

An interview with
WALTER GAUTSCHI

Conducted by Philip Davis
on
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at the
Department of Computer Science, Purdue University

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ABSTRACT

Walter Gautschi discussed his varied work in numerical analysis and a variety of prominent mathematicians that he has interacted with. Gautschi had fairly humble beginnings as the son of a shoe store manager with deep roots in Switzerland. Talented at mathematics from a young age, Gautschi nearly pursued his other passion, music, and continued taking piano and composition lessons as a student. Mathematics eventually emerged as his calling, and he attended the University of Basel, where his instructors included Andreas Speiser and Alexander Ostrowski, a dominating figure under whom Gautschi eventually completed a thesis in graphical methods. Gautschi recalls an early talk he gave at a GAMM (the German equivalent of SIAM) conference in which Richard Grammel, upon whose work his thesis built, made some disparaging remarks about his talk. Undaunted, Gautschi completed his thesis, broadening his work so that it became applicable to numerical methods as well.

Gautschi relates fascinating stories about his mentor, Ostrowski. Born in Ukraine, he left Kiev to study in Germany – first to Marburg under Kurt Hensel and later to Göttingen, where he worked with David Hilbert, Edmund Landau, and Felix Klein – and eventually landed a professorship in Basel in 1927. Ostrowski became interested in linear algebra and iterative methods in the U.S. when he met Herman Goldstine at the Bureau of Standards. Indeed, Gautschi tells about how Ostrowski, upon hearing a lecture by Goldstine on the use of iterative methods for solving eigenvalue problems for matrices, pointed out that Carl Jacobi had already done such work; henceforth it has been called the Jacobi Method.

Starting in 1956, Gautschi spent a few years at the Bureau of Standards, where he joined other Swiss mathematicians, including Peter Henrici and Eduard Stiefel. After a brief stint at American University, Alston Householder enticed him to Oak Ridge where he spent four years before moving to Purdue University in 1963. Gautschi has remained at Purdue ever since, advising a handful of Ph.D. students and postdocs, including Hiroki Yanagiwara, with whom Gautschi collaborated to solve a problem in Chebyshev quadrature. Gautschi also dedicated much of his time to editing the journal *Mathematics of Computation*, which he did for twelve years.

Gautschi's interest in three-term recurrence relations was initially sparked by the enthusiasm of Milton Abramowitz (who had tapped Gautschi to write two chapters for the National Bureau of Standards' Handbook of Mathematical Functions) for J.C. P. Miller's backwards recurrence. Gautschi's inclinations often lead him to uncover older, original work on a topic, which sometimes spark new ideas. In tracing the roots of Miller's ideas, for example, he went from Oskar Perron's book on continued fractions to Niels Nörlund's book from the 1920s on difference equations to a paper on hypergeometric functions by Salvatore Pincherle, where he found the theorem he was looking for. It is now widely used in difference equations.

Gautschi's work on orthogonal polynomials began, while he was at Oak Ridge, with an innocent request from a chemist. Gautschi thought the request would have a simple answer from Francis Begnaud Hildebrand's exposition of Gaussian quadrature and orthogonal polynomials, but it turned out to be far more complicated. The problem led him to explore the condition of confluent Vandermonde matrices and related problems. He has recently written a book and a Matlab software package on orthogonal polynomials and their computational aspects. His recent work also has touched on the circle theorem for Gauss-Lobatto, Gauss-Radau, and Gauss-Kronrod formulas. He has also collaborated extensively with Gradimir V. Milovanović on Gaussian quadrature, orthogonal polynomials, and moment-preserving spline approximation.

PHIL DAVIS: This is an interview held on December 7, 2004, with Professor Walter Gautschi of Purdue University, carried out in the Department of Computer Science in Purdue University. The interviewer is Phil Davis.

December 7 is an important date in American history...Correction: the interview is being held on December 6, not on December 7, but since I'm talking about December 7, it reminded me of what I was doing mathematically sixty-three years ago. Sixty-three years ago, I was a junior at Harvard and I was studying advanced calculus under George David Birkhoff (who is not such a great teacher by the way) and I studied mathematical logic with Willard Quine. I've spoken about this many times, in many places, but Quine just had a new text book on mathematical logic out in which J. Barkley Rosser had found an inconsistency in the axioms, right at the beginning. We spent most of the semester in correcting this equation and that equation and this other equation, so as to make a patch, to put a band aid on his stuff. This was one of the reasons that I fell out of love with mathematical logic and never thought that mathematics needed any sort of logical foundations. But I wanted to ask you, Walter, what were you doing on December 7, 1941?

WALTER GAUTSCHI: Forty-one. In 1941 I was still in high school.

DAVIS: Still in high school.

GAUTSCHI: I graduated from the gymnasium in Basel, in 1947.

DAVIS: Forty-seven.

GAUTSCHI: So that was six years before I, so I was a small kid.

DAVIS: Still a small kid. Did your mathematical talents come out very early in your life?

GAUTSCHI: Well, I always found it very easy to do mathematics. But it had a competitor, and that was music. And I was never very sure which way I should go, into music or into mathematics.

DAVIS: Isn't there some sort of a story on that about golf?

GAUTSCHI: Not that I know.

DAVIS: Some famous mathematician was perplexed when he was a young man as to whether he should go into math or something else.

GAUTSCHI: I'm not surprised. I mean, that happens.

DAVIS: There's a considerable relationship between math and music.

GAUTSCHI: Yeah, I wish I knew what it is.

DAVIS: Have you pursued music at all?

GAUTSCHI: Oh, as a young man, yes. Even when I was still studying mathematics I took piano lessons. I took composition lessons from a teacher who was himself a composer, but not very well known. But then, eventually, I neglected it. I still played the piano for a long time, especially playing chamber music.

DAVIS: Well, you're good enough to play chamber music – that's quite, that's quite proficient. And then after the gymnasium where did you go?

GAUTSCHI: Well, after the gymnasium I went to the University of Basel, to study mathematics. There was a rather small mathematics department there, two professors only. One was [Alexander] Ostrowski and the other was Andreas Speiser, who was well known also for his philosophical interests.

DAVIS: Speiser wrote a book on group theory, as a matter of fact.

GAUTSCHI: He wrote one of the first books on finite groups.

DAVIS: Finite groups.

GAUTSCHI: In fact, [Bartel Leendert] van der Waerden once even wrote that this is still the best book on group theory [laughter] He was one of the nicest –

DAVIS: I remember reading it. I never myself liked group theory, but I remember reading Speiser's book at an early stage. So, anyway –

GAUTSCHI: So, I went there in 1947. I took three major areas. One was mathematics, and we had to take two "Nebenfächer" –

DAVIS: That's related fields.

