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THE CHOICE OF LOT INSPECTION PLANS ON THE BASIS OF COST

BY

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I. Introduction

Cost is the common denominator of all industrial operations. The measure of management is its ability to make the ratio of value to cost as great as possible. In inspection management, the goal is to accomplish the required job at the least possible cost.

In this paper we shall set forth the cost factors involved in inspecting material submitted in lots. The inspector has a lot of, say, 1,000 pieces and he must decide whether to accept the lot or to reject it.

The usual single sampling plan for lot inspection is defined by two parameters.

n: The sample size
c: The acceptance number

The inspector takes a sample of size n from the lot. He accepts the lot if there are c or fewer defective pieces in the sample. He rejects the lot if there are more than c defective pieces in the sample.

In Section II we outline all the cost factors involved in the sampling inspection of lots. In Section II we develope the formulas to determine the most efficient single sampling inspection plan. In Section IV we give a work sheet summarizing all the calculations necessary to determine the most efficient plan for any set of actual cost factors.

II. The Cost Factors in a Lot Inspection Plan

There are many costs that may effect an inspection plan. In some practical inspection jobs, only a few of these costs will be important, and the rest can be ignored. But first the inspection planner must study the effects of all the costs and how they enter the problem. It is only by such study that he develops the judgement necessary to spot the situations where the assumptions and approximations of simplified plans cause large errors.

Inspection Cost: The most obvious cost of an inspection plan is the cost of inspection itself. We must hire inspectors, supervise them, give them equipment, and plan their operations. In most cases the cost of inspection can be rather accurately expressed as the sum of overhead items which do not vary with the amount of inspection, plus operating or running costs which are proportional to the number of items inspected. Overhead will usually include most equipment, supervision, and planning expenses. Running costs will usually include most labor charges. Also, if inspection is destructive, the cost of the pieces destroyed in inspection is a charge against running costs. Thus we are ready to define the following symbols:

J: Overhead cost of inspection per lot inspected
I: Running cost of inspection per piece inspected
n: Sample size (number of pieces inspected per lot)
J + nI: Total cost of inspection per lot submitted for inspection

The Cost of Defective Pieces Missed (Normal Quality): A sampling plan does not catch all the defective pieces produced. Some of the defective pieces are going to get past. They would not be defective if all or some of them were not going to cause complaints or trouble. A dollar sign can be placed on the complaints or trouble. What does a complaint cost you? It makes the Boss mad and you would pay something to prevent that. It hurts the reputation of the Company, and your Sales Department would be willing to pay something to prevent that. A defective piece may cause failure of an expensive assembly and you can evaluate that. A good deal of judgement may be necessary to evaluate the cost of a complaint which gets away. But don't go hog wild. Some defective pieces cost very little. For example, they may be automatically rejected in a future assembly operation. In the final analysis the answer you want is "How much would you be willing to spend to prevent a single complaint or trouble caused by a defect?" The cost should include the cost of repairing or replacing the piece if that will be involved in settling the complaint or trouble.

Normally, only a percentage of the defective pieces will cause complaints or trouble. Most plants would have been out of business long ago if every piece outside of specifications caused a complaint. Fortunately, only a more or less small percentage of defects causes trouble. Our first thought, when we realize this fact, is to condemn the engineers for making the specications too tight. A little reflection, and we see that this criticism is unjustified. All our experience has shown that one of the most effective ways (and often the only effective way) to obtain high quality at reasonable cost is to include safety factors in the specifications. I would not want to cross a bridge if the strength of the steel assumed by the designer was the ultimate strength, not a conservative design figure. Even more important, not all products are subject to the same usage. Not all refrigerators are going to Central America, where the weather is hot, and "110 volts" may be only 80 volts. The specifications must cover the most severe use. A defective piece may have only a slight chance of failing because it has only a slight chance of being subjected to the most severe possible use.

