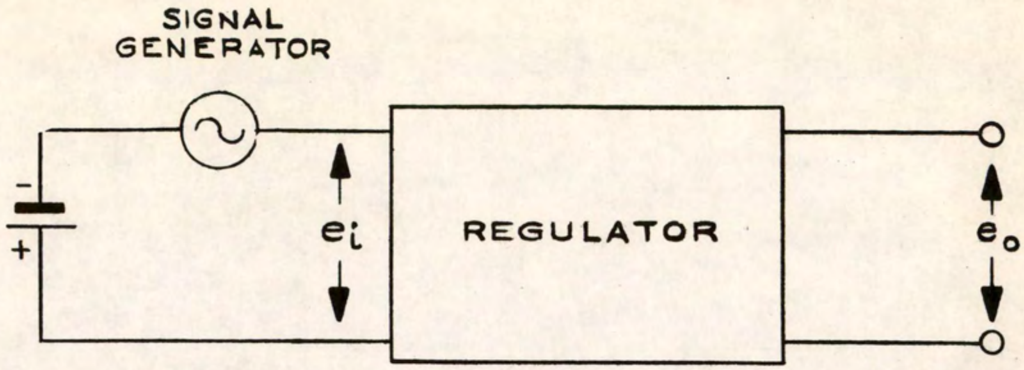
5.5 Variation of Performance Parameters with Frequency

The measured values of F and R_o given in Sect. 5.32 were obtained at zero frequency. Values at greater frequency are also of interest. Load and line transient disturbances can be expected and their effect on the performance of the regulator must be determined.

Figures 5-4(a) and 5-4(b) show methods of measuring stabilization factor and internal impedance as functions of frequency. In Fig. 5-4(b) an alternating current source is obtained by means of the signal generator and a large resistance, R. Voltages in both circuits are measured by an alternating voltmeter. Performance of the regulator at various frequencies is an indication of its behavior to rapid load and line variations.

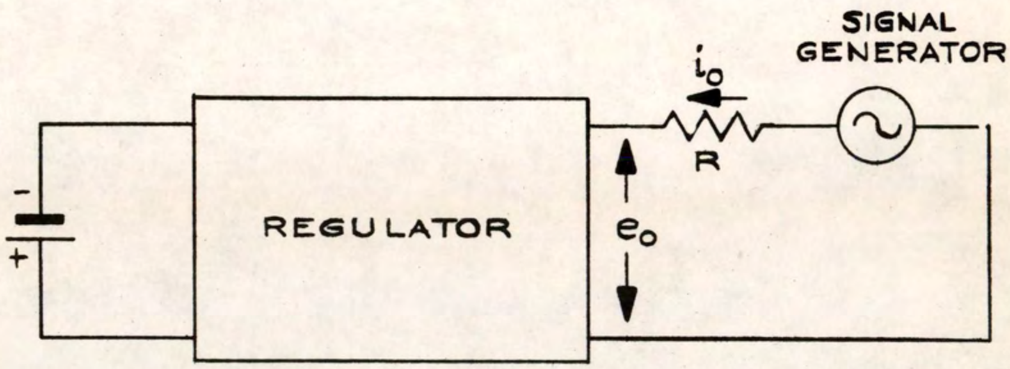
The circuit of Fig. 4-6 was tested in the manner shown in Fig. 5-4 and the results are shown in Fig. 5-5. The stabilization factor increases only slightly at frequencies up to 10 kilocycles but then begins to increase rapidly. In similar fashion, the internal impedance maintained its low frequency value up to 10 kilocycles and then increased sharply to the prohibitive value of 3 ohms at 50 kilocycles. This behavior can be explained by examining the frequency-sensitive parameters of the transistors used in the circuit.

Equation 3-9 predicts the decreasing transistor current amplification factor with increasing frequency and Figs. 3-14 and 3-15 show the experimental confirmation of this fact for the power transistors. One notices that at a frequency of about 10 kilocycles, α begins to decrease radically from its low frequency value. For the two units measured, alpha cut-off frequency occurred at 69 kilocycles for the type H-2 and 114 kilocycles for the type H-4.



(a)

$$F = \frac{e_o}{e_i}\bigg|_{i_o = 0}$$



(b.)

$$R_o = \frac{e_o}{i_o}\bigg|_{e_i = 0}$$

FIG. 5-4

PERFORMANCE PARAMETER MEASUREMENT CIRCUITS

A-63534

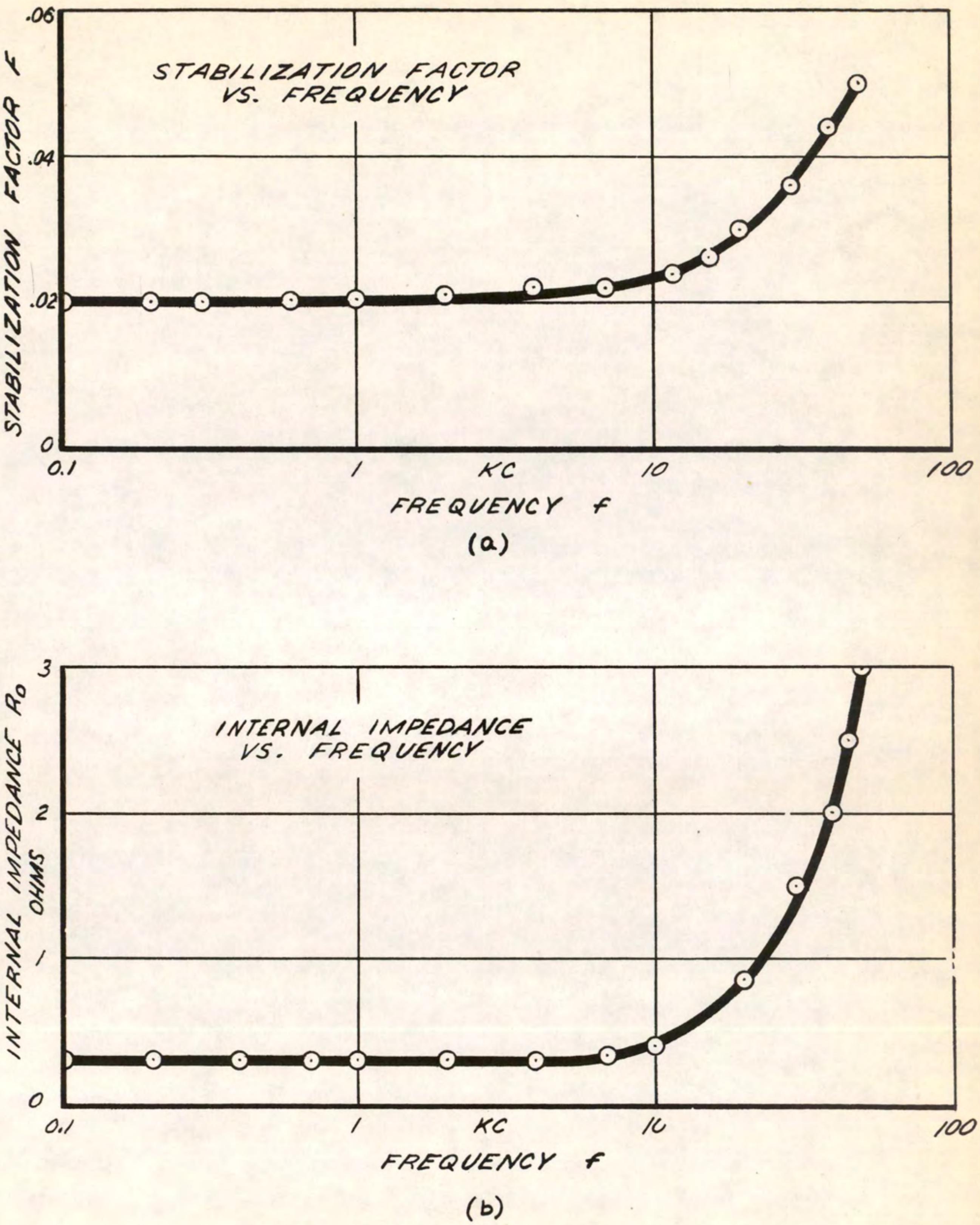


FIG. 5-5

PERFORMANCE PARAMETERS vs. FREQUENCY

A-63535

One should recognize that at high frequency the internal impedance can be maintained below its low frequency value of 0.3 ohm by shunting the output terminals with a capacitor whose reactance is 0.3 ohm at 10 kilocycles.

5.6 Rectifier Characteristics

A full-wave rectifier circuit was constructed using two type 1N347 diodes. The input transformer was a filament type with two 12.6 volt secondary windings. These windings were connected in series to provide 25.2 volts center tapped. A capacitor of 1000 microfarads was the only filter element. Figure 5-6 shows the experimental data obtained from tests performed on the rectifier-filter combination.