GAUTSCHI: Related fields. Three actually. One was theoretical physics, another was physical chemistry, and the third one was actuarial mathematics (just to be sure, in case I don't find a job I can always go into life insurance, which fortunately I didn't have to).

DAVIS: Well, as a matter of fact, my recollection is as follows: in the thirties, 1930s, for people that did mathematics, that was one of the areas in which they could be employed. Perhaps the only area in which they could be employed. This changed entirely with World War II.

GAUTSCHI: Oh, definitely, yes.

DAVIS: In World War II, mathematics became a very important thing in defense and offense and so on, and the mathematical opportunities just opened up.

GAUTSCHI: Yes, but more so in the U.S. than in Switzerland. Switzerland was not that much affected by it. It was still very difficult to find positions. Actually, there was a second area that was possible, teaching in high school or gymnasium.

DAVIS: Gymnasium.

GAUTSCHI: But then you need a special license, I didn't even bother getting that license because I didn't head in this direction.

DAVIS: What happened after the University of Basel, when you were still an undergraduate?

GAUTSCHI: Well, we don't have this system of undergraduates and graduates. You just start and doing exams and once you're finished with the exams you try to find a thesis advisor, you write a thesis, and then you're done; there's no final exam (only a thesis defense).

DAVIS: You found Ostrowski?

GAUTSCHI: I found Ostrowski, yes.

DAVIS: We're talking here about Alexander Ostrowski, whom I knew reasonably well. What was your thesis about?

GAUTSCHI: Well, that's a curious story. Ostrowski never lectured on numerical analysis or computing, although he was interested in the area, especially linear algebra, iterative methods, and so on. But there was another privatdozent [lecturer] at the University, and he was a high school teacher at the time, by the name of Eduard Batschelet.

DAVIS: Batschelet, oh yes.

GAUTSCHI: He later became a biostatistician and was at the Catholic University [of America] in Washington, D.C..

DAVIS: I didn't know he was Swiss, I always thought of him as French.

GAUTSCHI: The name may be French.

DAVIS: B-A-T-S-C-H-E-L-E-T.

GAUTSCHI: And that was a very good German, but it could well be of French origin. Anyway, he taught the course on what he called scientific computation. And he, among

other things, he talked about a graphical methods for solving ordinary differential equations [ODEs]. Due to a person named [Richard] Grammel, who was an engineer –

DAVIS: Grammel. It's a name that comes up in the early literature, I think.

GAUTSCHI: Well, he was an engineer in Stuttgart. But anyway, this is a strange method. You plot the reciprocal of the function on one ray, on the polar ray, and on another ray that's turned around by ninety degrees you plot the reciprocal of the derivative. And then the line connecting the end points is tangent to the first plot, so you know how to proceed. If you know the derivative, as you do in ODEs, you know how to proceed with the function. That's his idea. What puzzled me at the time was: why in the world was he taking the reciprocal of the function, why not the function itself? So I was thinking about it, and I derived some other method that works just as well. I told Batschelet about it and he was quite pleased with it. He must have told Ostrowski – he was a student of Ostrowski himself at one time – and at one point Ostrowski told me, when I asked for a thesis topic, he told me, "Well, you might as well continue with that method of yours and expand it into a thesis." I was very disappointed about this because I knew graphical methods were not the future. I knew numerical methods were the future, and I didn't want to spend all my time on graphical methods. But there was no way out. I mean, once he said that, I was stuck with it. And incidentally, he also suggested that I should talk about this at the GAMM meeting in Germany, at that time I was still a graduate student –

DAVIS: GAMM –

GAUTSCHI: GAMM, that's the equivalent of SIAM in Germany.¹

DAVIS: G-A-M-M.

GAUTSCHI: Right.

DAVIS: I do this spelling occasionally for the sake of the transcriber.

GAUTSCHI: Right. GAMM. And they met in Freiburg (which is near Basel) at that time, and Ostrowski told me I should talk about this. He forced me to, I didn't want to. I was too shy to do that, too inexperienced.

DAVIS: He was a dominating character.

GAUTSCHI: Oh, he was very overpowering, so there was no choice. So I went there and what made matters worse is that I discovered when I gave my talk that sitting right there in the first row was Professor [Richard] Grammel, he was sitting right there. And that made me very nervous –

DAVIS: Yes of course.

¹ GAMM stands for Gesellschaft für Angewandte Mathematik und Mechanik.

GAUTSCHI: At the end he stood up and made some rather disparaging remarks that put me into my place, after all I was a student at the time. Anyway, that was an interesting experience. But then I had to work on this and I decided to analyze the error of the method. I did it in a broad enough context so it applied to numerical methods as well. So that gave me my first experience in ODEs, in numerical analysis.

DAVIS: And thinking of the history of methods of computation, of course, in the pre-computer age there was a whole development of nomography. I would say that what you did was part of the theory of nomography. There's a huge text book on this thing, written by Maurice D'Ocagne, maybe in 1910 or so.² It was a huge book, and this material is now absolutely dead. It's absolutely dead.

GAUTSCHI: So are graphical methods for ODEs! Although you could think that maybe in graphics they have some applications.

DAVIS: Maybe so. Anyway, mentioning Ostrowski, since I knew him a little bit, tell me something of Ostrowski as a character, his personality and so on.

GAUTSCHI: You already mentioned one of the traits. He was very overpowering, he was dominating.

DAVIS: Dominating.

GAUTSCHI: Not only in appearance – in appearance, he was huge and solid – but then he also talked down to you. That's my experience, anyway. But on the other hand, once you knew him, and I got to know him quite well, he was very charming.

DAVIS: I found that out also. Once you penetrated this shell somehow.

GAUTSCHI: But only after he married. Before he married he was practically inaccessible.

DAVIS: I heard he married his psychoanalyst.

GAUTSCHI: Yes, that's right.

DAVIS: Is that correct?

GAUTSCHI: That's correct. I think that's correct.

DAVIS: I'm surprised at her! [laughter]

² This may be a reference to d'Ocagne's *Traite' de Nomographie* (1899 Gauthier-Villars, Paris) or to his *Le Calcul Simplifié par les Procédés Mécaniques et Graphiques* (1905 Gauthier-Villars, Paris).

GAUTSCHI: Well she's a very charming woman, in fact. I knew her quite well. She was a psychoanalyst from the Jung school –

DAVIS: Oh, she was a Jungian.

GAUTSCHI: Yeah, she was a Jungian psychologist.

DAVIS: I didn't know that. Now Ostrowski was born Ukraine wasn't he?

GAUTSCHI: He was born in Kiev, yes.

DAVIS: He was born in Kiev. Now the subject of much difficulty right now. And how did he come to Basel, do you know this story?

