SPACE RULE HANDBOOK

MARTIN

KAYTEN

THE MARTIN SPACE RULE

Handbook of Instructions and Basic Space Data for Scientists and Engineers

MARTIN

INTRODUCTION

Your *Martin Space Rule* is an instrument specifically designed to aid the student of astronautics and the skilled engineer in solving preliminary design problems that are most frequently encountered in several space flight technological areas.

For example, the *Space Rule* enables the user to determine very rapidly individual booster stage sizes and takeoff gross weights for single- or multiple-staged boosters, as well as the mission velocity requirements and associated flight parameters for most ballistic, orbital and interplanetary problems. The rapid methods for solving problems produce results with acceptable degrees of accuracy. However, applications of the *Rule* are not limited to quick answers. Techniques are presented which also enable the user to conduct various parametric and optimization studies to achieve a very high degree of accuracy without resorting to a computer. In addition, an insight into the effects of variation of the basic design parameters may be attained.

The only prerequisites for the use of the *Rule* are a familiarity with the basic slide rule and a rudimentary understanding of rocketry.

Your *Martin Space Rule* was originally conceived by Michael Stoiko and subsequently developed to its present status jointly with Werner Furth. Both men are engineers at Martin's Space Systems Division.

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I

YOUR SPACE RULE

The layout of the *Rule* and the grouping of the scales facilitate calculations in four space technology categories:

1. Booster design.
2. Exterior ballistics.
3. Orbital mechanics
4. Interplanetary travel.

Scales pertinent to each category are arranged together on the *Rule*, reducing the number of operations required in problem solving. The "front" of the *Rule* may be identified as the side having the D scale and the slide with the C scale showing. The "back" side of the *Rule* presents a completely solid face.

RATIONALE OF THE SPACE RULE

A man driving his car measures the magnitude of his trip (the mission) by the distance traveled, and the performance of his car by the distance traveled per gallon of gasoline. In modern rocketry, however, velocity rather than distance is the common denominator of a mission capability. A familiar example of this concept is the mission velocity normally associated with an escape from the earth's gravitational field, i.e., 36,700 fps.

Procedures for solving space problems with the *Space Rule* are derived from the relationship of velocity, mission, and performance. Four basic principles underlie the method:

1. It is possible to define a particular mission by a velocity.
2. It is possible to describe the performance capability of a vehicle by a velocity.
3. It is possible to account for performance losses due to gravity and aerodynamic drag, by a velocity.
4. Requirements for maneuvering, reserve performance, safety factors, stabilization, vehicle orientation, payload variations, and other criteria can also be expressed by a velocity.

The velocities described above are all necessary factors in determining the characteristic mission velocity. The characteristic mission velocity is the concept around which the construction of the *Space Rule* and the computational techniques outlined in this handbook are developed. The user of the *Space Rule* should understand and appreciate this fundamental concept.

CHARACTERISTIC MISSION VELOCITY

For each specific spaceflight mission there is a unique velocity associated with that mission. For a rocket booster to perform the required mission, it must achieve a velocity equal to that unique velocity at the end of its powered flight phase (burnout).

If an initially stationary booster is flown in a drag-free, gravity-free environment, the booster would be sized to carry only that amount of propellant required to produce the velocity increment equivalent to the burnout velocity. But since all boosters launched from earth incur velocity losses due to gravity, aerodynamic drag, and from other sources, these velocity losses must be added to the burnout velocity to determine the total velocity potential that the booster must have in order to successfully perform the given mission. This total velocity requirement is defined as the *characteristic mission velocity.*

EARTH'S ROTATIONAL VELOCITY

Determination of the total velocity requirements, for orbital missions, must also take into consideration the velocity component due to the rotation of the earth about its polar axis. Because of this rotation, a point on the equator moves due east at 1520 fps relative to the center of the earth. At latitudes other than the equator, the velocity component is also due east but is equal to 1520 cos λ fps.

Consequently, a booster launched due east inherits the earth's rotational velocity component, thereby reducing the total velocity requirement for the mission by an amount equal to the earth's rotational velocity.

If the booster is launched due west, then the total velocity requirements must be increased by the amount of the earth's rotational velocity. For polar orbits, the rotation of the earth may be neglected, since the direction of the earth's rotation is perpendicular to the final flight directions.

For initial booster design purposes, the effective initial velocity due to the rotation at the earth may be assumed to be

$$Vr = r_e\omega \cos \lambda \sin \beta$$

where:

r_e = radius of earth

ω = angular velocity of the earth

λ = launch latitude

β = launch azimuth, measured clockwise from north.

From Cape Canaveral, this velocity is 1340 sin β fps.

For ballistic missions, the rotational velocity of the earth is to be neglected.

EARTH'S ORBITAL VELOCITY

Just as the earth's rotational velocity applies to earth-centered orbits, the earth's orbital velocity applies to sun-centered orbits. The earth is moving in an orbit around the sun at a velocity of 97,760 fps. For problems involving transfer between coplanar outer planets, launching in the orbital direction—that is, taking advantage of the earth's orbital speed—is the only practical means of conducting interplanetary missions.

For travel to the inner planets, or even to the sun, it is advantageous to decrease the velocity of the spacecraft. This velocity is measured in respect to the sun. The decrease requires a velocity increment in the direction opposite to the starting velocity of 97,760 fps.

VELOCITY LOSSES

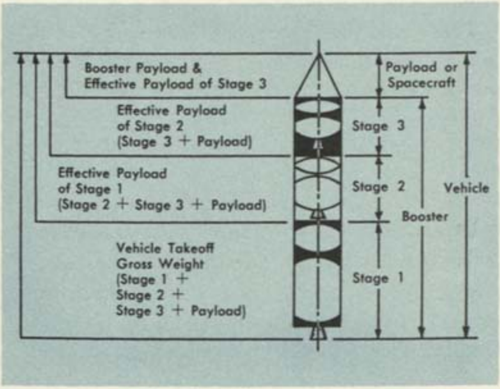
Raising a particle from the earth requires doing work against gravity. Therefore, if we add a fixed amount of energy to a particle (or payload), the final velocity depends on how high we raise the particle during the process of adding energy. The higher we raise the particle, the lower the velocity. The decrease in the final velocity of a booster due to the increase in altitude is referred to as a velocity loss due to gravity.

In the same manner, moving a particle

through the atmosphere works against aerodynamic forces such as drag. This, too, results in a slow down or velocity loss.

Appendix A presents a table of mission velocity loss estimates which can be applied with reasonable confidence in the absence of more precise information. The table makes no attempt to isolate individual velocity loss contributions. Rather, it uses representative criteria from the mission profile for estimating the losses, which are a composite of such factors as booster drag, flight path control, thrust-to-weight history and burnout conditions.