The complaint cost of a defect is not the cost of a complaint. It is the cost of a complaint times the chance that a complaint will occur. Thus, we will define the following symbols.

A: The cost of a complaint or other trouble resulting from a defect.
p: The chance that a defect will result in a complaint or other trouble.
$C = Ap$: The complaint cost per defective piece which gets past inspection.

The Process Average Percent Defective: When our production process is operating normally, it produces a percentage of defective pieces. This percentage of defective pieces is often called the process average percent defective, q_1. The process average percent defective, q_1, is the percentage of defective pieces produced when the process is producing normally. It is the percentage of defects that occurs when nothing is wrong.

The defective pieces in the uninspected portion of the lot will be accepted. The number of such defective pieces accepted will depend on the lot size. Thus we have:

N: Lot size; the number of pieces in the lot.
q_1: Process average percent defective; the percentage of defective pieces in the lot when the material is of normal quality.
$(N-n)q_1$: The number of defective pieces in the uninspected part of the lot.
$(N-n)q_1 C$: The complaint (or trouble) cost per lot of defective pieces in the uninspected portion of the lot.

Rework or Replacement Costs: When an inspector finds a defective piece, he rejects it. We must then rework or replace that piece and this rework or replacement is a cost. Therefore, we need the following symbols:

R: The cost of reworking or replacing a single defective piece.
nq_1: Number of defective pieces found per sample inspected
$nq_1 R$: Cost of reworking or replacing defective pieces per lot of q_1 quality submitted for inspection.

If inspection is destructive, the rework cost, R, equals zero, since we have already included in the cost of inspection the cost of replacing every piece inspected.

You will be surprised to discover how often R, the rework or replacement cost, is greater than C, the complaint or trouble cost of a defective piece. In such cases, it is more economical to let the defective piece go by than it is to reject it for repair or replacement. This is why most factories have a review area. Every day, the engineer allows the use of quantities of material which are outside of specifications. The material is not what is wanted, but it is cheaper to use it than to scrap it. When we use defective material, the rework cost, R, should be made equal to the complaint cost, C. The rework cost is replaced by the expected costs of complaints or trouble, which will result from use of the substandard material.

Lot Rejection Cost: A sampling plan will reject some lots, even though they are good lots. Each rejection introduces certain costs. These costs depend upon what is done with the rejected lots. Maybe we inspect the rejected lot 100% and such an inspection costs money. Maybe we scrap the lot and the cost of the pieces scrapped is a real cost. Maybe we return the lot to the supplier with the resulting transportation costs and the costs of any delays in getting a replacing lot.

The lot rejection cost has the same basic types of components as the inspection cost so that we shall use the same symbols with a subscript, d:

J_d: Overhead cost ~~per piece rejected~~ Total of a lot rejection(cost of setting up 100% inspection; returning to supplier; obtaining a substitute lot; inconvenience of going without the material;etc.)

I_d: Variable cost per piece of a lot rejection (inspection cost per piece for detaling; scrap cost per piece if the lot is scrapped; transportation cost per piece if the lot is returned to the supplier; etc.)

R_d: Rework cost per defective piece of a lot rejection (rework cost if defectives are reworked; replacement cost if defectives are replaced; etc.)

$J_d + (N-n)\, I_d + (N-n)\, q_1\, R_d$: Total lot rejection cost per lot rejected.

The formulas developed in this paper will give impossible answers if the rework cost, R_d, is greater than the complaint cost, C. If you are going to use all the defective pieces anyway, you cannot afford to inspect the lot to find the defective pieces. Actually, in practice, some inspection is justified so that you can put pressure on the supplier to improve future lots. This factor is not taken into account in our formulas.

Probability of Acceptance: For any sampling plan the probability that a lot will be accepted can be determined as a function of q, the percentage of defective pieces in the lot. This we define:

P_1: Probability of acceptance of a lot of process average percent defective q_1.

$1-P_1$: Probability of rejection of a lot of process average percent defective, q_1.