GAUTSCHI: Yeah, I know the story quite well. Actually, when he was only fifteen years old, he took a seminar under [Dmitry] Grave – a student of [Pafnuty] Chebyshev, actually – who was an algebraist at Kiev. He was admitted to Grave's seminar. So he was exceptionally gifted, and he wrote his first paper under the direction of Grave when he was only fifteen or sixteen years old. And then, when he applied to the University of Kiev, he was rejected. They wouldn't admit him apparently for the reason that he went to the wrong school: he went to the commercial school rather than to a lycée or a gymnasium or something of the sort. But it could have been anti-Semitic –

DAVIS: More likely.

GAUTSCHI: Very likely.

DAVIS: Very likely, yes.

GAUTSCHI: And so they didn't admit him. So Grave wrote two letters: one to [Edmund] Landau and one to (I forget) somebody in Marburg in algebra – [Kurt] Hensel, I think...yes, Hensel – as to whether they would take Ostrowski on. Both agreed, yes, they would be happy to have him in Germany. So he went to Marburg first, to Hensel, and that was just before the revolution.

DAVIS: This was Hensel of p -adic numbers,

GAUTSCHI: I think so, yes.

DAVIS: p -adic fields, and so on.

GAUTSCHI: I think so, yes. He was in algebra. So he went to Marburg and then World War I broke out and he was made a civil prisoner in Marburg. His movements were severely restricted, but he was allowed to visit the university library. And that's really all he needed, so he spent all his time at the university library and almost single handedly created his theory of valuations on fields, which is now very famous.

DAVIS: Yes. This came out of the p -adic analysis, I think.

GAUTSCHI: It's like a norm, the concept of a norm on a field. It could be, yeah.

DAVIS: He told me that in Marburg he was a fellow student of that famous Russian novelist –

GAUTSCHI: Novelist, [Boris] Pasternack.

DAVIS: Pasternack. But then he said that, well, he was in mathematics and Pasternack was in philosophy, so there was this gap between them. And then he came to –

GAUTSCHI: Okay, and then –

DAVIS: He was with [Felix] Klein in –

GAUTSCHI: And then after the war he could move again. He was restricted during World War I, he could not move, but after the war was over he moved to Göttingen and there he became a student of [David] Hilbert. He wrote a thesis under Hilbert and Landau, actually, and he was also associated with Felix Klein. He was an assistant to Felix Klein there. He was editing the collected works of Klein, the first volume of collected works –

DAVIS: Did he have something to do with Klein's encyclopedia on mathematical sciences?

GAUTSCHI: It could be. I don't know. He didn't write anything for it. But he wrote a very stunning thesis under Hilbert, solving part of one of the problems Hilbert suggested. And then he went to Berlin I think, or to Hamburg, he became a privatdozent at Hamburg. And then he went back to Göttingen and taught complex variables. He was a friend of [Ludwig] Bieberbach actually –

DAVIS: What was that word you used a moment, a fata – he became a fata-something –

GAUTSCHI: Privatdozent.

DAVIS: Oh, yes, yes, yes.

GAUTSCHI: That's just a title in Europe that allows you to teach at the university without salary.

DAVIS: D-O-Z-E-N-T.

GAUTSCHI: So he actually got that in Hamburg and then again in Göttingen, as far as I remember. He got it twice and spent maybe a semester in Göttingen, and then he

received a Rockefeller Fellowship to study in England. He went to Cambridge, Oxford, and Edinburgh.

DAVIS: I bet he met [Douglas] Hartree in –

GAUTSCHI: He could have met Hartree, that's quite possible.

DAVIS: Hartree was interested in that kind of thing. How did he get interested in numerical linear algebra and so on?

GAUTSCHI: Well, let me finish the story and then I will come back to that. After that fellowship was over he received a call from the University of Basel, as a full professor there, which he accepted, and he went to Basel. That was in 1927, when I was born.

DAVIS: He was there in Basel in 1927, well before World War II.

GAUTSCHI: Yes. That was a fortunate choice for him, absolutely. There were others who wished that they had gone to Switzerland at that time. So he got that professorship and essentially stayed there until he died, but he visited the U.S. and England.

DAVIS: He even visited Providence.

GAUTSCHI: Providence, yes, that's what he told me. Anyway, so to come back now about when he got interested in numerical analysis. In the 1930s, he was already interested in Newton's method, especially the convergence of Newton's method, not only for one equation and one variable, but also for two or more equations. And he had one of the early theorems about convergence of Newton's method, and, how do you say, predating [Leonid] Kantorovich, who is usually credited for –

DAVIS: Oh, Kantorovich made a big generalization of –

GAUTSCHI: All that too –

DAVIS: And Newton's –

GAUTSCHI: The first one was to systems of nonlinear equations –

DAVIS: Kantorovich went into nonlinear spaces and that kind of thing.

GAUTSCHI: Right. Ostrowski did something of that sort – all of it in the 1930s. So he was interested in computing, but I think his major interest in iterative methods, linear algebra, he picked up in the U.S., when he visited a numerical analysis group at the National Bureau of Standards when it was still in Los Angeles. (It was in Los Angeles first). So there he got into contact with [Herman] Goldstine and so on, and got interested in iterative methods. In fact there's an interesting story about this. Goldstine was lecturing there –

DAVIS: That's Herman Goldstine.

GAUTSCHI: Herman Goldstine from Princeton, who collaborated with John von Neumann –

DAVIS: Von Neumann.

GAUTSCHI: He was lecturing there on the new methods for solving eigenvalue problems for matrices, iterative methods. Well, at the end of the lecture Ostrowski, who was present, got up and said, "Well, this was already done by [Carl Gustav Jacob] Jacobi," and he gave the exact citation with page numbers and all. [laughter] And since then, the method actually was named after Jacobi. I think that's the merit of Ostrowski that he pointed that out, that it was Jacobi's method.

DAVIS: It's called the Jacobi Method –

GAUTSCHI: Yes, it's called the Jacobi Method –

DAVIS: Rotations, method of rotations, yes. And Herman Goldstine thought that he had –

GAUTSCHI: That he had discovered it independently, maybe –

DAVIS: Probably. You know independent discoveries in mathematics are frequent.

GAUTSCHI: Of course, it must have been in the air.

DAVIS: Yeah, yeah, it was in the air.

GAUTSCHI: Anyway, that started his interest in numerical linear algebra. Then, when I was his assistant in Basel, when he came back from the States, that's what he was working on. He was working on the convergence theory of the so-called....what was it called....this English guy...it was the method...it was similar to –

DAVIS: Least squares –

GAUTSCHI: No, no, no, it's not least squares. Maybe it will come to me...there was an English engineer –

DAVIS: Oh, yes.

GAUTSCHI: Do you remember the name?

DAVIS: Yes, [Richard V.] Southwell.

GAUTSCHI: Southwell, exactly. Relaxation methods.

DAVIS: Relaxation methods, yes.