The ripple voltage increases linearly with load current as is predicted by Eq. 4-29. The peak-to-peak ripple voltage at a load current of 700 milliamperes is 2.6 volts. The allowable maximum at this load current, as was discussed in Sec. 5.2, is three volts assuming the input voltage to the regulator is 20 volts. If large variations in line voltage, and therefore input voltage to the regulator, are experienced, the maximum current rating must be reduced to 600 milliamperes to assure regulation. One sees from Fig. 5-6 that the output voltage at 700 milliamperes is 19 volts. This value is one volt below the design center of 20 volts derived from the state diagram, Fig. 5-1. Because the constraints of Fig. 5-1 are conservative, it is believed that no trouble will be encountered from this below optimum value of input voltage to the regulator.

5.7 All Semiconductor Regulated Power Supply

With power supplied to the regulator of Fig. 4-6 by means of the rectifier just described, the power supply is complete. Figure 5-7 shows

A-63536

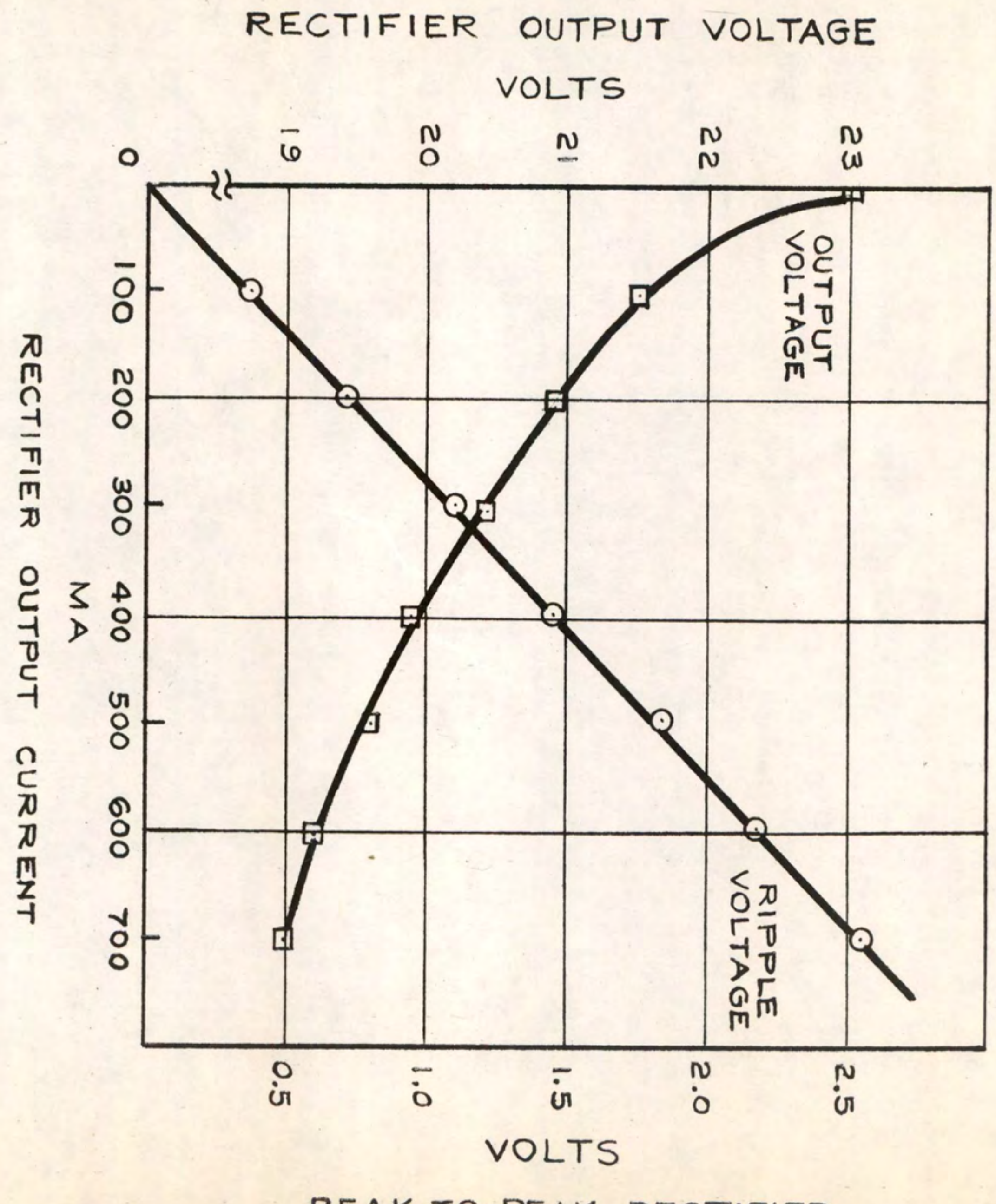
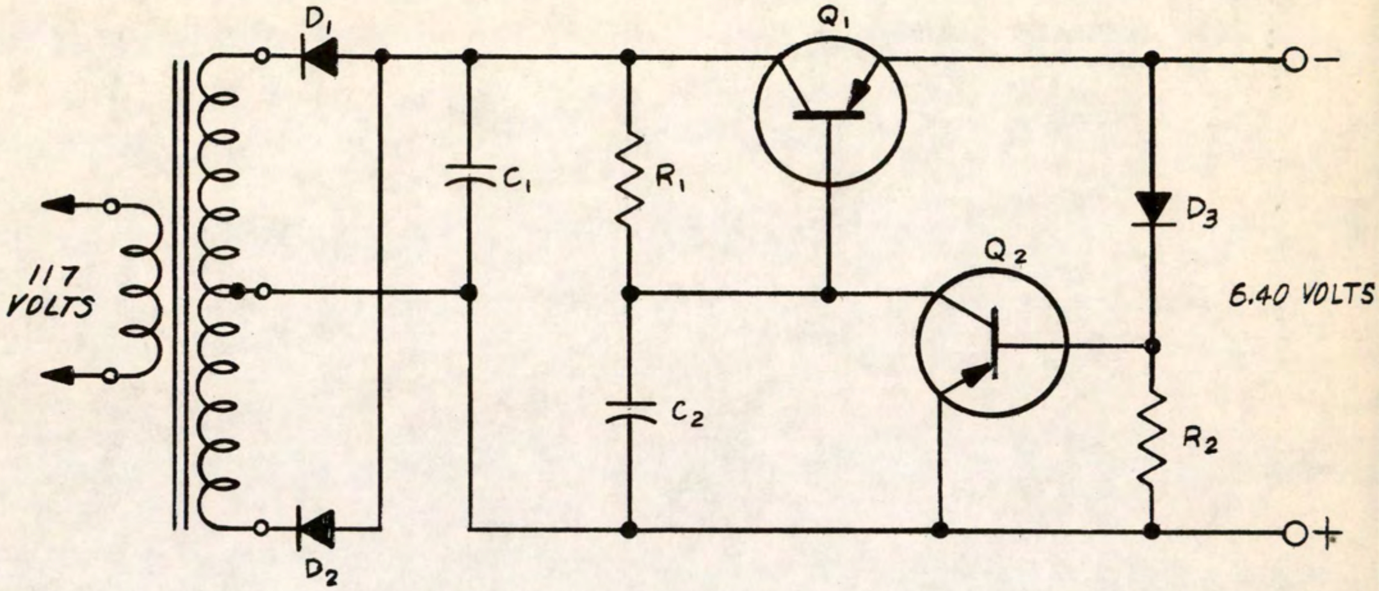


FIG. 5-6
RECTIFIER CHARACTERISTICS



R_1 = 140 OHMS
R_2 = 24 OHMS
C_1 = 1000 MFD
C_2 = 1000 MFD

D_1 − IN347
D_2 − IN347
D_3 − A5B
Q_1 − H-2
Q_2 − H-4

FIG. 5-7

SEMICONDUCTOR REGULATED
POWER SUPPLY

A-63537

the final circuit. An additional capacitor, C_2, has been added to reduce the output ripple. This capacitor forms a pi network together with R_1 and C_1, but notice that the load current is not drawn through the series resistor, R_1. The reduction in output ripple voltage is based on the findings of Sec. 5.41, viz., the gain from the base of transistor Q_1 to the output is much greater than the gain from the collector to the output. Capacitor C_2 reduces the ripple voltage at a point of low current drain, viz., the base of transistor Q_1 and therefore insures low ripple at the output.