$P_1\, (N-n)\, q_1\, C$: The complaint cost per lot of q_1 quality submitted for inspection.

$(1-P_1)\left[J_d + (N-n)\, I_d + (N-n)\, q_1\, R_d\right]$: The lot rejection cost per lot of q_1 quality submitted for inspection.

$T_1 = J + nI + nq_1\, R + P_1\, (N-n)\, q_1\, C.$
$+ (1-P_1)\left[J_d + (N-n)\, I_d + (N-n)\, q_1\, R_d\right]$:
Total cost per lot of q_1 quality submitted for inspection.

Lots of Poor Quality: If all lots submitted were of normal quality, q_1, percent defective, then there would be no need or use of inspection. If q_1 percent defective was satisfactory, we would accept all lots without inspection. If q_1 percent defective was unsatisfactory, we would reject all lots without inspection.

Thus we are interested in the case where some lots are (or may be) submitted which are of substandard quality. In order to simplify the analysis we shall assume that all substandard lots are of the same quality, q_2 percent defective. Of course in practice the substandard lots will not be of the same quality. It is thought, however, that the use of an average value for q_2 will not introduce a serious error. A rigorous analysis of the effect of this approximation will be a worthwhile project for some student.

The total cost for the substandard lots will be the same as that for normal lots with the appropriate changes in the percent defective. Thus we have:

q_2 = Percent defective pieces in the substandard lots.
P_2 = Probability of accepting a lot which is q_2 percent defective.
$T_2 = J + nI + nq_2 R + P_2 (N-n) q_2 C + (1 - P_2)\left[J_d + (N-n) I_d + (N-n) q_2 R_d \right]$: Total cost per lot of Q_2 quality submitted for inspection.

Probability That a Lot is Substandard: The average total cost for all lots, normal and substandard, will depend on the portion of substandard lots submitted for inspection. Past inspection records may give some indication of this portion. Howevr, in most cases it will have to be estimated on the basis of judgement and ones knowledge of the supplier, the product and the manufacturing process. What odds will you give (before inspecting the lot) that the lot will be substandard? Will you bet "10 to 1" or "100 to 1" that the lot is a good lot? Such betting odds probably sum up as completely as possible the probability that the lot will be substandard. Therefore, let -

f: Probability that the lot will be a substandard lot (q_2 percent defective)
1-f: Probability that the lot will be a normal lot (q_1 percent defective)
$T = (1-f)\, T_1 + f\, T_2$: Average total cost for all lots.

III. Mathematical Derivation:

It is suggested that non-mathematical readers skip this section.

The most efficient sampling inspection plan will be that plan which makes the total cost a minimum: The total cost is -

$$(1) \quad \begin{aligned} T &= (1-f)\,T_1 + f\,T_2 \\ &= J + nI + nR\left[(1-f)\,q_1 + fq_2\right] \\ &\quad + (N-n)\,C\left[(1-f)\,P_1\,q_1 + f\,P_2\,q_2\right] \\ &\quad + \left[J_d + (N-n)\,I_d\right]\left[(1-f)\,(1-P_1) + f\,(1-P_2)\right] \\ &\quad + (N-n)\,R_d\left[(1-f)\,(1-P_1)\,q_1 + f\,(1-P_2)\,q_2\right] \end{aligned}$$

This can be rearranged into the more convenient form.

$$(2) \quad T = J + NI + NR\,\bar{q} + J_d - \left[f\,(q_2 - Q)\,(C - R_d)/q_1\right] T'$$

where

(3) $\bar{q} = (1-f)\,q_1 + fq_2$: Average percent defective in all lots.

(4) $Q = \dfrac{\left[J_d/\,(N-n)\right] + I_d}{C - R_d}$: Break-even quality.