GAUTSCHI: So he essentially created the theory of convergence of relaxation methods with rather general steering mechanisms. That's what he was working on when I was his assistant. [For more on Ostrowski, see: <http://www.cs.purdue.edu/homes/wxg/AMOengl.pdf>].

DAVIS: I first met up with him when he spent some time in Washington in the National Bureau of Standards, but I would imagine that you were there at the time also.

GAUTSCHI: No, I came later. I came in 1956.

DAVIS: We had quite a few people in the Bureau of Standards.

GAUTSCHI: I was also after [Peter] Henrici. Henrici was there as well –

DAVIS: Yes, I shared an office with Henrici there for a while. There were quite a few mathematicians from Switzerland there.

GAUTSCHI: [Eduard] Stiefel, of course,

DAVIS: Stiefel was there. And the young man who –

GAUTSCHI: [Urs] Hochstrasser –

DAVIS: Hochstrasser who went to the Swiss Atomic Energy Commission –

GAUTSCHI: Yes, that's right.

DAVIS: Did he give up mathematics at all?

GAUTSCHI: Essentially, yes.

DAVIS: Essentially he gave it up.

GAUTSCHI: But before that he was at the [Swiss] embassy in Washington as a scientific attaché. From then on he gave up mathematics.

DAVIS: When did you come to the National Bureau of Standards?

GAUTSCHI: Well, as I said, it was 1956.

DAVIS: 1956. You were there a couple of years?

GAUTSCHI: About three years, until 1959.

DAVIS: 1959. Then you went where?

GAUTSCHI: Well, that's an interesting story too. I was Swiss. I didn't have my citizenship, and you know –

DAVIS: America citizenship?

GAUTSCHI: Yes. Actually officially, I worked at the American University under a contract. That's the only way I could work for the Bureau of Standards, and that was kind of illegal. So in 1961 or 1962, Congressional hearings were announced intending to investigate this illegal employment of foreigners working for the government in this way. And then the administration became scared of it and canceled all these contracts. So I had to go look for another job. That was in 1963. Anyway, fortunately Alston Householder –

DAVIS: That was in 1963?

GAUTSCHI: No, it wasn't in 1963; it was 1959.

DAVIS: 1959.

GAUTSCHI: So, Alston Householder knew about this problem and he often consulted at the Bureau, and I met him there. He asked me if I would be interested in going to Oak Ridge [National Laboratory] to join him on the mathematics panel, and I was more than happy to say yes.

DAVIS: You went to Oak Ridge?

GAUTSCHI: So I went to Oak Ridge and was there for another four years or so, before I came to Purdue.

DAVIS: When did you come to Purdue?

GAUTSCHI: That was in 1963.

DAVIS: In 1963, and you've been at Purdue since?

GAUTSCHI: I have been here since.

DAVIS: That's worked out very well, I take it.

GAUTSCHI: For me it did. It's interesting. It was the first computer science department that I joined, jointly with mathematics, and I had contacts with many people in

mathematics. So professionally it was quite pleasant. The lifestyle was not too comfortable, the Midwest.

DAVIS: And coming from Basel and from Washington. But how about Oak Ridge? Oak Ridge was –

GAUTSCHI: Oak Ridge was worse. [laughter]

DAVIS: Oak Ridge was fairly isolated, wasn't it?

GAUTSCHI: Yes, it was fairly isolated. But we went back to Europe every summer, anyway, so it was not such a bad place.

DAVIS: I must have met you at that time, at the Bureau of Standards.

GAUTSCHI: Oh, of course. That was before...and you know what?

DAVIS: What?

GAUTSCHI: John Todd gave this series of workshops for –

DAVIS: Yes, for directors of computer laboratories –

GAUTSCHI: For senior faculty members, to get them interested in numerical mathematics. And two years later you took over from John Todd.

DAVIS: I did the same thing two years later.

GAUTSCHI: And at that time you must have asked me to give the course on differential equations. The first time it was [Henry] Antosiewicz who gave that course, along with John Todd, but he didn't want to do it again. So you probably asked me to give that course. I was very pleased about that because it gave me an opportunity to do some more reading in ODEs.

DAVIS: And then you participated in this Bureau Handbook of Mathematical Functions? You wrote a chapter –

GAUTSCHI: I wrote two chapters.

DAVIS: You wrote two chapters. What were they about?

GAUTSCHI: One was on error functions and related functions. Well, I mean –

DAVIS: The error, the Gaussian error function.

GAUTSCHI: Yes. And the other one was exponential integrals. It was actually assigned to somebody else, but it didn't move ahead. So Milton Abramowitz asked me to write that chapter, too, jointly with the other.

DAVIS: You know that book has had an enormous influence, enormous success and so on.

GAUTSCHI: Yes, I'm very pleased about that. I use it constantly myself.

DAVIS: So, over the years you've maintained your interest in special function theory?

GAUTSCHI: That's when I got interested in special functions for the first time. In particular, Milton Abramowitz was very enthusiastic about the method of Miller.

DAVIS: J.C.P. Miller.

GAUTSCHI: J.C.P. Miller. He had an interesting method to compute special functions by recurring backwards instead of forward.

DAVIS: Backwards recurrence it's called, yes. That came from Miller?

GAUTSCHI: That came from Miller. And it was kind of considered a trick at the time. Anyway, I got interested in that too because I saw some other functions that could be treated that way. That started a long interest in my working on three-term recurrence relations and numerical uses of them; that was due to Milton's interest. Later, when I was already in Oak Ridge, I was thinking about the method again and it occurred to me that it had some connections with continued fractions and with what are called minimal solutions of difference equations. In fact, at the same time the third edition of [Oskar] Perron's book on continued fractions came out and he had a theorem on that.³

DAVIS: Oskar Perron.

GAUTSCHI: That is correct.

DAVIS: I remember that book.

GAUTSCHI: It's an old book, but a third edition came out. And there he had the theorem that confirmed what I had already conjectured, that there is a connection with minimal – he didn't call it minimal solutions but it's solutions that grow minimally at infinity compared to all the other solutions – and there's a connection with that and continued fractions. At that time, I thought that it must be much older. I liked – it might be characteristic of me – to try to go to the original sources. So I was looking for when this theorem first appeared. The way I went about this was to go to [Niels] Nörlund's

³ O. Perron, *Die Lehre von den Kettenbrüchen*, 3rd ed., Chelsea, New York, 1957.

book on difference equations, analytic theory of difference equations. It is an old book from the 1920s...1924 I think it came out.⁴

DAVIS: It was a unique book, because there wasn't much material in, sort of in a textbook fashion, except for Nörlund.

GAUTSCHI: Right, it was one of the first ones. So I read that book and there was a lot of stuff on difference equations. I was looking for asymptotic theory and minimal solutions, but couldn't find anything. But I found a lot of references to a mathematician by the name of [Salvatore] Pincherle –

DAVIS: P-I-N-C-H-E-R-L-E

GAUTSCHI: Very good. [laughter] He was an Italian mathematician from Bologna.