5.71 Testing and Evaluation

The slope of the output voltage curve of Fig. 5-6 is the internal impedance, R_o', of the rectifier. The average value measured was five ohms. From Eq. 4-27 one can predict the internal impedance, R, of the complete power supply.

$$R = FR_o' + R_o \tag{4-27}$$

$$R = (0.02)(5) + 0.3 = 0.4 \text{ ohm}$$

The output voltage of the power supply of Fig. 5-7 decreased from 6.40 volts at a load current of 100 milliamperes to 6.20 volts at 600 milliamperes. Equation 4-26 is used to calculate the internal impedance.

$$R = -\frac{dE_o}{dI_o} \tag{4-26}$$

$$R = -\frac{6.40 - 6.20}{0.1 - 0.6} = 0.4 \text{ ohm}$$

The turn ratio, n, of the input transformer to the rectifier is $\frac{117}{12.6} = 9.28$. Since the changes in line voltage are diminished by the turns ratio before they reach the regulator, one can expect the stabilization factor from line input voltage to output voltage to be $\frac{F}{n}$. The predicted over-all stabilization factor for the circuit of Fig. 5-7 is

$$\frac{F}{n} = \frac{0.02}{9.28} = 0.00215.$$

At a load current of 400 milliamperes, a change of output voltage of 0.1 volt was measured when the input voltage was varied from 100 to 135 volts. The stabilization factor calculated from these data is

$$\frac{dE_o}{dE_s} = \frac{0.1}{135-100} = 0.00286.$$

If Eq. 4-28 is solved for the peak-to-peak output ripple voltage, E_r, the following form is obtained:

$$E_r = FE_r' \tag{4-28}$$

From this equation one would predict an output ripple voltage of

$$E_r = (0.02)(2.6) = 0.052 \text{ volts}$$

at a load current of 700 milliamperes. The peak-to-peak output ripple voltage of the circuit of Fig. 5-7 measured 0.018 volt at the full load current of 700 milliamperes. This lower-than-predicted value of ripple voltage is attributable to the use of capacitor C_2. The ripple voltage at the base of transistor Q_1 is much less than that found at the collector of this transistor. Equation 4-28 is not valid for the circuit of Fig. 5-7. Instead the true expression is probably a weighted sum of two terms. The first is the ripple voltage found at the collector of transistor Q_1 times the gain from the collector to the emitter of this transistor. The second is the ripple voltage measured at the base of transistor Q_1 times the gain from the base to the emitter of this same transistor. Although these gains are known, the weighting factor can neither be predicted nor measured.

CHAPTER VI

CONCLUSION

6.1 Power Supply Performance Evaluation

The results of the semiconductor investigation of Chapter 3 and the power-supply design study of Chapter 4 have been utilized in the construction of the all semiconductor regulated power supply pictured in Fig. 6-1. Representing the performance of a voltage-regulated power supply on a percentage basis provides a convenient scale for comparison with other power supplies. The specifications of the power supply circuit of Fig. 5-7 are:

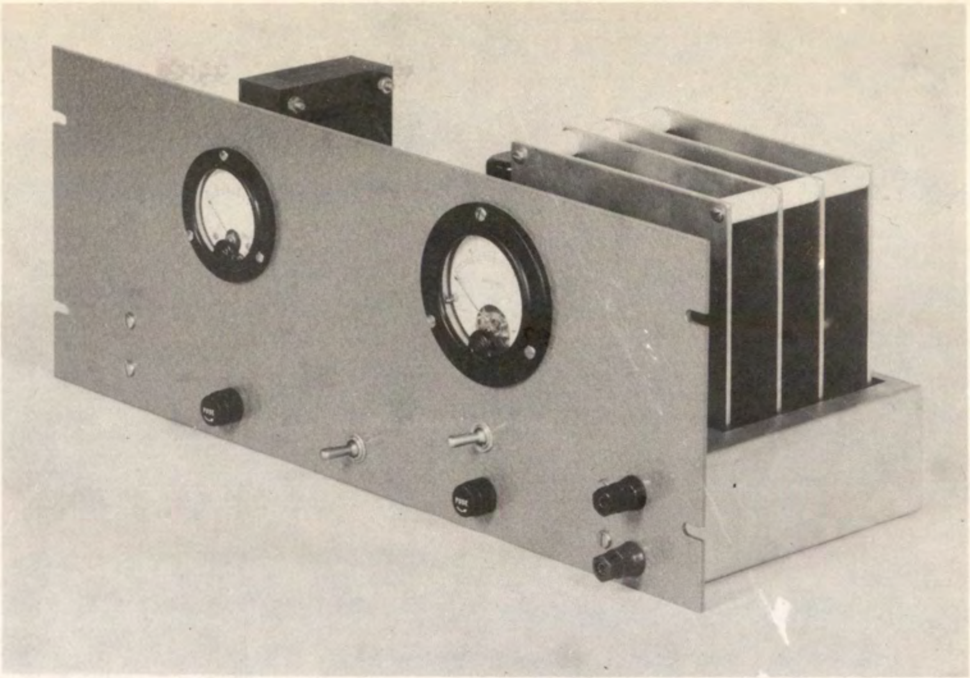
Input voltage	117 volts rms ± 15 percent
Output voltage	6.3 volts at 400 milliamperes
Output current	700 milliamperes maximum
Internal impedance	0.4 ohm
Load regulation	4.5 percent, no-load to full-load
Line regulation	1.6 percent for 30 percent variation in line voltage
Output ripple	0.3 percent
Efficiency	30 percent

Qualitatively, the advantages this supply has over vacuum-tube regulated supplies are: higher efficiency, smaller size, zero warm-up time, and greater reliability.

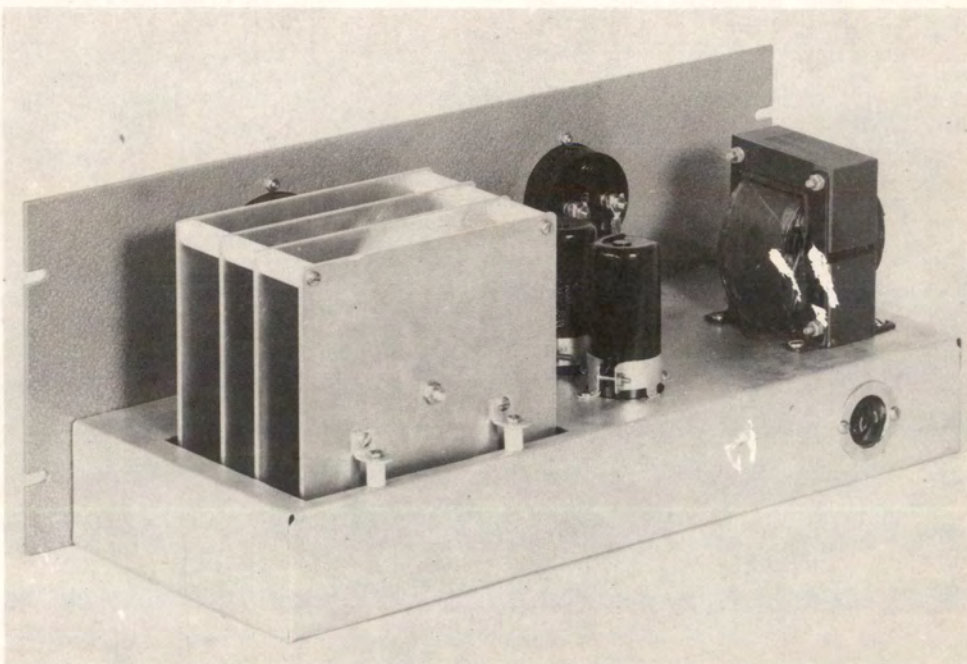
6.2 New Components

Since the experimental work of Chapter 3 was performed, new double-anode silicon diodes† have appeared on the market. These components consist of two diodes connected "back-to-back" in one capsule. The two diodes have been matched in the factory to produce nearly zero temperature coeffi-

† National Semiconductor Products Types A4C, A5C, and A6C.



FRONT VIEW



BACK VIEW

FIG. 6-1

POWER SUPPLY
WITH 19" RACK PANEL MOUNTING

A-63570

cient with small dynamic resistance. These double anode diodes are available with various avalanche voltages and have a maximum power dissipation of 150 milliwatts.

The maximum collector current rating of the type H-2 power transistor has recently been increased from 800 milliamperes to 1.4 amperes. Also available now in limited quantities are power transistors† rated at 60 watts with a collector current maximum of five amperes.

6.3 Suggestions for Further Investigation

In some regulator applications, large changes in input voltage may be encountered. By using the results of Sec. 5.43 a circuit may be designed to keep the variations of the output voltage small under such conditions. Such a circuit is shown in Fig. 6-2. Diode D_1 could be a silicon power rectifier. When operated in the avalanche region of its reverse characteristics, it would provide a constant voltage independent of variations of the input voltage. As was shown in Appendix C, a constant voltage supply for the collector of transistor Q_2 causes the circuit to have low stabilization factor. With large variations of input voltage, diode D_1 must be capable of dissipating several watts. Silicon power rectifiers which would furnish the necessary current have too high an avalanche voltage for this application. This high avalanche voltage is purposely designed into these diodes because the peak inverse voltage rating of the diode must be less than the avalanche voltage. Several companies are believed to be capable of manufacturing a power diode with a specified avalanche voltage. If such a diode were available, the circuit of Fig. 6-2 could easily be designed and constructed.