NOMENCLATURE — TYPICAL THREE-STAGE VEHICLE



II

PROPELLANT MASS FRACTION

Estimating the propellant mass fraction of a stage is difficult in the initial phases of design. In lieu of an engineering basis for this estimate, the curves presented in Appendix B may be used. These curves express an empirical relationship between λ, payload and the propellant mass fraction. Since stage performance is extremely sensitive to the propellant mass fraction, an accurate value for this parameter should be ascertained as quickly as possible from a detailed weight analysis.

There are three auxiliary scales on the front face of the *Space Rule* which are applicable to the problem of determining the stage's mass fraction.

DEFINITION OF SCALES

Utilization of these scales will determine the following design parameters:

$\%W_{pr}$—percentage of the propellant loaded that remains in the stage at burnout.

$\%W_d$—stage dry weight divided by the total stage weight (payload excluded). Ratio is expressed as a percentage.

MF—propellant mass fraction of a stage, defined as the ratio of the stage propellants consumed for power divided by the gross weight of the stage (payload excluded). Thus, the mass fraction is a number greater than zero and less than one.

OPERATION OF SCALES

The *Rule* is constructed so that if any two of the values for $\%W_{pr}$, $\%W_d$ or MF are known, the third value is fixed and read directly. Furthermore, the physical limits within which these parameters can vary are established by one setting of the *Space Rule*.

EXERCISE ONE: Assuming that the weight of a stage structure is 12.5% of the stage weight and that the weight of the residual propellant is 7.5% of the total loaded propellant weight, find the stage mass fraction.

(1) On the $\%W_d$ scale, place the hairline over 12.5.

(2) Adjust the slide so that 7.5 on the $\%W_{pr}$ scale lies under the hairline.

(3) With the slide stationary, move the hairline to the indicator, ↓MF.

(4) Under the hairline, read:

MF—propellant mass fraction: 0.809.

EXERCISE TWO: Determine, by the *Space Rule*, the limits of $\%W_{pr}$ and $\%W_d$. Assume that the booster stage has a propellant mass fraction of 0.9.

(1) On the MF scale, place the hairline over 0.9.

(2) Adjust the slide so that the indicator, ↓MF, lies under the hairline. With the slide stationary, move the hairline to the left index on the $\%W_{pr}$ scale.

(3) Under the hairline, read:

$\%W_d$—percentage dry weight: 10% (assuming no residual propel lants).

(4) On the $\%W_d$ scale, move hairline to 0.

(5) Under the hairline, read:

$\%W_{pr}$—residual propellants: 10% (the maximum even for a perfect weightless structure).

In practice, the $\%W_{pr}$ normally does not exceed 2%. For first approximations in design work, 1% is an acceptable mean value. Design judgment must be exercised in an assumption of $\%W_d$ or MF, but generally it is easier to find a working value for MF.

The propellant mass fraction may be specified or it may be calculated by the use of the *Rule* and the procedures established above. If the required information is not available, the curves in Appendix B may be utilized. These curves relate λ, payload and the propellant mass fraction.

λ is found at the same time as the K' value, and therefore, it varies as K', and is a function of the velocity of the stage. The definition of λ and K' are found in Section V. Payload, generally, is a known or given value for most problems one encounters in design.

III

EXTERIOR BALLISTICS

The scales grouped on the lower back or solid face of the *Space Rule* are used for exterior ballistics as applied to ballistic missile flight. In developing these scales, it was assumed that the earth is a stationary sphere without atmosphere, that burnout velocity is reached at zero altitude, that the ballistic trajectory, after burnout, is unpowered for the remainder of the flight, and that the burnout flight path angle is optimum for maximum range.

DEFINITION OF SCALES

The definitions of the exterior ballistic scales are:

V_2—velocity at burnout of booster (10^3 fps).

R_1—range from burnout at low altitude to impact on earth surface (10^3 stat mi—downrange).

γ_{bo}—flight path angle at burnout (deg—from horizontal).

T_F—time of flight from burnout to impact (min).

H_a—maximum altitude of flight (stat mi).

OPERATION OF SCALES

Determining the related flight parameters on the ballistic missile scales is simply and conveniently accomplished with one setting of the hairline. If either the burnout velocity (V_2) or the range (R_i) is known, then setting the hairline on either of the known values will determine the remaining vehicle performance parameters.

EXERCISE ONE: For a 5000-statute-mile ballistic missile, determine the characteristic mission velocity and the related flight parameters for this mission.

(1) On the R_i scale, set the hairline over 5 (5000 stat mi).

(2) Under the hairline, read:

V_2—required burnout velocity: 22.36 (22,360 fps).

γ_{bo}—burnout angle: 26.9 deg.

T_F—time of flight: 28.1 min.

H_a—height at apogee: 787 stat mi.

The 22,360 fps is the burnout velocity required to achieve the range of 5000 statute miles and does not include the losses incurred by the vehicle during powered flight. Typical flight losses (given in Appendix A) must be added to the burnout velocity. In this mission, the losses are 4000 fps. Therefore, at launch, the booster must have a capability of delivering a total or characteristic mission velocity of 26,360 fps.

IV

EARTH ORBITAL MECHANICS

The earth orbital mechanics scales are contained on the solid face of the *Rule* and are grouped primarily in the upper half. These scales utilize one important assumption: that all orbits found on these scales have a 200-mile perigee. The velocities relate to a stationary, homogeneous sphere (therefore, to inertial space), and external forces other than the earth's gravity (e.g., atmospheric effects, influence of other bodies) are neglected.

DEFINITION OF SCALES

The orbital parameter scales on the upper part of the *Rule* are:

ε—eccentricity of the orbit.

V_a—velocity at apogee (10^3 fps).

h_a—altitude at apogee (10^3 stat mi).

h_m—mean altitude of the orbit (10^3 stat mi)

τ—orbital period (hr).

V_1—velocity at perigee (10^3 fps).

At the bottom of the *Rule*, on the same face, is an additional orbital mechanics scale:

h_c—altitude of circular orbit (10^3 stat mi).

This scale is used in conjunction with the V_2

scale, which then shows the corresponding circular orbit velocity (in 10^3 fps) at any desired altitude (h_c).

OPERATION OF SCALES

One setting of the hairline suffices to determine the several correlated orbital parameters, just as it did for exterior ballistics.