DAVIS: An Italian mathematician.

GAUTSCHI: There were lots of references that to me looked very suspicious – maybe there was something in there. So the next thing I did was I went to the collected works of Pincherle, two volumes. Indeed, there in a long paper on hypergeometric functions I found precisely that theorem. Then, when I wrote that stuff up I made reference to Pincherle's theorem. That was the underpinning of Miller's idea, that was the theoretical background. It was not a trick but had some roots in classical analysis. Since then Pincherle has become very well known and was applied by many people.

DAVIS: Is this a current method now for computing solutions –

GAUTSCHI: It still is very much in use, yes.

DAVIS: Of difference equations and ODEs?

GAUTSCHI: Oh, not so much ODEs, but for difference equations.

DAVIS: Difference equations.

GAUTSCHI: Very much so. In fact, [Frank] Olver was one of them who also wrote –

DAVIS: Backward-recurrence –

GAUTSCHI: That's the paper that he published in the Bureau of Standards actually, and I still occasionally make use of it.

DAVIS: To go from the sublime to a little bit of the ridiculous. You did a real job on the difference equation that I set up for the Spiral of Theodorus.

⁴ Niels Nörlund. (1924). *Vorlesungen über Differenzenrechnung*. Berlin: Springer Verlag.

GAUTSCHI: Yes, that's an interesting difference equation.

DAVIS: It must have interested you.

GAUTSCHI: Well, it was not so much the difference equation as the infinite series.

DAVIS: The infinite series.

GAUTSCHI: You remember how this came about?

DAVIS: No, tell me.

GAUTSCHI: Well, I was visiting Brown at that time, giving a lecture (I think it was on Gauss quadrature). Anyway, I visited your office, because we knew each other for a long time, and then you scribbled an infinite series on the back of an envelope and gave it me and said, "Compute it." [laughter] You must have tried –

DAVIS: You said, "I have a program in my briefcase."

GAUTSCHI: Essentially, yes. So it happened that soon thereafter I discovered the method that I worked on together with [Gradimir V.] Milovanović (maybe we'll talk about him later) that applied to this series. In fact, I was able to send you back the result to, I don't know, fifteen decimal digits.

DAVIS: Yes, that's right. But you also identified it in terms of special functions.

GAUTSCHI: That's right, yes.

DAVIS: Which in that case was quite a trick.

GAUTSCHI: Yes, that case was part of the theory that we wrote. They were named Einstein functions.

DAVIS: Yes, Einstein functions.

GAUTSCHI: Einstein was one of them, and another one was Dawson's –

DAVIS: Dawson's Integral.

GAUTSCHI: And we still don't know who Dawson really was.

DAVIS: I think he was a schoolteacher.

GAUTSCHI: That's what I suggested to you.

DAVIS: He was a school teacher in England who did only that, as far as I can tell.

GAUTSCHI: Are you sure?

DAVIS: I think so –

GAUTSCHI: It could be, because he didn't do much other than that.

DAVIS: I met a mathematician in Vienna, from Graz, I think his name was Gronau (but I'm not sure) and I told him about this progress, that you had wrapped it up except to make a uniqueness theorem.⁵

GAUTSCHI: Oh, yes, yes.

DAVIS: And he sent me, he worked out a uniqueness theory –

GAUTSCHI: That's right, I noticed that paper.

DAVIS: The way there is a characterization of these classical gamma functions in terms of convexity, and I think he worked on something like that.

GAUTSCHI: He did, yes. I saw the paper.

DAVIS: It was rather nice.

GAUTSCHI: A nice paper, yes. I'm sure you saw it.

DAVIS: And then of course Iserles in Cambridge took off, and he did the thing for matrix formulation of the difference equation and he wrapped it up for, I don't know, hypergeometric functions or whatever.

GAUTSCHI: Yes, that was an interesting problem there.

DAVIS: Well, it seems that the way you get into your material is by working on things that are already there but are incomplete.

GAUTSCHI: Yeah, that's a fair statement, yes.

DAVIS: And how would you say you get your ideas? Where do they come from?

GAUTSCHI: Well, it's always a mystery to me. I have a habit of carrying things around in my head for some time before I sit down and actually work on it, just to get ideas and think what approaches to take and so on. Sometimes they come to me that way, but often I

⁵ Davis is referring to Detlef Gronau, from the University of Graz: "The Spiral of Theodorus". *American Mathematical Monthly* 111(2004), 230-237; "Normal solutions of difference equations, Euler's functions and spirals". *Aequationes Mathematicae* 68 (2004), 230-247.

also go back to the literature. I look up the literature and see what has been done in this particular area – not just the recent literature but also way back to the nineteenth century literature. This often gave me ideas on how to attack a problem. But the specific ideas, somehow they just come to me, somehow or other, it's probably a mystery to many people how they get their ideas.

DAVIS: Who are the mathematicians, either of the past or ones that you know or knew, who are particularly influential in your professional work?

GAUTSCHI: Well, you know I gave that lecture at the Christoffel Symposium for which you are also responsible, I think. Remember?

DAVIS: Yeah, I bowed out of it.

GAUTSCHI: Well, I was approached to give the lecture, I was happy to do so. But at any rate, it was at that time that I started to look into the classical literature on orthogonal polynomials and Gauss quadratures and so on. Jacobi was one of the persons that impressed me most because of his clarity.

DAVIS: You read some of the original papers in their –

GAUTSCHI: Oh, yes, I read them. Of course, I read Gauss's paper and Jacobi's paper and, as I mentioned, this paper by Pincherle.

DAVIS: Was Gauss's paper written in Latin?

GAUTSCHI: Yes, originally, but it has been translated.

DAVIS: Oh, it has been translated.

GAUTSCHI: There's a German translation.

DAVIS: Now you've done a tremendous amount of work in orthogonal polynomial theory –

GAUTSCHI: That's when it started, most of it, yes, that's right –

DAVIS: Let's talk about that a little bit, your engagement with orthogonal polynomials.

GAUTSCHI: Well, to tell you the truth, it goes back further, to Oak Ridge. I was approached there by a chemist who came to my office and wanted me to compute an integral, a definite integral. It had some kind of functional peculiarity, he couldn't compute it accurately enough. I thought, "Oh, I know how to do that, I read [Francis Begnaud] Hildebrand very carefully, and I knew there was something called Gaussian quadrature and orthogonal polynomials –

DAVIS: Hildebrand's book on numerical analysis, Hildebrand.⁶

GAUTSCHI: *Introduction*. He was at MIT –

DAVIS: MIT, right.