† Minneapolis-Honeywell Regulator Company Type P-11.

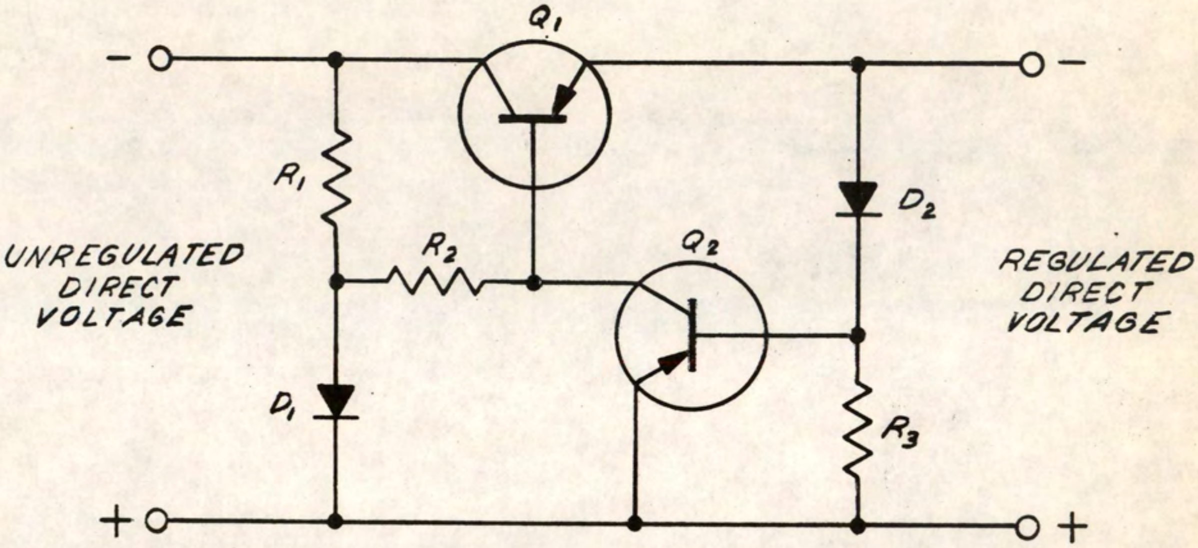


FIG. 6-2

CIRCUIT A PROPOSED FOR INVESTIGATION

A-63571

Figure 5-1 pointed out the limitations imposed by the maximum allowable power dissipation in the reference diode and the power transistor. A voltage regulator of much higher power handling capabilities than the one discussed in this report could be built by using the larger power transistors now available and by using a reference element made from one of the aforementioned power rectifiers.

The internal impedance of the circuit of Fig. 5-7 could be reduced by increasing the amplification of the comparison circuit. Additional transistors could be inserted between the error signal and the control element, thereby increasing the degenerative loop gain. Complementary symmetry could be used to provide direct coupling between these new stages of amplification. Such a circuit is shown in Fig. 6-3. A battery provides a good reference element because of the small current drain. The life of this battery would be only slightly less than shelf life.

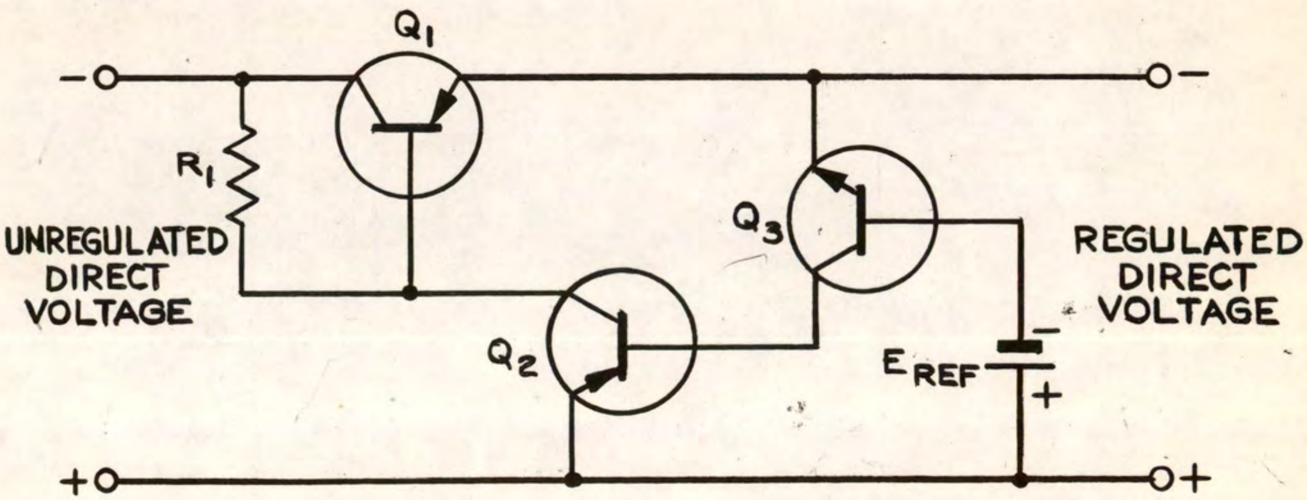
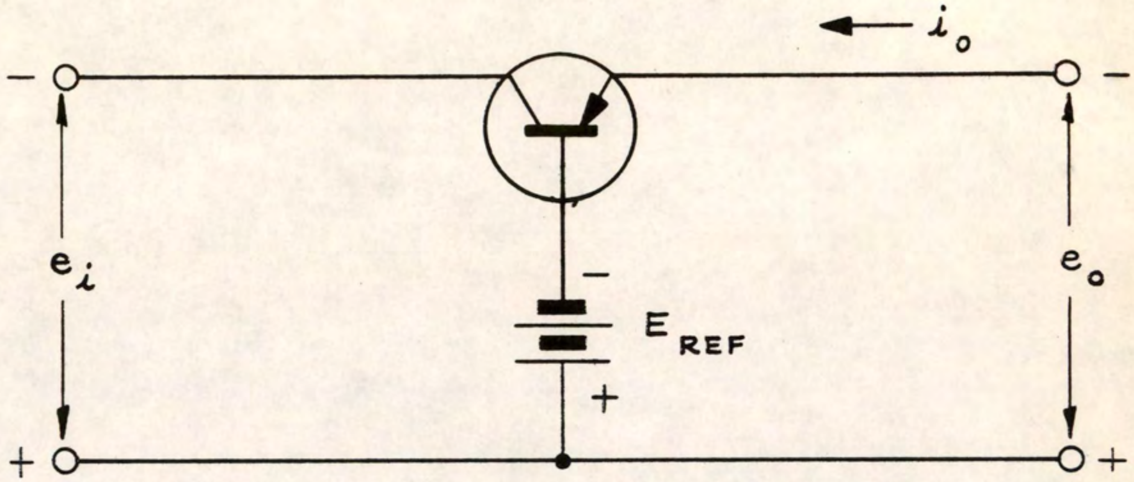


FIG. 6-3
CIRCUIT B PROPOSED
FOR INVESTIGATION

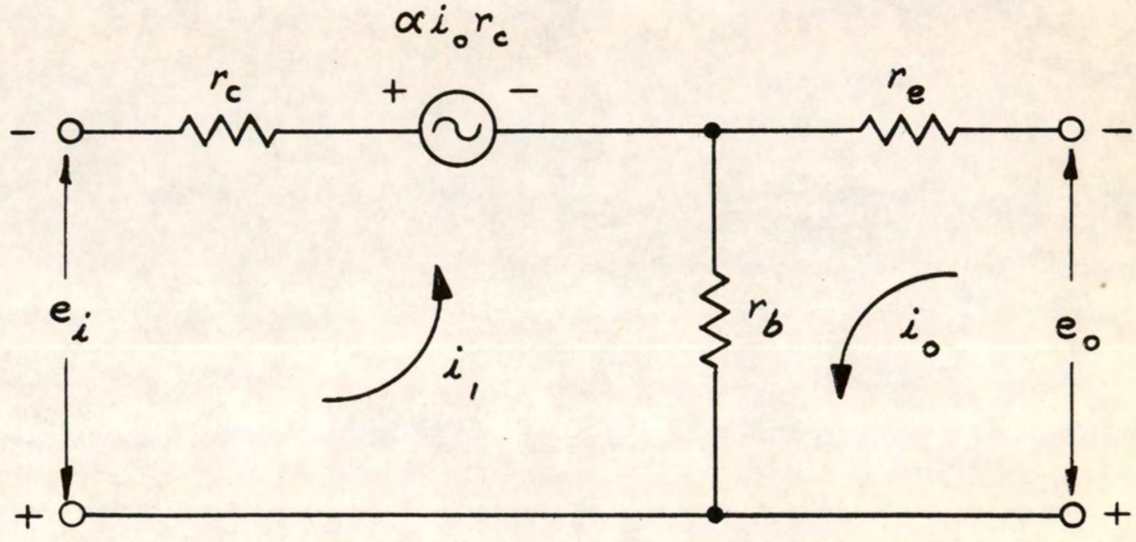
APPENDIX A

ANALYSIS OF ELEMENTARY VOLTAGE REGULATOR



(a)