In determining the period (τ) of a circular orbit, the h_m scale rather than h_a scale is used as the orbit altitude. Use of the h_m and τ scales also defines the period of any orbit, since all orbits (elliptical or circular) sharing a common mean altitude have identical periods. In particular, the period of a circular orbit is immediately determined since the mean altitude is equal to the orbital height.

EXERCISE ONE: Find the circular velocity for a 1000-statute-mile orbit.

(1) On the h_c scale, set the hairline over 1 (1000 stat mi).

(2) Under the hairline, read:
V_2—orbital circular velocity: 23.2 (23,200 fps).

EXERCISE TWO: Find the period of a 1000-statute-mile circular orbit.

(1) On the h_m scale, set the hairline over 1 (1000 stat mi).

(2) Under the hairline, read:

τ—orbital period: 1.97 hr.

EXERCISE THREE: A satellite is injected with a zero flight path angle into orbit at an altitude of 200 miles with a velocity of 33,280 fps. Find the orbital parameters.

(1) On the V_1 scale, set the hairline over the injection velocity: 33.28 (33,280 fps).

(2) Under the hairline, read:

τ—orbital period: 10.6 hr.

h_m—mean altitude: 11.3 (11,300 stat mi).

h_a—apogee altitude: 22.4 (22,400 stat mi).

V_a—velocity at apogee: 5.24 (5240 fps).

ε—orbital eccentricity: 0.727.

It should be noted that with an injection velocity of 35,808 fps the orbital payload would have reached an altitude at apogee of infinity and would have escaped the earth's gravitational field. For injection velocities higher than 35,808 fps, the orbit is hyperbolic, rather than elliptical. The terms τ, h_m, h_a, and V_a are not defined. However, the eccentricity of the orbit is a meaningful quantity, even though greater than 1, and is shown for velocities higher than the escape velocity.

EXERCISE FOUR: For Exercise Three, find the velocity which must be added to the satellite to establish a circular orbit at the apogee altitude of 22,400 statute miles.

(1) On the h_c scale, set the hairline over 22.4 (22,400 stat mi).

(2) Under the hairline, read:

V_2—orbital circular velocity: 10.05 (10,050 fps).

(3) Subtract the satellite velocity at apogee from the velocity required to maintain a circular orbit. This is the velocity increment which must be added to the satellite at apogee to inject into a circular orbit. Thus:

$$\Delta V = 10{,}050 - 5240 = 4810 \text{ fps.}$$

EXERCISE FIVE: In Exercise Four above, find the period of the circular orbit.

(1) On the h_m scale, set the hairline over 22.4 (22,400 stat mi).

(2) Under the hairline, read:

τ—orbital period: 24.2 hr.

In the preceding example, it is well to remember that when the satellite is launched from earth, it will take 5.3 hours to reach apogee. This is one-half of the orbital period of the elliptical transfer orbit. At this time, the ΔV of 4810 fps is added to the satellite to inject it into a circular orbit at the apogee altitude of 22,400 statute miles. This final velocity increment will be added when the satellite is halfway around the world, in reference to the 200-statute-mile perigee.

V

BOOSTER DESIGN

The booster design scales relating the physical characteristics of a vehicle to the vehicle's performance are located on the front of the *Rule* (slide side). For these calculations the slide must be inserted with the C scale showing.

DEFINITION OF SCALES

On the upper half of the *Rule* are the scales used to establish the stage weight of an n-stage booster:

λ—ratio of the initial weight of a stage at launch to its final weight at burnout.

K_4—ratio of the (n - 3rd) stage weight to the payload weight.

K_3—ratio of the (n - 2nd) stage weight to the payload weight.

K_2—ratio of the (n - 1st) stage weight to the payload weight.

K_1—ratio of the nth stage weight to the payload weight.

Scales on the "slide" are:

I_{sp}—engine overall specific impulse (sec)*.

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* For engines operating from sea level to vacuum conditions, an average value I_{sp} should be used. For engines operating in a vacuum, the I_{sp} value at this condition is used.

K_o—numerically equal to K′.

$\%W_{pr}$—see Section II.

C—conventional slide rule C scale.

The following scales are on the lower half of the *Rule:*

D—conventional slide rule D scale.

K′—$\dfrac{\lambda}{\lambda-1}$.

$\%W_d$—see Section II.

The standard slide rule C and D scales permit multiplication and division. The D scale has an additional function in that it is used as a velocity scale for stage sizing calculations.

OPERATION OF SCALES

The operation of the booster design scales requires positioning the slide twice and moving the hairline four times. These operations are performed in the following order.

(1) When the characteristic mission velocity for the stage or the booster is established, the hairline is set to this value on D, the velocity scale.

(2) The slide is moved so that the stage specific impulse as shown on the I_{sp} scale is also under the hairline.

(3) The values of K′ and λ are now determined by moving the hairline to the in-

dex on the C scale.* The value of K′ is recorded.

(4) On the MF scale, the hairline is next moved to the stage propellant mass fraction value, and the right index of the C scale is moved under the hairline.

(5) With this setting of the slide, the previously found value of K′ is transferred to the K_o scale, i.e., the hairline is moved over this value on the K_o scale.

(6) Then the value of K_1 is read. This value is the ratio between the first-stage weight and the payload weight.

For *simplified and rapid determination* of the stage weight and launch weight of an n-stage vehicle, the relative sizes can be found from Step (6) above by just reading the values of K_1, K_2, K_3, etc. The only provision is that the characteristic mission velocity on the D scale is divided by the number of stages and that the average values are used for the specific impulse and the mass fraction. For more detailed design, the stage parameters may be calculated individually. This involves substituting the appropriate mass fractions for each stage, beginning with the top stage (for which the payload is known) and continuing down (the payload for each stage being the weight of all

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* In some problems, the opposite index must be used in the same manner as a standard slide rule.

higher stages plus the vehicle payload). The variation between the basic and detailed methods is slight, however, particularly when the stage mass fractions are not grossly different.

The ratios obtained by using the K_1, K_2, K_3, and K_4 scales are summarized in Table 1 (on the next page) for multiple-stage boosters. The K factors represent the ratio of the stage weight to the payload or spacecraft weight. This table also provides the expressions for determining the booster gross weight (W_o) and gross weight-to-payload ratio in terms of the payload weight (W_{PL}), K, and stage weight (W_{s_n}).

EXERCISE ONE: Find the stage weight, launch weight, propellants consumed, burnout weight, and the ratio of the launch weight to the burnout weight of a single-stage booster which is to deliver a velocity increment of 15,000 fps. Assume a stage mass fraction of 0.9, an effective specific impulse of 300 seconds, and a payload weight of 1000 pounds. The velocity losses are zero.