(5) $T' = (Nq_1 - nq_1)(G + FP_1 - P_2)$

(6) $G = \dfrac{(I + R\bar{q}) - (I_d + R_d\bar{q})}{f(q_2 - Q)\,(C - R_d)}$

(7) $F = \dfrac{(1 - f)}{(\;f\;)}\,\dfrac{(Q - q_1)}{(q_2 - Q)}$

Now T will be a minimum if T' is a maximum so that we shall restrict ourselves to T' from here on.

In this section we shall study single sampling plans only. Such plans are defined by -

n: Sample size.
c: Acceptance number.

We shall also assume that the q's and the c's are small enough to use the Poisson approximation to the binomial distribution. Thus -

$$(8) \quad P_{i,n,c} = e^{-nq_i}\left\{1 + \frac{nq_i}{\lfloor 1} + \frac{(nq_i)^2}{\lfloor 2} + \dots + \frac{(nq_i)^c}{\lfloor c}\right\}$$

$$(9) \quad P_c - P_{c-1} = e^{-nq_i}\,\frac{(nq_i)^c}{\lfloor c}$$

For a fixed n we can determine the value of c which will make T' a maximum by letting:

$$(10) \quad \begin{aligned} &T'_c - T'_{c-1} = 0 \\ &(Nq_1 - nq_1)\left\{G - G + F\,(P_{1,c} - P_{1,c-1}) - (P_{2,c} - P_{2,c-1})\right\} = 0 \end{aligned}$$

(11) $F e^{-nq_1} \frac{(nq_1)^c}{\lfloor c} = e^{-nq_2} \frac{(nq_2)^c}{\lfloor c}$

(12) $F e^{n(q_2 - q_1)} = \left(\frac{q_2}{q_1}\right)^c$

(13) $\log_e F + n(q_2 - q_1) = c \log_e \left(\frac{q_2}{q_1}\right)$

(14) $nq_1 = \frac{\log_e (q_2/q_1)}{(q_2/q_1) - 1} c - \frac{\log_e F}{(q_2/q_1)-1}$

(15) $= Hc - L$

where

(16) $H = \frac{\log_e (q_2/q_1)}{(q_2/q_1)-1}$

(17) $L = \frac{\log_e F}{(q_2/q_1)-1}$

For each value of the acceptance number, c, this formula gives the value of nq_1 for which $T_c = T_{c-1}$. For values of nq_1 between Hc-L and H(c+1)-L the most efficient acceptance number is c. Note that for c=0, nq_1 is negative. It may also be negative for values of c greater than zero. If c' is the smallest c for which Hc-L is positive, then c'-1 is the smallest acceptance number which can be efficient, even for small samples.

We can now restrict our considerations to sampling plans where c is determined as above. In fact for the moment we shall restrict ourselves to plans for which $nq_1 = Hc-L$, $c = c', c'+1, c'+2, \ldots$. For these plans we have -

(18) $T_c'' = (Nq_1 - nq_1)(G + F P_{1,c} - P_{2,c})$

(19) $= \left[Nq_1 - (Hc-L)\right] (G + F - S_c)$

(20) $S_c = F (1 - P_{1,c}) + P_{2,c}$

We can determine which of these plans is most economical by setting -

(21) $T''_c = T''_{c+1}$

(22) $(Nq_1 - Hc + L) (G + F - S_c) = (Nq_1 - H(c+1)+L)(G + F - S_{c+1})$

(23) $(Nq_1 - Hc + L)(S_{c+1}-S_c) = - H(G + F - S_c+1)$

(24) $\frac{Nq_1 + L}{H} = \frac{G + F - S_{c+1}}{S_c - S_{c+1}} + c$

The left side of (24) is a constant and the right side is a monotonic increasing function of c. The right side can be calculated for two or three trial values of c and the results interpolated to obtain the c" for which the right side equals the left.

If for c' (the smallest c for which $nq_1 = Hc - L$ is positive) the right side of (24) is larger than the left side, we know that

(1) c" is between c' - 1 and c' or
(2) We are in the region where no inspection is justified.