ELEMENTARY VOLTAGE REGULATOR



(b)

INCREMENTAL EQUIVALENT CIRCUIT

A-63572

FIG. A-1

The two loop equations are

$$-e_i + (i_1 - i_o)r_b - \alpha i_o r_c + i_1 r_c = 0 \qquad \text{(A-1)}$$

$$e_o + i_o r_e + (i_o - i_1)r_b = 0 \qquad \text{(A-2)}$$

Rearrangement of terms gives

$$(r_b + r_c)i_1 - (r_b + \alpha r_c)i_o = e_i \qquad \text{(A-1a)}$$

$$-r_b i_1 + (r_e + r_b)\, i_o = -e_o \qquad \text{(A-2a)}$$

If Eq. A-1(a) is solved for i_1, one obtains

$$i_1 = \frac{(r_b + \alpha r_c)i_o + e_1}{r_b + r_c} \qquad \text{(A-3)}$$

Substitution in Eq. A-2(a) yields

$$\frac{-r_b\left[(r_b + \alpha r_c)i_o + e_i\right]}{r_b + r_c} + (r_e + r_b)i_o = -\, e_o \qquad \text{(A-4)}$$

Collecting terms give:

$$-\frac{r_b}{r_b + r_c}\, e_i + \frac{(r_e + r_b)(r_b + r_c) - r_b(r_b + \alpha r_c)}{r_b + r_c}\, i_o = -e_o \qquad \text{(A-5)}$$

Simplification yields the final equation

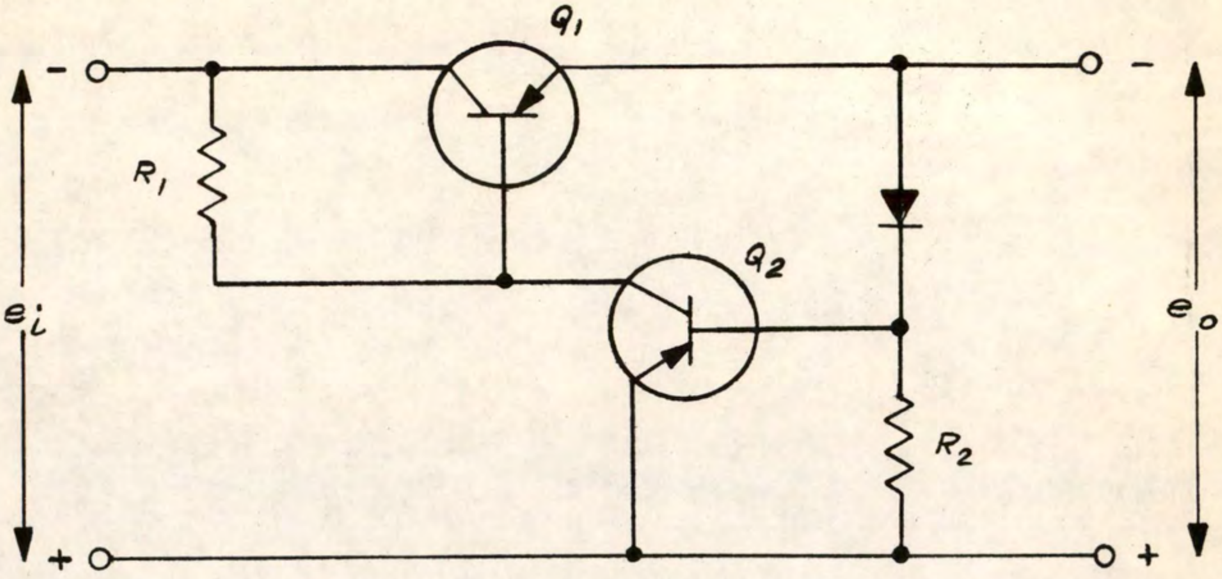
$$e_o = \frac{r_b}{r_b + r_c}\, e_i - \frac{r_e r_b + r_c\left[r_e + r_b(1-\alpha)\right]}{r_b + r_c}\, i_o \qquad \text{(A-6)}$$

One recognizes Eq. A-6 to be of the same form as Eq. 2-5b; therefore

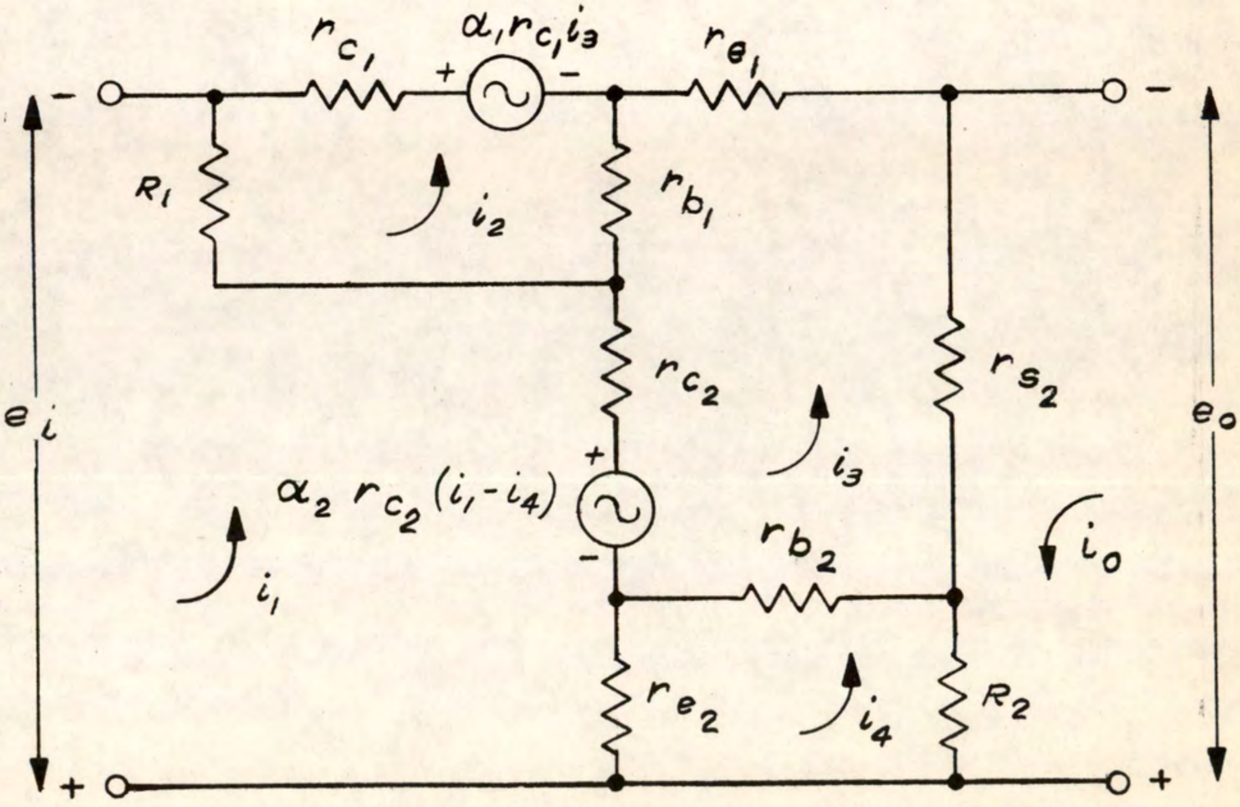
$$F = \frac{r_b}{r_b + r_c} \quad \text{and} \qquad \text{(A-7)}$$

$$R_o = \frac{r_e r_b + r_c\left[r_e + r_b(1-\alpha)\right]}{r_b + r_c} \qquad \text{(A-8)}$$