(1) On the D scale, set the hairline to 1.5 (15,000 fps) and move the slide until 300 on the I_{sp} scale is under the hairline. Now index the hairline at the end of the C scale.

(2) Under the hairline, read:

$K' = 1.268.$

TABLE 1
K Factor Definition

Scale or Term	Number of Booster Stages			
	1	2	3	4
K_4				W_{s_1}/W_{PL}
K_3			W_{s_1}/W_{PL}	W_{s_2}/W_{PL}
K_2		W_{s_1}/W_{PL}	W_{s_2}/W_{PL}	W_{s_3}/W_{PL}
K_1	W_{s_1}/W_{PL}	W_{s_2}/W_{PL}	W_{s_3}/W_{PL}	W_{s_4}/W_{PL}
W_0	$W_{PL}+W_{s_1}$	$W_{PL}+W_{s_1}+W_{s_2}$	$W_{PL}+W_{s_1}+W_{s_2}+W_{s_3}$	$W_{PL}+W_{s_1}+W_{s_2}+W_{s_3}+W_{s_4}$
W_0/W_{PL}	$1+K_1$	$1+K_1+K_2$	$1+K_1+K_2+K_3$	$1+K_1+K_2+K_3+K_4$

With the same setting, one can read that $\lambda = 4.73$, which means that the launch weight is 4.73 times heavier than the burnout weight.

In some problems, the ratio of takeoff gross weight to burnout weight, λ, will be known or assumed at the outset. In this case, the value of K′ is found immediately by setting the hairline over the λ value (4.73) and directly reading 1.268 under the K′ scale.

(3) On the MF scale, set the hairline to 0.90. Move the right index of the C scale under the hairline. Then on the K_o scale set the hairline to 1.268 (K′ value).

(4) Under the hairline, read:

$$K_1 = 7.08.$$

(5) The weight of the stage loaded is:

$$W_s = W_{PL} \times K_1 = 1000 \times 7.08$$
$$= 7080 \text{ lb.}$$

(6) The launch weight of the booster is:

$$W_o = W_s + W_{PL} = 7080 + 1000$$
$$= 8080 \text{ lb.}$$

(7) The weight of propellants consumed is:

$$W_{pu} = W_s \times MF = 7080 \times 0.9$$
$$= 6372 \text{ lb.}$$

(8) The burnout weight is:

$$W_{bo} = W_o - W_{pu} = 8080 - 6372$$
$$= 1708 \text{ lb.}$$

(9) The launch weight to burnout weight ratio is:

$$W_o/W_{bo} = 8080/1708 = 4.73.$$

The value 4.73 is identical to the value of λ found initially.

If a mass fraction of 0.79 had been used instead of 0.9, notice that $K_1 = \infty$. This means that no matter how large the stage is, it will never accomplish the mission. Physically, the burnout weight of the stage with zero payload is $1/\lambda$ or $1/4.73$ of the stage weight. Consequently, all the permissible weight is in the stage, none being left for the payload.

EXERCISE TWO: Find the range of a single-stage IRBM with a launch weight of 46,500 pounds carrying a 3000-pound payload. The stage has a propellant mass fraction of 0.91 and an average I_{sp} of 270 seconds.

(1) Find K_1, the ratio of booster weight to payload weight:

$$K_1 = \frac{46{,}500 - 3000}{3000} = 14.5.$$

(2) On the MF scale, set the hairline over 0.91 and move the slide so that the right-hand C index is under the hairline. Then, on the K_1 scale, set the hairline over 14.5.

(3) Under the hairline, read:

$$K_o = 1.175.$$

(4) On the K′ scale, set the hairline over 1.175 and move the slide so that the right index is under the hairline. Then, on the I_{sp} scale, set the hairline over 270.

(5) Under the hairline, read:

$D = 1.66$ (16,600 fps). This is the characteristic mission velocity.

(6) From Appendix A, find 3000 fps as the estimated velocity loss for an IRBM-class vehicle. Therefore, the burnout velocity is:

$V_{bo} = 16{,}600 - 3000 = 13{,}600$ fps.

(7) On the V_2 scale, set the hairline over 13.6 (13,600 fps).

(8) Under the hairline, read:

$R_1 = 1.27$ (1270 stat mi).

EXERCISE THREE: It is desired to have a three-stage vehicle deliver a 1000-pound payload to a 45,000-fps velocity, assuming no flight velocity losses. The specific impulse of each stage is 450 seconds and the mass fraction of each stage is 0.9.

(1) The average mass fraction is:

$(0.9 + 0.9 + 0.9)/3 = 0.9.$

(2) The average specific impulse is:

$(450 + 450 + 450)/3 = 450$ sec.

(3) The velocity delivered per stage is:

$$45{,}000/3 = 15{,}000 \text{ fps.}$$

(4) On the D scale, set the hairline on 1.5 (15,000 fps) and move the slide until the specific impulse value of 450 is under the hairline; then move the hairline to the right index on the C scale.

(5) Under the hairline, read:

$$K' = 1.55.$$

(6) On the MF scale, set the hairline on 0.90 and move the slide until the right index on the C scale is under the hairline. Now, on the K_o scale, move the hairline to $K_o = 1.55$.

(7) Under the hairline, read:

$$K_1 = 2.53;\ K_2 = 8.9;\ K_3 = 31.5.$$

(8) Compute the stage weights:

$$W_{s_3} = K_1 \times W_{PL} = 2.53 \times 1000 = 2530 \text{ lb.}$$

$$W_{s_2} = K_2 \times W_{PL} = 8.90 \times 1000 = 8900 \text{ lb.}$$

$$W_{s_1} = K_3 \times W_{PL} = 31.50 \times 1000 = 31{,}500 \text{ lb.}$$

(9) The booster weight at launch is:

$$W_o = W_{s_1} + W_{s_2} + W_{s_3} + W_{PL}$$

$$W_o = 31{,}500 + 8900 + 2530 + 1000 = 43{,}930 \text{ lb.}$$

EXERCISE FOUR: Determine the payload weight of a three-stage vehicle with a launch weight of 1,000,000 pounds which must deliver a payload into a 173-statute-mile circular polar orbit. Assume specific impulses of 305, 425, and 423 and mass fractions of 0.92, 0.90, and 0.88 for Stages 1 through 3, respectively.

(1) On the h_c scale, set the hairline to 0.173 (173 stat mi).

(2) Under the hairline, read:

$V_2 = 25.4$ (25,400 fps). This is the circular orbital velocity at that altitude.

(3) From Appendix A, estimate the velocity losses due to gravity and aerodynamic drag as 5960 fps.