GAUTSCHI: Anyway, so I was quite confident, I knew how to generate these orthogonal polynomials, starting from the moments of the weight function, which you could compute. I programmed it and I ran it, and lo and behold this thing completely went to pieces. I may have gotten the first eight or so Gaussian quadrature rules, up to order eight, but after that it completely got nonsensical. That was my first experience with orthogonal polynomials and naturally I was depressed because I had to admit to this person, to this chemist, “Well, I’m sorry. I’ll try the trapezoidal rule or Simpson's rule, that may work.” I didn’t know about [Werner] Romberg at the time yet – that would have been a better suggestion. But anyway, I got interested to find out what was the reason why this fell apart. And so I suspected there was ill-conditioning of the basic problem, taking the first $2n$ moments of your weight function and generating from them the n -point Gauss quadrature, which also has $2n$ parameters.

DAVIS: Are these moments given in terms of integrals of special functions –

GAUTSCHI: No, no, no. You just multiply the weight functions by the n^{th} power and integrate.

DAVIS: And the weight functions were analytic, or were they experimental?

GAUTSCHI: No, they were analytic.

DAVIS: They were analytic.

GAUTSCHI: In this case, it was a weight function with a logarithmic-type singularity. So you could compute these moments, and that’s what I did. But after this experience I suspected that this was an ill-conditioned method. So I looked into the sensitivity of this method – how do small perturbations in the moments affect the Gauss points and the Gauss weights. That led me into confluent Vandermonde matrices. I discovered that it’s a confluent Vandermonde matrix that’s responsible for the sensitivity of the map. The Fréchet derivative essentially was this Vandermonde matrix, this linear transformation. So I began studying Vandermonde matrices from the point of view of conditioning, not just confluent ones but also ordinary ones. Later on I was interested in the conditioning of all sort of maps with Vandermonde –

DAVIS: But Vandermonde is just terrible in terms of conditioning.

⁶ Hildebrand, Francis B., *Introduction to Numerical Analysis*. (1956 Livermore, CA: Golden Valley Publications).

GAUTSCHI: It's terrible if the nodes are real. If the nodes on the real line can be projected on the circle

DAVIS: There we come to how to balance again –

GAUTSCHI: Well, yes. On the circle, if you take the n^{th} roots of unity then their condition number is one, which is optimal. But then, later on, I was also interested in points on the circle that are a van der Corput sequence. Van der Corput sequence is an infinite sequence in which usually each partial sequence is more or less equally distributed on the unit circle, something like that. So I was looking at the condition number of a Vandermonde matrix when the end nodes are the first end numbers of that random van der Corput sequence. That was one of my hardest papers to work on. Fortunately, [Stephan] Ruscheweyh was visiting us – Ruscheweyh from Würzburg, a rather famous name –

DAVIS: I don't know that name –

GAUTSCHI: Ruscheweyh. There's a Ruscheweyh derivative that people write about a lot –

DAVIS: How do you spell that?

GAUTSCHI: It's R-U-S-C-H-E-W-E-Y-H –

DAVIS: I just learned something –

GAUTSCHI: He was visiting us and I told him about the problem. We got together and solved it. The problem was to find singular values and the spectral condition number of these matrices. We were able to show that, asymptotically, they go like square root of n , which is very good.⁷

DAVIS: That's not bad at all, that's very good.

GAUTSCHI: That's very good, yes. So that was one of my favorite papers. That was very difficult.

DAVIS: What's another favorite paper?

GAUTSCHI: Oh, well, the one on three-term recurrence relations I liked very much.⁸ Another one is the conditioning of this moment map, but if you start not from the

⁷ Córdova Yévenes, Antonio; Gautschi, Walter; Ruscheweyh, Stephan. "Vandermonde matrices on the circle: spectral properties and conditioning." *Numerische Mathematik*. 57 (1990) 577-591.

⁸ It is not clear which of the following two articles Gautschi is referring to here. Gautschi, Walter. "Computational aspects of three-term recurrence relations." *SIAM Review*. 9 (1967) 24-82. Gautschi, Walter. "Minimal solutions of three-term recurrence relations and orthogonal polynomials." *Mathematics of Computation*. 36 (1981) 547-554.

ordinary moments but from what I call modified moments.⁹ The idea there is that you don't multiply the weight functions into a power but into any polynomial you can select – Legendre polynomial, Chebyshev polynomial –

DAVIS: Multiply them by some orthogonal, usually by some well known orthogonal –

GAUTSCHI: Well known that you can compute and that may be very close to what you are looking for. And there's a moment map from these modified moments to the Gauss formula. There I was able to actually write down the condition number in terms of the Frobenius norm, exactly. That's my second favorite.

DAVIS: Oh, that's wonderful. So you do work with fairly arbitrary weight functions?

GAUTSCHI: Yes, I was interested – in fact that was my main interest – in generating orthogonal polynomials for any weight function –

DAVIS: How about weight functions of mixed sign, did they come up?

GAUTSCHI: Okay, they come up too, yes, and I was looking into some cases. Of course, there things may break down after a while. But very recently I was looking into a kind of sine of 1 over x, and x goes to zero. So that's an oscillatory weight function, it has a dense oscillation near the origin with an interval from zero to one. But you make it positive; I simply added one to it, and it's positive. And there the moments exist, so there exists a system of moments with these densely oscillating weight functions, and I was able to actually compute them using high precision computation. It turns out that I used ordinary moments, because modified moments would be too difficult to compute. Ordinary moments can be expressed in terms of special functions, exponential integrals, Bessel functions, and the like. So in order to overcome the ill-conditioning that was mentioned before, I used arbitrary precision arithmetic. In fact I used Matlab and I used an algebraic toolbox called Symbolic Toolbox of Matlab that allows you to do things to any precision whatsoever.

DAVIS: How high a precision did you go to?

GAUTSCHI: Up to a hundred.

DAVIS: Two hundred.

GAUTSCHI: One hundred and something. You get about thirty four decimal digits.

DAVIS: That is ill-conditioned.

GAUTSCHI: That was ill-conditioned, that's true.

⁹ Gautschi, Walter. "On generating orthogonal polynomials". *SIAM Journal of Scientific and Statistical Computing* 3 (1982) 289-317.

DAVIS: But it's interesting that it is possible now.

GAUTSCHI: It is possible to do this –

DAVIS: This wasn't possible some years ago –

GAUTSCHI: No, definitely not. Even nowadays it's not so well known that you can do that. I just wrote a book on orthogonal polynomials and the computational aspects, and I gave reference to a Matlab software package, that's easily available now on the Internet and people can now –

[End of Tape 1, Side A – Start of Tape 1, Side B]

DAVIS: Working the second side of the tape.

GAUTSCHI: We were just talking about the book I wrote and the software package available now. So I hope that, based on that, people become aware of it and they use it. Then, maybe ten or twenty years from now, maybe that's a matter of routine, everybody just computes –

DAVIS: Does this mean that the questions of ill conditioning become irrelevant?