The second condition will usually be the case. This can be verified by checking that the total cost is less for n = c'-1 than for n = c' when the acceptance number is c' - 1.

The Poisson approximation does not hold in this case so we revert to the binomial distribution. Then

(27) $P = 1 \; ; \; n = c'-1, \quad c = c'-1.$

(28) $P = 1-q_1^{c'}; n = c' \quad , \quad c = c'-1.$

(29) $T'_{c} = (Nq_1 - c'q_1)\left[G - F\,(1 - q_1^{c'}) - (1 - q_2^{c'})\right]$

(30) $T'_{c'-1} = \left[Nq_1 - (c'-1)\,q_1\right]\left[G + F - 1\right]$

(31) $T'_{c'} - T'_{c'-1} = q_1\,(G + F - 1) - (Nq_1 - c'q_1)\,(Fq_1^{c'} - q_2^{c'})$

Thus no sampling will be indicated where

(32) $T'_{c'} - T'_{c'-1} < 0$

or

(33) $G + F - 1 < (N - c')(Fq_1^{c'} - q_2^{c'})$

Since c' will usually be small, this is readily rarified. If (33) does not hold, n has some value between c' and $(Hc'-L)/q_1$. It would appear that no great harm will be done by using in every case:

(34) $n = (Hc' - L)/q_1$

(35) $c = c' - 1$

IV. Routine Calculations:

The attached work sheet has been drawn up to bring in one place all the factors necessary to determine the most efficient single sampling plan for lot inspection under any particular set of cost factors. In using the work sheet, be sure to keep in mind the explanationsgiven in Section II. The short titles given on the work sheet may not be a complete description of all the factors involved in each item.

The following notes may be helpful in carrying out these calculations:

(7) n_E is an estimate of the sample size which will be used. The final answer will not be changed much even though n_E is rather far off. Pick a number out of the air and forget about it.

(20) H can be calculated with one setting on a log log slide rule. Set $(q_2/q_1)-1$ on the c scale opposite q_2/q_1 on the LL2 (or LL3) scale. Read the answer on the D scale opposite 1 on the C scale. On regular slide rules, you can read the mantissa of $\log_{10}(q_2/q_1)$ on the L scale opposite (q_2/q_1) on the D scale.

(21) See (20) above.

(30) The values of P_1 (the probability of accepting a lot of quality q_1 percent defective) can best be read from a chart of the Poisson distribution such as that given by Dodge and Romig (Page 44). Enter the chart with nq_1 on the bottom scale and read P_1 on the vertical scale at the point (nq_1, c).

(31) Note that if the chart for P_1 has nq plotted on a log scale, then the points for nq_2 are a constant distance to the right of the points for nq_1. These can then be very easily marked off on the chart if you mark off the distance "1" to "q_2/q_1" on the lower edge of a triangle. If a second triangle is placed on the vertical line through nq_1, then the first triangle can be slid up or down against it until the marked point coincides with the line for the desired c.

(32) See Above.

(34) Note that in calculating ΔS_c that you pick up S_{c+1} from the column for the next larger c.

(35) See (34)

(37b) We have not found any convenient way to solve for c" directly. We have to use successive approximation. Guess a value of c and try it. From the result, make a better guess and try it. Keep this up until you have narrowed it down between two successive integers.

V. Conclusion

The calculations involved here are not simple enough to use wholesale. Their usefulness will probably be largely in drawing up sampling tables for particular sampling jobs which occur regularly and on jobs where inspection is expensive or quality is critical. The methods proposed here provide a standard against which all other methods for choosing single sampling inspection plans can be measured.

This work should be extended to include double sampling plans. Whenever c is more than 5 or 10 by these calculations, it is probable that a double sampling plan will result in worthwhile savings.

This same approach should also be applied to sequential sampling plans. There is some hope that sequential plans may have a simpler arithmetic than single and double sampling plans.

WORK SHEET

SINGLE SAMPLING PLAN HAVING SMALLEST POSSIBLE COST

(1) I = ________: Inspection cost per piece excluding overhead.