(4) The characteristic mission velocity is:

$V = 25{,}400 + 5960 = 31{,}360$ fps.

(5) The average specific impulse is:

$(305 + 425 + 425)/3 = 385$ sec.

(6) The average mass fraction is:

$(0.92 + 0.90 + 0.88)/3 = 0.90$.

(7) Divide the characteristic mission velocity by 3, the number of stages ($31{,}360/3 = 10{,}453$), to find the velocity delivered per stage.

(8) On the D scale, set the hairline to 10.453 (10,453 fps), move the slide until 385 on the I_{sp} scale is under the hairline, and then move the hairline to the right index on the C scale.

(9) Under the hairline, read:

$K' = 1.755.$

(10) On the MF scale, set the hairline over 0.90 and bring the right index of the C scale under the hairline. Then, on the K_o scale, move the hairline to 1.755.

(11) Under the hairline, read:

$K_1 = 1.72$; $K_2 = 4.70$; $K_3 = 12.80.$

(12) The ratio of launch weight to payload is:

$$W_o/W_{PL} = K_1 + K_2 + K_3 + 1$$
$$= 20.22.$$

(13) Therefore, the payload is:

$$W_{PL} = W_o/20.22 = 1{,}000{,}000/20.22$$
$$= 49{,}444 \text{ lb.}$$

VI

STAGE OPTIMIZATION

With the basic method for determining take-off gross weight (TOGW), explained in Section V, it was possible to estimate quickly a TOGW very close to the optimum value for conventional vehicles. This procedure is exact when the stages have the same specific impulse and the same propellant mass fraction. For boosters whose stages have dissimilar specific impulses or mass fractions, a more elaborate technique can be employed, if desired, to arrive at the optimum staging.

DEFINITION

Optimization of an n-stage vehicle is defined as determining the minimum TOGW for a specific payload and mission.

OPERATION

The key to the optimization procedure is first to allocate the characteristic mission velocity among the stages in proportion to the specific impulses of the stages. Then, around this proportional velocity distribution, assign arbitrary stage velocity distributions, both larger and smaller, and calculate the booster TOGW for each of these combinations.

In these calculations, each stage is treated individually as having its own specific impulse and mass fraction. The parameters of the n^{th} stage are determined first, which in turn establishes the payload for the stage below it.

The results of the calculations are tabulated (or plotted) and the velocity distribution combination producing the minimum TOGW is determined. In particular, the effect of changing the relative stage sizes is demonstrated.

EXERCISE ONE: Optimize a two-stage IRBM with a burnout velocity of 12,000 fps. Assume that the payload is 1000 pounds, the effective (or average) specific impulse of the first stage is 270 seconds, and that of the second stage is 310 seconds.

(1) From Appendix A, establish flight losses as 3000 fps. Add the flight losses to the burnout velocity (12,000 + 3000) to obtain a characteristic mission velocity (V) of 15,000 fps.

(2) The average specific impulse for the booster is:

$$I_{sp_{(avg)}} = (270 + 310)/2 = 290 \text{ sec.}$$

(3) Find the velocity increment of each stage in proportion to its specific impulse by:

$$V_s = (V/n) \times (I_{sp}/I_{sp_{(avg)}})$$

$$V_1 = 6980 \text{ fps; } V_2 = 8020 \text{ fps.}$$

(4) Determine the second stage weight:

(a) On the D scale, set the hairline on 8.02 (8020 fps). Move the slide until 310 on the I_{sp} scale is under the hairline, and then move the hairline to the left index C scale.

(b) Under the hairline, read:

$K' = 1.81$ and $\lambda = 2.24$.

(c) By using the payload weight of 1000 lb and $\lambda = 2.24$, determine the mass fraction to be 0.818 from Appendix B.

(d) On the MF scale, set the hairline on 0.818 and move the slide to the right index C scale. Then, on the K_o scale, set the hairline on 1.81 (value of K').

(e) Under the hairline, read:

$K_1 = 2.08$.

(f) The second stage weight (without payload) is:

$$W_{s_2} = K_1 \times W_{PL} = 2.08 \times 1000$$
$$= 2080 \text{ lb.}$$

(5) Determine the weight of the first stage in the same manner as Step (4). The following values are used or found: $V_1 = 6980$ fps; $K' = 1.81$; $\lambda = 2.24$; $W_{PL} = 3080$ lb. Therefore, we have: $MF = 0.852$, $K_1 = 1.85$, and $W_{s_1} = 5700$ lb.

(6) Now, vary the velocity distribution of the second stage by multiplying the ini-

tial velocity by arbitrary factors—for example, V_2 (8020 fps) × 1.4, 1.2, 0.8, 0.6, etc. For each of these assumed new velocities, there is a corresponding velocity for the first stage such that the sum of each pair equals the characteristic mission velocity.

(7) Determine the weights of the second and first stages for each velocity distribution in the same manner as Steps (4) and (5). Prepare a table, as shown below, and identify the velocity distribution which yields the lowest TOGW. Here, it is shown to be 8700 lb.

V_1	V_2	MF_1	MF_2	W_{s_1}	W_{s_2}	TOGW
0	15,000	—	0.869	0	8,600	9,600
2,200	12,800	0.823	0.850	2,310	5,600	8,910
3,800	11,200	0.840	0.843	3,080	4,050	8,730
5,390	9,610	0.850	0.830	4,750	2,950	8,700
6,980	8,020	0.852	0.818	5,700	2,080	8,780
8,600	6,400	0.860	0.805	6,650	1,450	9,100
10,290	4,710	0.865	0.787	8,000	950	9,950

(8) For comparison, determine the TOGW by the rapid method, using the average specific impulse of 290 sec and an average MF of 0.835. The latter value is obtained from the fractions established

for the original velocity distribution where $MF_2 = 0.818$ and $MF_1 = 0.852$. Following the procedure established in Section V, we find that $K_1 = 1.95$ and $K_2 = 5.75$. Therefore,

(a) $W_{s_2} = 1.95 \times 1000 = 1950$ lb.

(b) $W_{s_1} = 5.75 \times 1000 = 5750$ lb.

(c) $TOGW = 5750 + 1950 + 1000$
$= 8700$ lb.

This TOGW is identical to the weight found by the optimization procedure. Depending on the spread of the specific impulses and mass fractions between stages, normally one can expect differences no greater than 5%.

THREE-STAGE OPTIMIZATION

The principle of optimizing a three-stage vehicle is to divide the three-stage booster into a two-stage and a one-stage design problem.

Assume that the velocity distribution for the upper two stages is proportional to their specific impulse. Then, for this two-stage problem, the second and third stages are sized to minimize their total weight. This minimum weight, plus the final payload, is the gross payload weight for the first stage.