GAUTSCHI: To some degree, yes. First of all because modified moments give rise to much better conditioned problems; not always, but they do. But there are other methods based on discretization of the inner-product, discrete inner-product, that generate discrete orthogonal polynomials which approximate the continuous ones. That usually works much better, even for very ill conditioned situations. So that's often the way to go, anyway. I hope in the future this becomes a routine matter.

DAVIS: Well, let's hope that it does.

GAUTSCHI: But usually it takes about twenty years, in my experience, until people become aware of the availability.

DAVIS: Sometimes these things have to be rediscovered originally. You know, like your story of Jacobi and Goldstine –

GAUTSCHI: Yes.

DAVIS: The name by the way is G-O-L-D-S-T-I-N-E. But let's talk very briefly about something that flatters me, to my ego. [laughter] Talk about the circle theorem. Say a few words about the circle theorem.

GAUTSCHI: Yes, that's an interesting theorem. If you plot the Gaussian weights, suitably normalized, over the nodes, it often happens that they come to lie on a half circle over the interval of interest. You and [Philip] Rabinowitz were the first, I think, to point

this out, in 1961 I think it was.¹⁰ And how I got interested in that again after all these years? Well, I'll tell you why. I'm working on exercises for my book, *Orthogonal Polynomials*¹¹ –

DAVIS: It s just been published –

GAUTSCHI: That's correct. Just published this year, yes, by Oxford University Press.

DAVIS: Oxford.

GAUTSCHI: Anyway, I'm working on exercises and I thought of problems to do. One of them, I remember, maybe I should look into this circle theorem again and try it out for other weight functions, make it an exercise, which I did. And then, to my astonishment, I found out that it works very well not only for Gauss formulas, for Gauss-Lobatto formulas too, which you already mentioned I think, and also for Gauss-Radau, and even to some extent for Gauss-Kronrod formulas. So that's how I became interested in that again. Actually, I'm in the process of writing a paper, because I went to a meeting in Atlanta very recently and many people there were interested in potential theory and equilibrium measures and all that kind of thing. One of the participants there was Vilmos Totik from Hungary. He told me, "This circle is nothing but the reciprocal of the density of the equilibrium measure of the interval $[-1, 1]$. That's all to it." And so the circle theorem holds for much more general weight functions, essentially weight functions in this so-called Szegő class, which are quite general –

DAVIS: Had Totik been thinking about this?

GAUTSCHI: He has been thinking about the Christoffel function. He formulated this in terms of the Christoffel function. And, as you know, if you evaluate the Christoffel function at the Gauss nodes, you get the Gauss weights. And that's the connection with potential theory.

DAVIS: I see. So he knew about –

GAUTSCHI: He knew about the Christoffel function. He consulted [Paul] Nevai, actually.

DAVIS: Paul Nevai.

GAUTSCHI: Paul Nevai, yes. So now I am trying to get this connection with potential theory and then hopefully publish it soon.

DAVIS: So this little theorem, this circle theorem, of mine has very deep roots.

¹⁰ Davis, Philip J. and Rabinowitz, Philip. "Some geometrical theorems for abscissas and weights of Gauss type." *Journal of Mathematical Analysis and Applications* 2 (1961) 428-437.

¹¹ Walter Gautschi. *Orthogonal Polynomials*. (2004, Oxford: Oxford U.P.).

GAUTSCHI: Yes, definitely yes.

DAVIS: That's interesting –

GAUTSCHI: Yes, it has roots in potential theory, which you would not expect.

DAVIS: No, I didn't expect that. Let's talk about something quite different, let's talk about your surname Gautschi. It seems a little bit strange for someone born in Basel to have such a name. Can you say something about that?

GAUTSCHI: Yes. Actually, I can pretty much give you the history of it. It goes back to the Middle Ages. You probably know that in the Middle Ages, in the twelfth century, thirteenth century, the business of dealing with money, particularly lending money at exorbitant interest rates and so on, was almost exclusively the prerogative of Jews.

DAVIS: Yes, I knew this because the church, the Catholic church, had a prohibition –

GAUTSCHI: Exactly, it was the Pope Leo the Great in the first century after Christ¹² –

DAVIS: This is called usury, you see, in English The interest is called usury.

GAUTSCHI: The what? Oh, the interest –

DAVIS: The interest is called...you're not supposed to charge interest –

GAUTSCHI: Yes, that was not considered to be Christian.

DAVIS: So the widespread use of credit cards today is very much anti-Christian in the sense of the Pope.

GAUTSCHI: Exactly. But it was the Pope of the first century –

DAVIS: The Pope of the first century –

GAUTSCHI: That was a long time ago. Anyway, so it was Jews who did engage in that business. But as time went by, later on, two groups of Christians evolved that engaged in this illegal business of lending out money. One group was called the Gauwertschis, or sometimes Galwertschis, and the other was the Lombards –

DAVIS: Lombards, oh yes.

GAUTSCHI: Now they were called Gauwertschi because they came from a town called Cahors–

DAVIS: How is that Gauwertschi spelled, do you know?

¹² Actually, Leo the Great (Leo I) was Pope from 440-461 A.D.

GAUTSCHI: Well, it had many different spellings at the time. But one is G-A-U-W-E-R-T-S-C-H-I, that's one spelling, but there were others at the time.

DAVIS: But the Lombards were famous for being bankers.

GAUTSCHI: Yes, still today a Lombard is a loan.

DAVIS: A Lombard is a loan?

GAUTSCHI: Yes, that comes from those.

DAVIS: I didn't know that.

GAUTSCHI: Yeah, or some kind of loan.¹³ Anyway, there were two groups that evolved that did these things illegally. One came from Cahors, from southern France, and the other from northern Italy, from Lombardia. And now, as you know, names of professions – this was a sort of profession: the Gauwertschi's they were dealing in money, lending – but as often happens with such names like Baker or Miller or Shoemaker, they became family names eventually. That's what happened with that Gauwertschi, it eventually became a family name. They were found all over Europe, but between 1550 and the end of the sixteenth century, four of those Gauwertschis settled down in Switzerland, in the town of Reinach, central Switzerland near Lucerne.

DAVIS: R-E-I-N-A-C-H –

GAUTSCHI: R-E-I-N-A-C-H, yes. Four of them settled there. I don't know, they probably were not engaging in money anymore. Or maybe they were thrown out of France because of that. Anyway, they became farmers eventually, and had many offspring, and most of them stayed in Reinach. So after a while it was said that there are more Gauwertschis in Reinach than there is sand at the beach. [laughter] And even today it's like that, if you look up a telephone book –

DAVIS: In Reinach?