(2) I_d = ________: Rejection cost per piece excluding overhead.

(3) J_d = ________: Rejection cost per lot (overhead only)

(4) R = ________: Rework cost per defective piece found in samples.

(5) R_d = ________: Rejection cost per defective piece in rejected lots.

(6) N = ________: Size of lot.

(7) n_E = ________: Estimate of sample size (does not have to be at all accurate)

(8) A = ________: Complaint cost per complaint

(9) p = ________: Probability that a defective piece will cause a complaint

(10) C = ________: (8) x (9): Complaint cost per defective piece

(11) Note: If C is less than R_d, then all lots should be accepted without inspection.

(12) Q = ________ $= \left\{ \left[J_d/(N-n_E)\right] + I_d \right\} /(C-R_d)$: Break-even quality.

(13) q_1 = ________: Average percent defective of lots which are better than Q%.

(14) q_2 = ________: Average percent defective of lots which are poorer than Q%.

(15) f = ________: Probability that a lot will be poorer than Q%.

(16) F = ________: $\left[(1-f)/f\right]\left[(Q-q_1)/(q_2-Q)\right]$

(17) $\bar{q}$ = ________: $(1-f)\ q_1 + f\ q_2$: Average percent defective in all lots.

(18) G = ________: $\left[(I+R\bar{q}) - (I_d + R_d\ \bar{q})\right] / \left[(f)(q_2-Q)(C-R_d)\right]$

(19) q_2/q_1 ________: Operating ratio.

(20) H = ________: $\left[\log_e\ (q_2/q_1)\right]/\left[(q_2/q_1) - 1\right] = 2.3\ \left[\log_{10}\ (q_2/q_1)\right]/\left[(q_2/q_1)-1\right]$

(21) L = ________: $\left[\log_e F\right]/\left[(q_2/q_1) - 1\right] = 2.3\ \left[\log_{10} F\right]/\left[(q_2/q_1) - 1\right]$

(22) W = ________: $(Nq_1 + L)/H$

(23) V = ________: $G + F$

(24) c' = ________: L/H: Smallest value of c (use next larger integer).

(25) U = ________: $(N-c')(Fq_1{}^{c'} - q_2{}^{c'})$

(26) Note: If U is greater than V-1, it is economical to accept the lot without any inspection at all.

(27) Make calculation (28) to (33) for c' and $c'+1$ and make calculations (34) to (36) for c'.

(28) c = Acceptance No.

(29) $nq_1 = Hc - L$

(30) $1-P_1$: at (c, nq_1)

(31) $nq_2 = nq_1 \, (q_2/q_1)$

(32) P_2 : at (c, nq_2)

(33) $S_c = F(1-P_1) + P_2$

(34) $\triangle S_c = S_c - S_{c+1}$

(35) $(V-S_{c+1})/\triangle S_c$

(36) (35) + C - W

(37) (a) If (36) is positive for $c = c'$, use the sampling plan:
$n = Hc' - L$: sample size
$c = c' - 1$: acceptance number.
(b) If (36) is negative for $c = c'$, repeat calculation (28) to (36) for an estimated c greater than c'.

(38) (a) If (36) is positive, repeat (28) to (36) for a smaller c.
(b) If (36) is negative, repeat (28) to (36) for a larger c.
(c) Continue to repeat the calculations until two successive c's are found which give opposite signs on line (36).

(39) c'' ________: Interpolate for a fractional c which will give zero on line (36).

(40) Use the sampling plan:
n = ________: $[H(c''+ 0.5) - L]/q_1$: Sample size.
c = ________: Nearest integer to c'': Acceptance number.

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WORK SHEET

SINGLE SAMPLING PLAN HAVING SMALLEST POSSIBLE COST

(1) I = ________ : Inspection cost per piece excluding overhead.

(2) I_d = ________ : Rejection cost per piece excluding overhead.