The first stage velocity increment is equal to the characteristic mission velocity minus the velocity increment assumed for upper stages. Thus, the first stage weight and TOGW are computed.

A range of TOGW can then be determined by varying the assumed velocity increment of the second and third stages and repeating the above steps. TOGW may be plotted, as a function of first stage velocity increment, with the understanding that the second and third stages are properly chosen. The minimum TOGW can then be determined.

A set of tables like the following can be constructed. In each, the sum of $V_2 + V_3$ equals an assumed constant value, as shown in form below.

V_2	V_3	MF_2	MF_3	W_{s_2}	W_{s_3}	$W_{s_2} + W_{s_3} + W_{PL}$
......	.....					
10,000	5,000					
8,000	7,000					
6,000	9,000					
......	.....					

For an assumed $V_{2,3}$ (the sum of $V_2 + V_3$), the minimum $W_{s_2} + W_{s_3} + W_{PL}$ is determined by first varying the second stage (V_2) velocity increment and then computing the MF_2 and MF_3 for each variation. The minimum $W_{s_2} + W_{s_3} + W_{PL}$ is determined by plotting $W_{s_2} + W_{s_3} + W_{PL}$ as a function of V_2.

After determining the minimum $W_{s_2} + W_{s_3} + W_{PL}$ for several values of $V_{2,3}$, Table 2 may be used to determine V_1 and TOGW. TOGW is then plotted as a function of first stage velocity increment to determine the minimum.

FOUR-STAGE OPTIMIZATION

A four-stage vehicle is optimized basically in the same manner as a three-stage vehicle. The procedure is to solve the problem as a series of two-stage vehicles in which:

(1) A division of the characteristic mission velocity between the upper and lower two stages is assumed.

(2) The upper two stages are optimized to a spread of arbitrary velocity increments within the originally assumed division.

(3) The lower two stages are optimized in the same manner, using the minimum weight of the upper two stages for the payload. The best TOGW, therefore, is determined for the assumed division of the characteristic mission velocity.

(4) A different division of the characteristic mission velocity between the lower and the upper two stages is assumed and Steps (2) and (3) are repeated.

(5) The minimum TOGW is computed from the different TOGW's of Step (4).

TABLE 2

Table Form for Determining TOGW

$V_{2.3}$	$V_1 = V_t - (V_2 + V_3)$	$W_{s_2} + W_{s_3} + W_{PL}$	MF_1	W_{s_1}	TOGW
17,000	8,000 = (25,000 − 17,000)	For each $V_{2.3}$, choose only the minimum $W_{s_2} + W_{s_3} + W_{PL}$. This is payload for W_{s_1}.	Treat as a series of single stage design problems and find the minimum TOGW.		
16,000	9,000 = (25,000 − 16,000)				
15,000	10,000 = (25,000 − 15,000)				
14,000	11,000 = (25,000 − 14,000)				
13,000	12,000 = (25,000 − 13,000)				

VII
INTERPLANETARY MISSIONS

Scales applicable to interplanetary flight, or sun-centered orbits, are contained on the reverse side of the "slide". Complete removal of the slide reveals a reference of planetary symbols, escape velocities, and planetary radii.

Data obtained from the interplanetary scales are subject, by assumption, to the following conditions:

(1) The flight path from the earth follows a minimum energy path.

(2) The gravitational field of the earth acts only for a short distance compared to the interplanetary distances traveled.

(3) The gravitational field of the sun is the only force acting upon the vehicle after escape from the earth, except during soft landing on a target planet.

(4) The time of flight is not affected by the gravitational fields of the earth or other planets.

(5) Planetary and lunar orbits are circular and coplanar.

These assumptions do not materially affect the gross results presented on the scales.

DEFINITION OF SCALES

The following scales are used for interplanetary flight calculations:

V_3 (Impact Landing)—burnout velocity required to leave earth and coast to aphelion or the orbit of the target planet (10^3 fps).

V_3 (Soft Landing)—required velocity to leave earth and coast to the target planet, and to counteract that planet's gravitational attraction on landing (10^3 fps).

Time of travel—time to coast from earth to interplanetary aphelion (years). Applicable only to the outer planets.

R_a/R_e—aphelion distance divided by the earth's mean orbital radius around the sun (A.U.). Applicable only to outer planets.

V_{circ}—velocity of circular planetary orbit around the sun (10^3 fps).

R/R_e—radius of orbit around the sun divided by radius of the earth's orbit from the sun (A.U.).

OPERATION OF SCALES

The operation of the interplanetary scales is analogous to that of orbital mechanics, and the procedures are similar. In essence, the sun replaces the earth as the orbital center. Simply setting the hairline on the interplanetary scale defines the parameters in adjacent scales.

V_3

The V_3 scale is in two parts for determining characteristic mission velocity requirements, one for impact landing and the other for soft landing a spacecraft on a planetary body.

In the impact landing, the spacecraft, at the apogee of its orbit around the sun, is captured by the gravitational field of the target planet and impacts upon it. Therefore, the spacecraft reaches the planet surface with a finite velocity in relation to the planet. This velocity is due to the combined effects of an initial approach velocity and the gravitational pull of the planet. Thus, the term "impact landing" is used.

In a soft landing, the relative velocity due to the approach velocity and the gravitational pull must be neutralized. Neutralization is achieved by providing an additional velocity increment to the spacecraft during the landing phase. Since the final velocity relative to the planet will be zero, the term "soft landing" is used. Planetary atmospheric drag is neglected.

For both the impact landing and the soft landing velocity requirements, it is assumed that a Hohmann transfer is used to go from the earth to the planet. The orbits of planets are assumed to be coplanar and circular.

Time of travel (years)

The "Time of travel" scale can be used with either the V_3 (Impact Landing) scale or the R_a/R_e scale. Used in conjunction with the planetary symbols on the V_3 scale, the time to travel to any outer planet of the solar system can be read directly from the *Rule*. In conjunction with the R_a/R_e scale, the time to reach apogee at any distance (in astronomical units) for sun-centered orbits is also read directly from the scales.

R_a/R_e

The R_a/R_e scale presents a ratio in which R_e, the mean distance of the earth's orbit around the sun, is a constant, or 92.9×10^6 statute miles (1 astronomical unit). This means that the orbital radius of any planet is easily determined and the radius at apogee (R_a) of any sun-centered orbit originating from the earth can be calculated.

V_{circ}

The V_{circ} scale is used primarily with the R/R_e scale. For example, by placing the hairline on the R/R_e scale, the velocity required for any circular orbit about the sun can be read directly on the V_{circ} scale as a function of the mean distance from the sun.