APPENDIX B
ANALYSIS OF SEMICONDUCTOR REGULATOR



(a.)
SEMICONDUCTOR REGULATOR



(b.)
INCREMENTAL EQUIVALENT CIRCUIT

FIG. B-1

A-63573

The five loop equations are:

$$-e_i + (i_1 - i_4)r_{e2} - \alpha_2 r_{c2}(i_1 - i_4) + (i_1 - i_3)r_{c2} + (i_1 - i_2)R_1 = 0 \quad \text{(B-1)}$$

$$(i_2 - i_3)r_{b1} - \alpha_1 r_{c1} i_3 + i_2 r_{c1} + (i_2 - i_1)R_1 = 0 \quad \text{(B-2)}$$

$$(i_3 - i_4)r_{b2} + (i_3 - i_o)r_{s2} + i_3 r_{e1} + (i_3 - i_2)r_{b1} + (i_3 - i_1)r_{c2} + \alpha_2 r_{c2}(i_1 - i_4) = 0 \quad \text{(B-3)}$$

$$(i_4 - i_o)R_2 + (i_4 - i_3)r_{b2} + (i_4 - i_1)r_{e2} = 0 \quad \text{(B-4)}$$

$$(i_o - i_3)r_{s2} + (i_o - i_4)R_2 + e_o = 0 \quad \text{(B-5)}$$

Rearrangement of terms gives:

$$\left[r_{e2} + r_{c2}(1-\alpha_2) + R_1\right] i_1 - R_1 i_2 - r_{c2} i_3 + (\alpha_2 r_{c2} - r_{e2}) i_4 = e_i \quad \text{(B-1a)}$$

$$-R_1 i_1 + (r_{b1} + r_{c1} + R_1) i_2 - (r_{b1} + \alpha_1 r_{c1}) i_3 = 0 \quad \text{(B-2a)}$$

$$- r_{c2}(1-\alpha_2) i_1 - r_{b1} i_2 + (r_{b2} + r_{s2} + r_{e1} + r_{b1} + r_{c2}) i_3 - (r_{b2} + \alpha_2 r_{c2}) i_4 - r_{s2} i_o = 0 \quad \text{(B-3a)}$$

$$-r_{e2} i_1 - r_{b2} i_3 + (r_{e2} + r_{b2} + R_2) i_4 - R_2 i_o = 0 \quad \text{(B-4a)}$$

$$-r_{s2} i_3 - R_2 i_4 + (r_{s2} + R_2) i_o = - e_o \quad \text{(B-5a)}$$

These equations are solved for i_3 and i_4. The expressions for i_3 and i_4 are then substituted into Eq. B-5a, and the following equation is obtained.

$$e_o = \frac{A}{D} e_i - \frac{B}{D} i_o \quad \text{(B-6)}$$

where
$$
\begin{aligned}
A = {} & (r_{b1} + r_{c1} + R_1) \Big\{ \big[r_{e2} + r_{c2}(1-\alpha_2) \big] \big[r_{b2}(R_2 + r_{s2}) + R_2 r_{s2} \big] + \\
& R_2 r_{e2} r_{e1} + r_{c2} r_{e2}(R_2 + r_{s2}) \Big\} + R_1 r_{b1} \big[(R_2 + r_{s2})(r_{e2} + r_{b2}) + R_2 r_{s2} \big] \\
& + R_2 r_{e2} r_{b1} r_{c1}(1-\alpha_1)
\end{aligned}
$$

$$
\begin{aligned}
B = {} & (R_2 + r_{s2}) \Big\{ r_{b2}\big[r_{e2} + r_{c2}(1-\alpha_2) \big] + r_{e2} r_{c2} \Big\} \Big\{ r_{e1}(r_{b1} + r_{c1} + R_1) + \\
& (R_1 + r_{b1}) r_{c1}(1-\alpha_1) \Big\} - R_1(r_{b1} + r_{c1}) \Big[r_{s2}^2 (r_{e2} + r_{b2} + R_2) + \\
& R_2^2 (r_{s2} + r_{e1}) + R_2 r_{s2} r_{b2} \Big] - R_1 R_2 r_{s2} r_{e2}(r_{b1} + \alpha_1 r_{c1}) + \\
& R_1 R_2 r_{s2} r_{c2}(1-\alpha_2) r_{c1}(1-\alpha_1) + R_1 r_{b1}(r_{b2} + r_{e2})\ (R_2 + r_{s2}) r_{c1}(1-\alpha_1)
\end{aligned}
$$

$$
\begin{aligned}
D = {} & \Big\{ r_{b2}\big[r_{e2} + r_{c2}(1-\alpha_2) \big] + r_{e2} r_{c2} \Big\} \Big[(r_{b1} + r_{c1} + R_1)(R_2 + r_{s2} + r_{e1}) + \\
& (R_1 + r_{b1}) r_{c1}(1-\alpha_1) \Big] + R_2 \big[r_{e2} + r_{c2}(1-\alpha_2) \big] \Big[(r_{b1} + r_{c1} + R_1) \cdot (r_{s2} + r_{e1}) \\
& + r_{b1} r_{c1}(1-\alpha_1) \Big] + R_1 r_b\ (r_{b2} + r_{e2}) \big[R_2 + r_{c1}(1-\alpha_1) \big] + \\
& R_1 R_2 \big[r_{c1} r_{b2} + r_{c1}(1-\alpha_1)(r_{c2} + r_{b1}) + r_{c2} \alpha_2 r_{c1} \alpha_1 \big]
\end{aligned}
$$

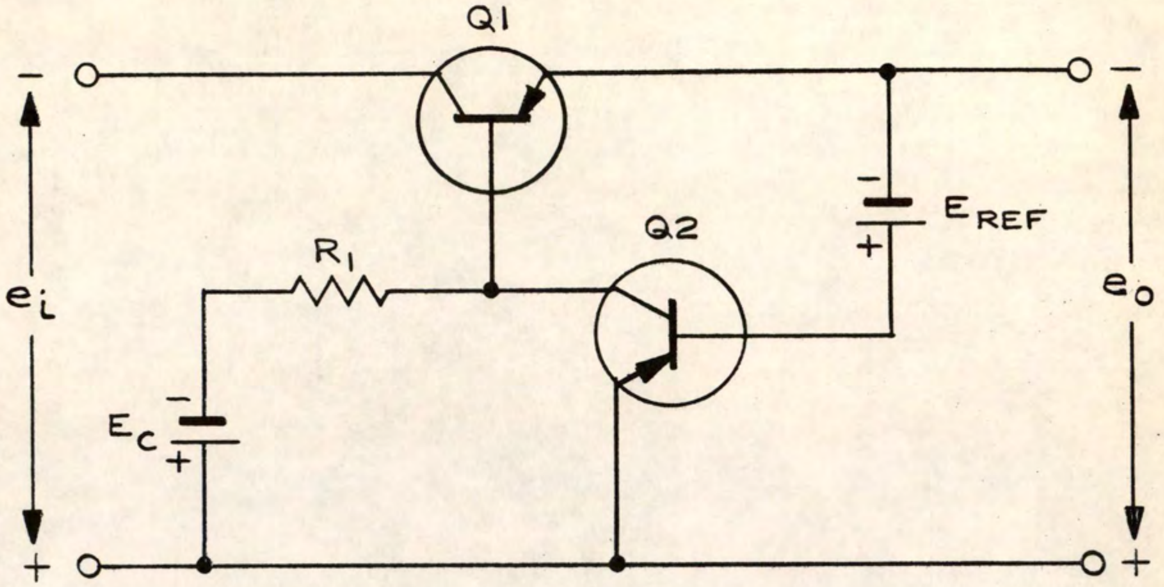
From B-6, one obtains the following expressions:

$$F = \frac{A}{D} \tag{B-7}$$

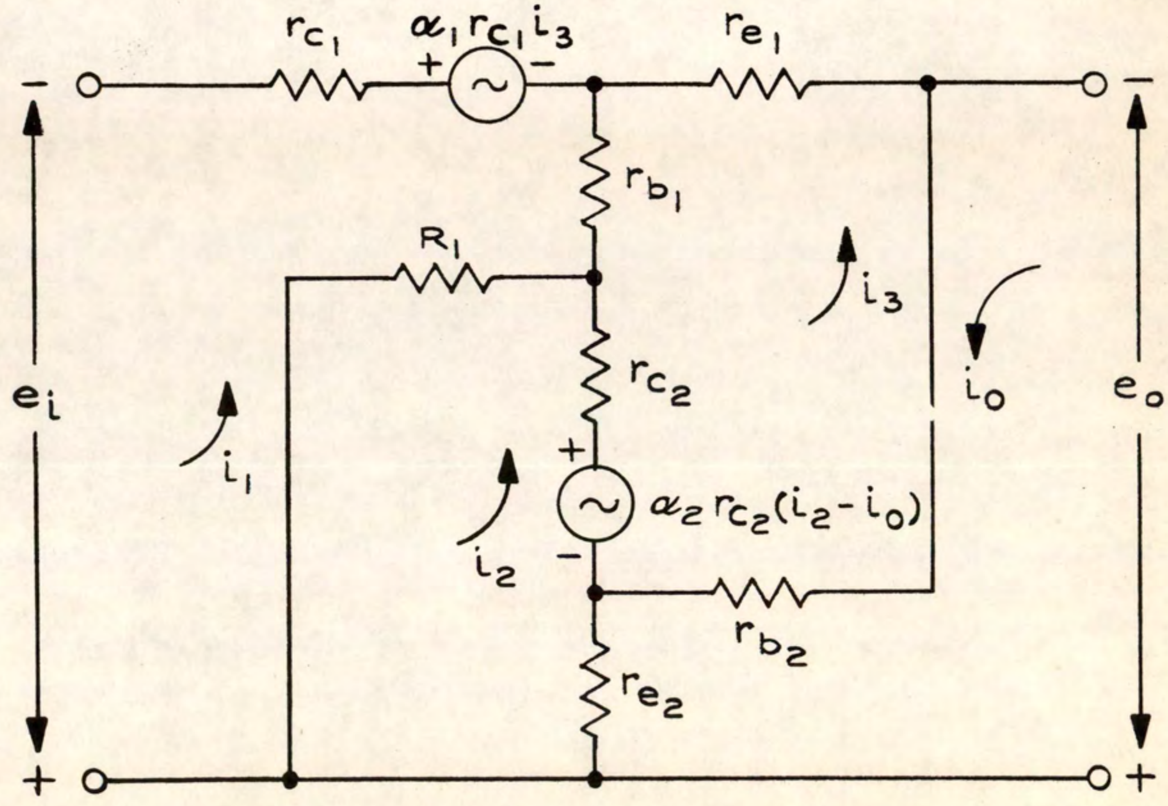
$$R_o = \frac{B}{D} \tag{B-8}$$

Equations B-7 and B-8 can be simplified considerably by use of Eq. 4-3. Different approximate expressions may result from different operating points of the transistors. Careful consideration should be given to the exact expressions before making approximations which are to apply throughout a wide range of operation.

APPENDIX C
IMPROVEMENT ANALYSIS, CIRCUIT A



a) EXPERIMENTAL CIRCUIT A



b) INCREMENTAL EQUIVALENT CIRCUIT

FIG. C-1

A-63574

The four loop equations are:

$$-e_i + (i_1 - i_2)R_1 + r_{b1}(i_1 - i_3) - \alpha_1 r_{c1} i_3 + i_1 r_{c1} = 0 \qquad \text{(C-1)}$$

$$(i_2 - i_o)r_{e2} - \alpha_2 r_{c2}(i_2 - i_o) + (i_2 - i_3)r_{c2} + (i_2 - i_1)R_1 = 0 \qquad \text{(C-2)}$$

$$i_3 r_{e1} + (i_3 - i_1)r_{b1} + (i_3 - i_2)r_{c2} + \alpha_2 r_{c2}(i_2 - i_o) + (i_3 - i_o)r_{b2} = 0 \qquad \text{(C-3)}$$

$$(i_o - i_3)r_{b2} + (i_o - i_2)r_{e2} + e_o = 0 \qquad \text{(C-4)}$$

Rearrangement of terms gives:

$$(r_{b1} + r_{c1} + R_1)i_1 - R_1 i_2 - (r_{b1} + \alpha_1 r_{c1})i_3 = e_i \qquad \text{(C-1a)}$$

$$-R_1 i_1 + \left[r_{e2} + r_{c2}(1 - \alpha_2) + R_1\right] i_2 - r_{c2} i_3 - (r_{e2} - \alpha_2 r_{c2})i_o = 0 \qquad \text{(C-2a)}$$

$$-r_{b1} i_1 - r_{c2}(1-\alpha_2)\, i_2 + (r_{e1} + r_{b1} + r_{c2} + r_{b2})i_3 - (\alpha_2 r_{c2} + r_{b2})i_o = 0 \qquad \text{(C-3a)}$$

$$-r_{e2} i_2 - r_{b2} i_3 + (r_{b2} + r_{e2})i_o = -e_o \qquad \text{(C-4a)}$$

These equations are solved for i_3 and i_4. The expressions for i_3 and i_4 are then substituted into Eq. C-4a and the following equation is obtained.

$$e_o = \frac{A}{D} e_i - \frac{B}{D} i_o \qquad \text{(C-5)}$$

where
$$A = -R_1 r_{e2}(r_{e1} + r_{b1} + r_{c2} + r_{b2}) - r_{b1} r_{c2} r_{e2} - R_1 r_{b2} r_{c2}(1-\alpha_2) - r_{b1} r_{b2}\left[r_{e2} + r_{c2}(1-\alpha_2) + R_1\right]$$

$$B = (r_{b1} + \alpha_1 r_{c1})\Big\{ R_1 r_{e2}\left[r_{b2} + r_{c2} + r_{b1}\right] + r_{b1} r_{b2}\left[r_{e2} + r_{c2}(1-\alpha_2) + R_1\right] + R_1 r_{b2} r_{c2} + r_{e2} r_{b1} r_{c2}\Big\} + (r_{b1} + r_{c1} + R_1)\Big\{ -r_{e2}(r_{e1} + r_{b1})\left[R_1 + r_{b2} + r_{c2}\right] - r_{b2}(r_{e1} + r_{b1})\left[R_1 + r_{c2}(1-\alpha_2)\right. - r_{b2} R_1 r_{c2}(1-\alpha_2) - r_{e2} R_1 (r_{b2} + r_{c2})\Big\} + R_1 r_{b1} r_{b2}\left[r_{e2} + r_{c2}(1-\alpha_2) + R_1\right] + R_1^2 r_{b2}\left[r_{e1} + r_{c2}(1-\alpha_2)\right] + R_1^2 r_{e2}(r_{e1} + r_{b1} + r_{c2} + r_{b2}) + R_1 r_{c2} r_{e2} r_{b1}$$

and $D = (r_{b1} + \alpha_1 r_{c1})\left\{ R_1 r_{c2}(1-\alpha_2) + r_{b1}\left[r_{e2} + r_{c2}(1-\alpha_2) + R_1\right]\right\} +$

$(r_{b1} + r_{c1} + R_1)\left\{ r_{c2}^2(1-\alpha_2) - \left[r_{e2} + r_{c2}(1-\alpha_2) + R_1\right](r_{e1} + r_{b1} + r_{b2} + r_{c2})\right\}$

$+ r_{b1}R_1 r_{c2} + R_1^2(r_{e1} + r_{b1} + r_{c2} + r_{b2})$

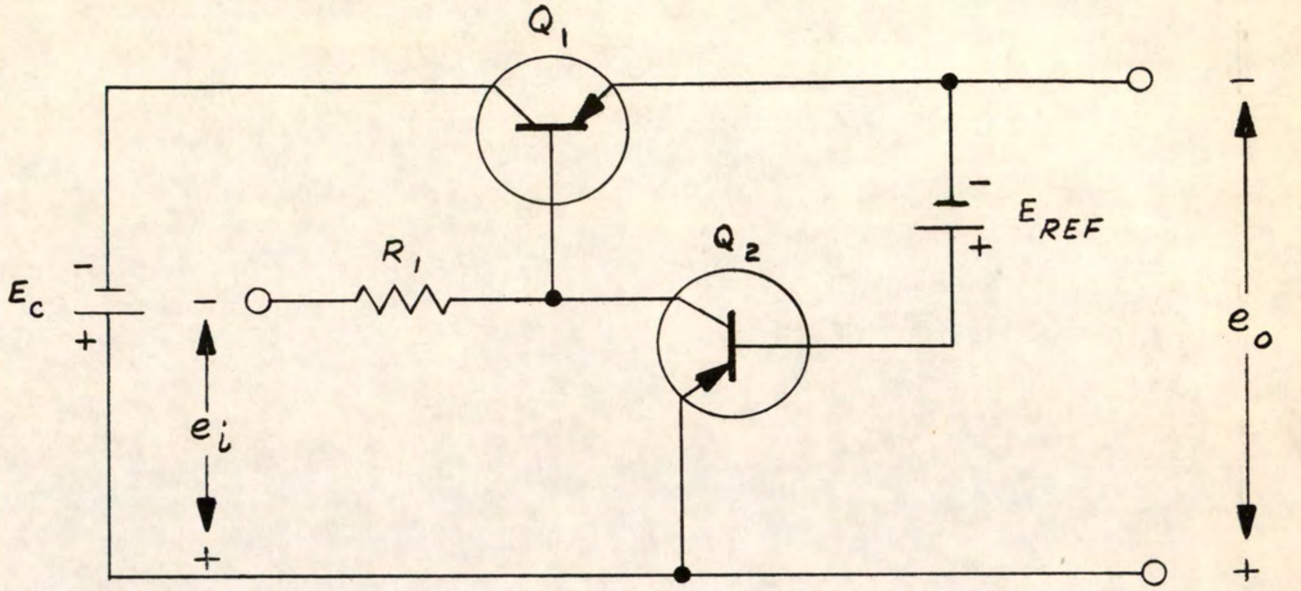
From Eq. C-5, one obtains the following expressions:

$$F = \frac{A}{D} \qquad \text{(C-6)}$$

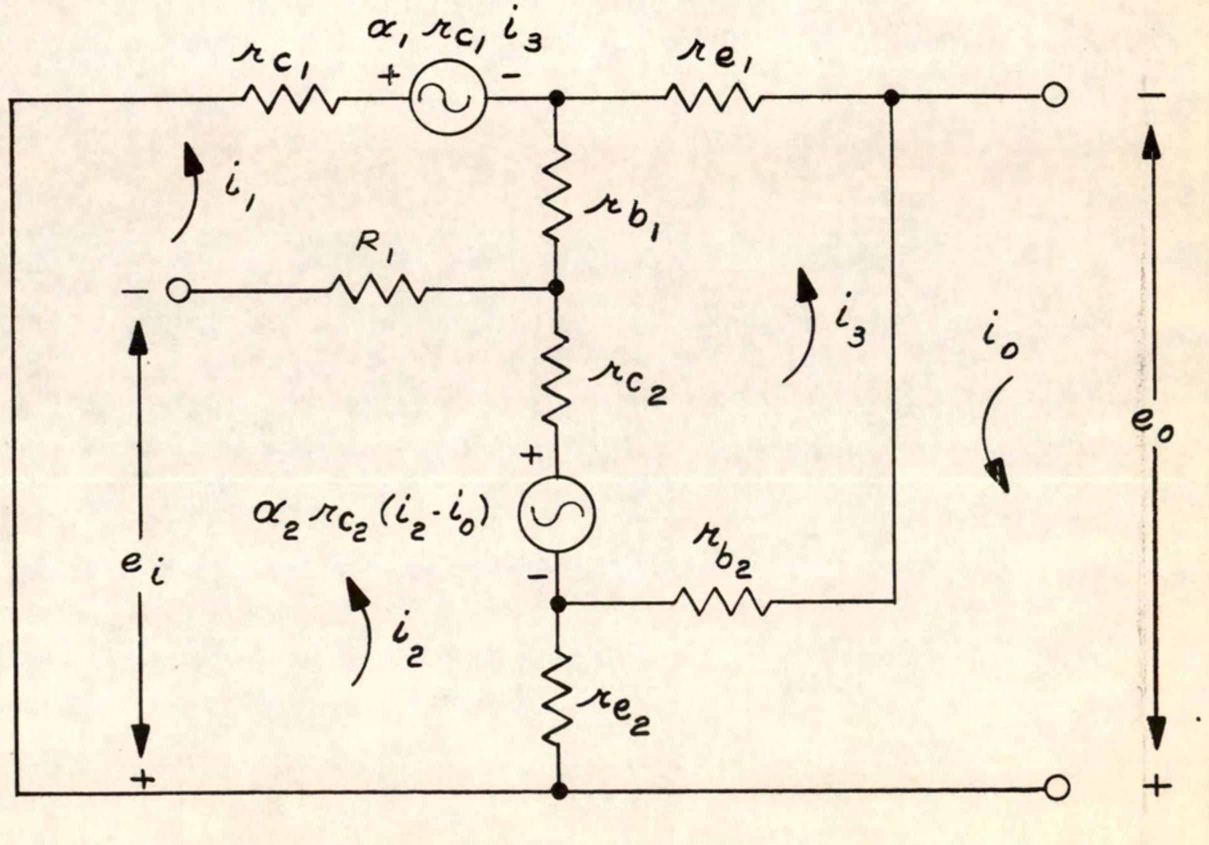
$$R_o = \frac{B}{D} \qquad \text{(C-7)}$$

Equations C-6 and C-7 can be simplified considerably by use of Eq. 4-3. Different approximate expressions may result from different operating points of the transistors. Careful consideration should be given to the exact expressions before making approximations which are to apply throughout a wide range of operation.

APPENDIX D
IMPROVEMENT ANALYSIS, CIRCUIT B



(a) EXPERIMENTAL CIRCUIT B



(b) INCREMENTAL EQUIVALENT CIRCUIT

FIG. D-1

A-63575

The four loop equations are:

$$(i_1 - i_2)R_1 + (i_1 - i_3)\, r_{b1} - \alpha_1 r_{c1} i_3 + i_1 r_{c1} + e_i = 0 \qquad \text{(D-1)}$$

$$-e_i + (i_2 - i_o) r_{e2} - \alpha_2 r_{c2}(i_2 - i_o) + (i_2 - i_3) r_{c2} + (i_2 - i_1) R_1 = 0 \qquad \text{(D-2)}$$

$$i_3 r_{e1} + (i_3 - i_1) r_{b1} + (i_3 - i_2) r_{c2} + \alpha_2 r_{c2}(i_2 - i_o) + (i_3 - i_o)\, r_{b2} = 0 \qquad \text{(D-3)}$$

$$(i_o - i_3) r_{b2} + (i_o - i_2) r_{e2} + e_o = 0 \qquad \text{(D-4)}$$

Rearrangement of terms gives:

$$(r_{b1} + r_{c1} + R_1) i_1 - R_1 i_2 - (r_{b1} + \alpha_1 r_{c1}) i_3 = -e_i \qquad \text{(D-1a)}$$

$$-R_1 i_1 + \left[r_{e2} + r_{c2}(1-\alpha_2) + R_1 \right] i_2 - r_{c2} i_3 - (r_{e2} - \alpha_2 r_{c2}) i_o = e_i \qquad \text{(D-2a)}$$

$$-r_{b1} i_1 - r_{c2}(1-\alpha_2) i_2 + (r_{e1} + r_{b1} + r_{c2} + r_{b2}) i_3 - (\alpha_2 r_{c2} + r_{b2}) i_o = 0 \qquad \text{(D-3a)}$$

$$-r_{e2} i_2 - r_{b2} i_3 + (r_{b2} + r_{e2})\, i_o = -e_o \qquad \text{(D-4a)}$$

These equations are solved for i_3 and i_4. The expression for i_3 and i_4 are then substituted into Eq. D-4a and the following equation is obtained.

$$e_o = \frac{A}{D} e_i - \frac{B}{D} i_o \qquad \text{(D-5)}$$

where

$$\begin{aligned} A = {} & r_{e2} r_{b1}(r_{b1} + \alpha_1 r_{c1}) - r_{b1} r_{b2} R_1 - (r_{b1} + r_{c1} + R_1) \Big[r_{e2}(r_{e1} + \\ & r_{b1} + r_{c2} + r_{b2}) + r_{b2} r_{c2}(1-\alpha_2) \Big] \end{aligned}$$

$$\begin{aligned} B = {} & -(r_{b1} + r_{c1}) r_{e1} r_{b2} \left[R_1 + r_{c2}(1-\alpha_2) + r_{e2} \right] - R_1 r_{e1} \Big\{ r_{b2} \big[r_{c2}(1-\alpha_2) \\ & + r_{e2} \big] + r_{e2}(r_{b1} + r_{c1}) \Big\} - R_1 r_{c1}(1-\alpha_1) \Big[r_{b1}(r_{b2} + r_{e2}) + r_{e2}(r_{c2} \\ & + r_{b2}) + r_{b2} r_{c2}(1-\alpha_2) \Big] - r_{b1} r_{c1}(1-\alpha_1) \Big[r_{e2}(r_{c2} + r_{b2}) + r_{b2} r_{c2} \\ & (1-\alpha_2) \Big] \end{aligned}$$

and

$$\begin{aligned} D = {} & (r_{b1} + \alpha_1 r_{c1}) \Big\{ R_1 R_{c2}(1-\alpha_2) + r_{b1} \left[r_{e2} + r_{c2}(1-\alpha_2) + R_1 \right] \Big\} + \\ & (r_{b1} + r_{c1} + R_1) \Big\{ r_{c2}^2 (1-\alpha_2) - \left[r_{e2} + r_{c2}(1-\alpha_2) + R_1 \right] (r_{e1} + r_{b1} + r_{c2} + r_{b2}) \Big\} \\ & + r_{b1} R_1 r_{c2} + R_1^2 (r_{e1} + r_{b1} + r_{c2} + r_{b2}) \end{aligned}$$

From Eq. D-5, the following expressions may be obtained:

$$F = \frac{A}{D} \tag{D-6}$$

$$R_o = \frac{B}{D} \tag{D-7}$$

Equations D-6 and D-7 can be simplified considerably by use of Eq. 4-3. Different approximate expressions may result from different operating points of the transistors. Careful consideration should be given to the exact expressions before making approximations which are to apply throughout a wide range of operation.

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