(3) J_d = ________ : Rejection cost per lot (overhead only)

(4) R = ________ : Rework cost per defective piece found in samples.

(5) R_d = ________ : Rejection cost per defective piece in rejected lots.

(6) N = ________ : Size of lot.

(7) n_E = ________ : Estimate of sample size (does not have to be at all accurate)

(8) A = ________ : Complaint cost per complaint

(9) p = ________ : Probability that a defective piece will cause a complaint

(10) C = ________ : (8) x (9): Complaint cost per defective piece

(11) Note: If C is less than R_d, then all lots should be accepted without inspection.

(12) Q = ________ $= \left\{ \left[J_d/(N-n_E) \right] + I_d \right\} / (C-R_d)$: Break-even quality.

(13) q_1 = ________ : Average percent defective of lots which are better than Q%.

(14) q_2 = ________ : Average percent defective of lots which are poorer than Q%.

(15) f = ________ : Probability that a lot will be poorer than Q%.

(16) F = ________ : $\left[(1-f)/f\right]\left[(Q-q_1)/(q_2-Q)\right]$

(17) $\bar{q}$ = ________ : $(1-f)\, q_1 + f\, q_2$: Average percent defective in all lots.

(18) G = ________ : $\left[(I+R\bar{q}) - (I_d + R_d\,\bar{q})\right] / \left[(f)(q_2-Q)(C-R_d)\right]$

(19) q_2/q_1 ________ : Operating ratio. .1404

(20) H = ________ : $\left[\log_e (q_2/q_1)\right] / \left[(q_2/q_1) - 1\right] = 2.3 \left[\log_{10} (q_2/q_1)\right] / \left[(q_2/q_1) - 1\right]$

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(21) L = ________ : $\left[\log_e F\right] / \left[(q_2/q_1) - 1\right] = 2.3 \left[\log_{10} F\right] / \left[(q_2/q_1) - 1\right]$

(22) W = ________ : $(Nq_1 + L)/H$

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(23) V = ________ : $G + F$

(24) c' = ________: L/H: Smallest value of c (use next larger integer).

(25) U = ________: $(N-c')(Fq_1^{c'} - q_2^{c'})$

(26) Note: If U is greater than V-1, it is economical to accept the lot without any inspection at all.

(27) Make calculation (28) to (33) for c' and c'+1 and make calculations (34) to (36) for c'.

(28) c = Acceptance No.

(29) $nq_1 = Hc - L$

(30) $1-P_1$: at (c, nq_1)

(31) $nq_2 = nq_1 (q_2/q_1)$

(32) P_2 : at (c, nq_2)

(33) $S_c = F(1-P_1) + P_2$

(34) $\Delta S_c = S_c - S_{c+1}$

(35) $(V-S_{c+1})/\Delta S_c$ - q1'

(36) (35) + C - W

(37) (a) If (36) is positive for c = c', use the sampling plan:
$n = \frac{Hc' - L}{q_1}$: sample size
$c = c' - 1$: acceptance number.
(b) If (36) is negative for c = c', repeat calculation (28) to (36) for an estimated c greater than c'.

(38) (a) If (36) is positive, repeat (28) to (36) for a smaller c.
(b) If (36) is negative, repeat (28) to (36) for a larger c.
(c) Continue to repeat the calculations until two successive c's are found which give opposite signs on line (36).

(39) c" ________: Interpolate for a fractional c which will give zero on line (36).

(40) Use the sampling plan:
n = ________: $[H(c'' + 0.5) - L]/q_1$: Sample size.
c = ________: Nearest integer to c": Acceptance number.