R/R_e

R/R_e is the distance (in A.U.) from the sun where the *circular* velocity is V_{circ}. In the R/R_e scale, R may also stand for the mean distance from the sun of any *elliptic* orbit. The two definitions are consistent because the mean velocity of an elliptic orbit occurs at the mean distance of that orbit.

EXERCISE ONE: Soft land a spacecraft on Mars. Determine the flight parameters.

(1) On the V_3 (Soft Landing) scale, place the hairline over the symbol (♂) for Mars.

(2) Under the hairline, read:

V_3—characteristic mission velocity for soft landing on Mars: 55.9 (55,900 fps).

(3) On the V_3 (Impact Landing) scale, place the hairline over the symbol for Mars.

(4) Under the hairline, read:

V_3—characteristic mission velocity for departure from earth to achieve a transfer orbit to the orbit of

Mars: 37.04 (37,040 fps).

Time of travel—flight time from earth to aphelion at orbit of Mars: 0.71 year.

R_a/R_e—maximum distance of the spacecraft from the sun: 1.52 times the distance of the earth from the sun.

(5) The difference between the 55,900 fps and the 37,040 fps (18,860 fps) is the velocity that must be applied when the spacecraft reaches aphelion at the orbit of Mars.

EXERCISE TWO: Place a spacecraft into an orbit around the sun with a period of four years. Determine the characteristic mission velocity and the maximum distance of the spacecraft from the sun.

(1) On the "Time of travel" scale, place the hairline over 2. (The time to travel from perihelion to aphelion is half the orbital period.)

(2) Under the hairline, read:

V_3 (Impact Landing)—characteristic velocity required to leave the earth and its orbit: 44.05 (44,050 fps).

R_a/R_e—maximum distance of the spacecraft from the sun: 4.05

times the distance of the earth to the sun.

The minimum distance or perihelion will be the same as the earth-sun distance.

EXERCISE THREE: Design a four-stage booster capable of soft landing a 10,000-pound spacecraft on the moon. Use the first two stages to achieve an easterly launch to a 200-mile earth orbit, the third stage to transfer from the earth orbit to the moon's orbit, and the fourth stage to soft land the spacecraft on the moon. Use a specific impulse of 435 seconds for all four stages and a propellant mass fraction of 0.93, 0.93, 0.90 and 0.88 for the first, second, third and fourth stages, respectively.

(1) Determine the characteristic mission velocity for each stage.

(a) On the h_c scale, set the hairline over 200 miles.

(b) Under the hairline, read:

$V_2 = 25.32$ (25,320 fps) orbital circular velocity.

(c) From Appendix A, assume that the first two stages require 6500 fps to overcome gravity and drag losses. Since the vehicle is launched east, it starts with a finite velocity, i.e., the velocity of the earth, in the intended direction. The first two stages must therefore be capable of delivering a characteristic mission velocity of 25,320 + 6500 — 1400 = 30,420 fps.

(d) On the V_3 (Impact Landing) scale, set the hairline over the moon symbol.

(e) Under the hairline, read:

V_3 (Impact Landing) = 35.5 (35,500 fps) required to escape the earth and reach the moon's orbit.

Since the spacecraft has an orbital velocity of 25,320 fps and the characteristic mission velocity to impact on the moon is 35,500 fps, the third stage must be capable of delivering a velocity of 35,500 + 1000 (losses) — 25,320 = 11,180 fps.

(f) On the V_3 (Soft Landing) scale, set the hairline over the moon symbol.

(g) Under the hairline, read:

V_3 (Soft Landing) = 43.2 (43,200 fps) required to soft land on the moon.

Therefore, to overcome the lunar approach velocity and the lunar gravitational attraction for soft landing on the moon, the fourth stage must be capable of delivering a velocity of

43,200 − 35,500 = 7700 + losses = 9000 fps.

(2) Size the individual stages. The procedure is to work backwards. That is, first the fourth (or last) stage is sized. This stage plus the payload is considered as the net payload for the third stage, which can now be sized. This simple procedure is applicable because the characteristic mission velocity for each of the upper two stages is known.

When the weights of the third and fourth stages are determined, then the lower two stages, for which only the total characteristic mission velocity is specified, should be optimized to minimize the total vehicle weight.

(a) By following the procedure for a single-stage vehicle (Chapter V, Exercise One) it can be shown that the fourth stage weight to payload ratio is 1.17. Therefore, with a 10,000-lb payload, we have:

$$W_{s_4} = 1.17 \times 10{,}000 = 11{,}700 \text{ lb.}$$

$$W_{s_4} + W_{PL} = 11{,}700 + 10{,}000 = 21{,}700 \text{ lb.}$$

This weight is used as the payload for the third stage.

(b) The third stage weight to payload

ratio is found to be 1.57. Therefore, with a 21,700-lb payload, we have:

$$W_{s_3} = 1.57 \times 21{,}700 = 34{,}200 \text{ lb.}$$

$$W_{s_3} + W_{s_4} + W_{PL} = 34{,}200 + 11{,}700 + 10{,}000 = 55{,}900 \text{ lb.}$$

(c) By following the outlined procedure for a two-stage configuration, we can show that:

$K_1 = 2.5$ (second stage to payload ratio).

$K_2 = 8.5$ (first stage to payload ratio).

Therefore, the weights of the stages are:

$$W_{s_2} = 2.5 \times 55{,}900 = 140{,}000 \text{ lb.}$$

$$W_{s_1} = 8.5 \times 55{,}900 = 475{,}000 \text{ lb.}$$

(3) The detail weights of the vehicle design can be summarized as follows:

STAGE	WEIGHT DETAILS (LB)			
	W_s	W_{pu}	W_{bo}	W_{PL}
4	11,700	10,300	1,400	—
3	34,200	30,800	3,400	—
2	140,000	130,000	10,000	—
1	475,000	442,000	33,000	—
—	—	—	—	10,000
TOTALS	660,900	613,100	—	10,000

The vehicle TOGW is 660,900 + 10,000 = 670,900 lb.

(4) Compare the exact results derived above with the approximate technique outlined in Chapter V, Exercise Three.

(a) $\Delta V = 30{,}420 + 11{,}180 + 9000 = 50{,}600$ fps.

(b) $I_{sp_{(avg)}} = (435 + 435 + 435 + 435)/4 = 435$ sec.

(c) $MF_{(avg)} = (0.93 + 0.93 + 0.90 + 0.88)/4 = 0.91$.

(d) $K_1 = 1.87$; $K_2 = 5.4$; $K_3 = 15.5$; and $K_4 = 45$.

(e) The TOGW is found by:

$$TOGW = W_{PL}\,(1 + K_1 + K_2 + K_3 + K_4).$$

$$TOGW = 10{,}000 \times 68.8 = 688{,}000 \text{ lb.}$$

The approximate method differs by less than 2.5% from the total vehicle weight found in Step (3).

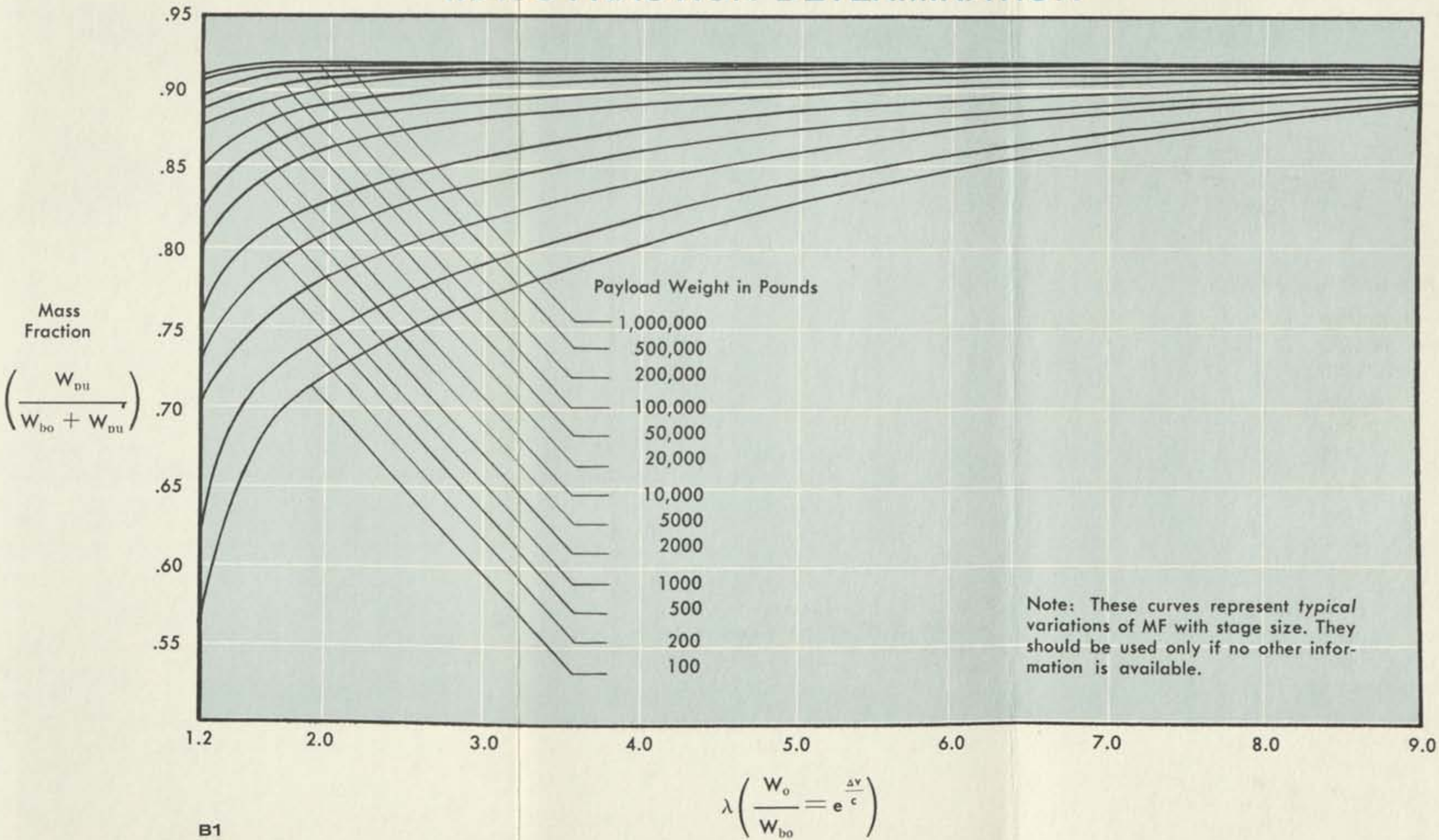
APPENDIX A

TYPICAL MISSION VELOCITY LOSSES

This information is provided only for initial estimates and preliminary evaluations. For a specific problem, when preliminary work is carried beyond the preliminary design stage, values should be replaced with more accurate trajectory data.

Mission	*Mission Losses (fps)*
Orbit to escape [These losses are due to gravity (200 fps) plus allowance for margin and guidance corrections.]	1000
IRBM	3000
ICBM	4000
Boost glide	4500
Low orbits (100 miles) [For direct injection into orbit, at altitudes greater than 100 stat mi, add 20 fps to the velocity losses for every statute mile higher than 100.]	4500
Lunar and interplanetary missions	6000

APPENDIX B
MASS FRACTION DETERMINATION



APPENDIX C
ELLIPTICAL EQUATIONS

Semimajor axis:

$$a = \frac{r_a + r_p}{2}$$

Semiminor axis:

$$b = a\sqrt{1 - e^2}$$

Semilatus rectum:

$$p = r_a\,(1 - e); = a\,(1 - e^2)$$

Eccentricity:

$$e = \frac{r_a - r_p}{r_a + r_p}; = 1 - \frac{p}{r_a}$$

Radius at apogee:

$$r_a = r_p\left(\frac{1+e}{1-e}\right); = a\,(1 + e); = r_p\left(\frac{V_p}{V_a}\right)$$

Radius at perigee:

$$r_p = \frac{p}{1+e}; = a\,(1 - e); = r_a\left(\frac{V_a}{V_p}\right)$$

Acceleration—gravity:

$$g = \frac{\mu}{r^2}$$

Circular velocity:

$$V_c = \sqrt{\frac{\mu}{r}}$$

Escape velocity:

$$V_e = V_c \sqrt{2}$$

Velocity at any radius:

$$V = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

Velocity at apogee:

$$V_a = V_p\left(\frac{r_p}{r_a}\right); = V_{c_a}\sqrt{(1-e)}$$

Velocity at perigee:

$$V_p = V_a\left(\frac{r_a}{r_p}\right); = V_{c_p}\sqrt{(1+e)}$$

Orbital period:

$$\tau = \frac{2\pi}{\sqrt{\mu}}\,a^{3/2} = \frac{2\pi\,r_p}{V_{c_p}}(1-e)^{-3/2}$$

NOTES