File #136

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3/16/49

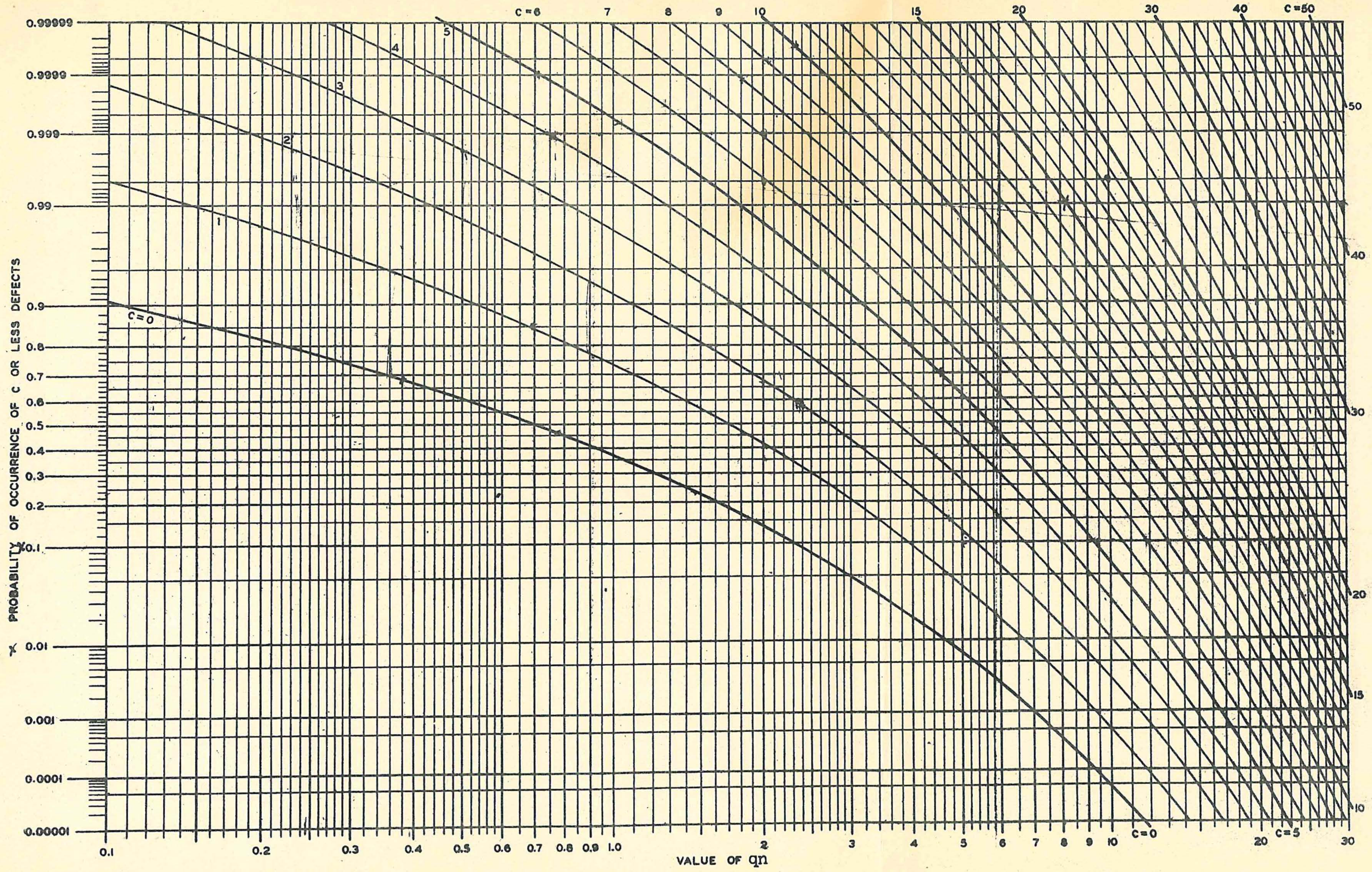

Fig. 6—Cumulative probability curves—Poisson exponential. For determining probability of occurrence of *c* or less defects in a sample of *n* pieces selected from an infinite universe in which the fraction defective is q. (A modification of chart given by Miss F. Thorndike *B. S. T. J.*, October 1926).