GAUTSCHI: Yes, in Reinach about half the telephone book is Gautschi [erived from Gauwertschi]. Anyway, there were many offspring and one was my father, who was born in Reinach. He was one of the offspring, but unlike most of them he left Reinach. He got an apprenticeship in a Bally shoe factory. Do you know Bally?

DAVIS: No. In Naples?

GAUTSCHI: Bally, B-A-L-L-Y. It's a famous shoe factory in Switzerland. Anyway, he got an apprenticeship there. Then he was sent to the French part of Switzerland to learn

¹³ A Lombard loan is a type of loan issued by a central bank that is secured by collateral such as stock and bonds.

French there. That's where he met my mother, who was quite a different type of person. She was a city girl from the eastern part of Switzerland, from the town of St. Gallen. Her father was in the silk business, which was flourishing at the time because of the fashion. Anyway, they were two totally different kinds of people. My mother was artistically inclined, intellectually inclined, and my father was a farmer's boy. They got engaged and they married, and then they moved to Basel. There, my father was given a store, a Bally shoe store, to manage, which he did for most of his life.

DAVIS: St. Gallen is a place where there is a famous monastery if I remember –

GAUTSCHI: Yes, and a famous library. That's correct, yes.

DAVIS: Yes, well that's very interesting, and you've traced it back to the Middle Ages –

GAUTSCHI: Yes, there is a history available, actually.

DAVIS: Really. Since we still have some time, and if you're not too tired shall we go on?

GAUTSCHI: Sure.

DAVIS: You have collaborated with many people over the years. Do you find that an easy thing to collaborate?

GAUTSCHI: Well, most of the time it was because I met these people and I suggested some open questions that I was working on at the time. Most of the time that's how it started, but not always. The first time, actually, was a Japanese mathematician who came to me for a sabbatical at Purdue. That was 1972. His name was [Hiroki] Yanagiwara from Japan. It was difficult because he didn't speak English very well. He was able to read English, but in school they never learned how to pronounce it, so the only way we could actually conduct a conversation, a mathematical conversation, was at the blackboard writing down formulas. So I –

DAVIS: Mathematics is the universal language.

GAUTSCHI: It is, yes. And so that I finally found out what he was interested about, and it was an interesting problem about Chebyshev quadrature. Perhaps you remember – of course, you remember because you wrote a book about quadrature – these are quadratures that have equal coefficients. [Sergei] Bernstein did a lot of work on that and showed that it's not always possible to find them. So this guy Yanagiwara was interested in finding out, if it's not possible in the original way, can you modify the formula so we can still get Chebyshev-type quadrature? And he had some minimization problems that he formulated. That got me interested in Chebyshev quadrature. I worked quite a bit in the area for a while and I had students working on that too. So that was one encounter, Yanagiwara. The second important encounter I've already mentioned, Paul Butzer in Aachen. He invited me to give that Christoffel festival lecture that you were originally scheduled to give. That was very fortunate –

DAVIS: Well, I thought myself incompetent at that stage, since I'd gotten out of working on it. But anyway, I was delighted that you took over.

GAUTSCHI: You are certainly not incompetent. And you would probably have given a much more interesting lecture than I eventually gave. But at any rate, that started my work on orthogonal polynomials, my serious work, my more recent work. Another person you know too, Richard Varga –

DAVIS: Varga, Dick Varga –

GAUTSCHI: Dick, "der dicke" Varga, we called him. Anyway, he came here, gave a lecture. Then I mentioned to him an open problem that had to deal with (which you were also interested in) error analysis for analytic functions, Gauss formulas for analytic functions, the contour integral representations of the error. I mentioned to him an open problem that I was interested in at the time. Apparently when he flew back to Kent State he was able to see how to solve that problem. He wrote back to me and then we started on joint work on precisely this topic. That was quite interesting. Another person was [Gradimir V.] Milovanović. He was from Yugoslavia, at that time. The way we got together was also interesting, a new way of starting collaboration. Actually I was refereeing one of his papers which involved Gaussian quadrature with weight functions, which are called the Einstein weight function and the Fermi weight function. But he had a very old fashioned approach in this paper, this manuscript. So essentially I rewrote the paper for him, and then we agreed that we should publish it jointly. So that was my first contact with Milovanović. Then he invited me to visit him in Yugoslavia, which I did. Then after dinner at his apartment we sat together and started to talk about all kinds of problems. It was a very inspiring conversation. We threw out a lot of problems that we were interested in and were somehow related to what I was working on. That started our collaborations, especially on moment-preserving spline approximation, for example, which is related to Gaussian quadrature, and on other orthogonal polynomials on the semi-circle, and so on.

DAVIS: So, I take it that collaboration has been fruitful and very enjoyable.

GAUTSCHI: It has been for me, yes.

DAVIS: What about in the other direction, with Ph.D. students and so on? You've had many I'd assume.

GAUTSCHI: That is not correct. I had only seven.

DAVIS: Seven.

GAUTSCHI: Seven Ph.D. students, and two postdoctoral students: one was Yanagiwara and another one was [Giovanni] Monegato from Italy, who became a professor in Turin. He was essentially my student although he didn't write the thesis [with me]. But I

advised him and sent him to Stanford, and we worked together in Madison on my sabbatical, and so on. So I had only seven students. The first one was Zahar, who incidentally edited the book I mentioned [the festschrift in my honor]. I gave him the problem of extending this work on three-term recurrence to difference equations of arbitrary degree, linear difference equations. He did a marvelous job on that – he wrote a thesis on that and published it later on. Then I had other students. Some of them worked on quadratures and some others worked on so-called Sobolev orthogonal polynomials, but I didn't have many students. Part of the reason was that I was busy managing the journal *Mathematics of Computation* for about twelve years and that was practically a full time job, so I couldn't really do much else. I still carried on with my research but I didn't want to take on students during that time.

DAVIS: When you were doing this editorial work, were you reading the submitted papers carefully or did you send them out to referees?

GAUTSCHI: Well, what I did is I sent most of them out to referees [actually to associate editors]. Those that were rejected I didn't bother reading; those that were accepted I always read again and edited before I sent them back. Then I read them again when they were in proof stage. I was very careful, especially language-wise because many of the authors came from overseas, many came from China and so on, and their English was sometimes quite bad. Incidentally, that started my interest in style, language, English-style. I had to study the style the Chicago, what is it called?

DAVIS: Style Manual, Style Book –

GAUTSCHI: *Manual of Style*, something like that.¹⁴

DAVIS: That's a big book isn't it?

GAUTSCHI: That's a big book, but it's very interesting. So I studied that and was very careful in all these papers. I did much more than was called for, really. I enjoyed it too because I learned a lot about it, what people did, especially in partial differential equations.

DAVIS: I think that I'm a bit exhausted now, and I think I'll bring this session to a close with thanking you very much, and I'm sure you're a bit exhausted. You've done a wonderful job as far as I can see.

GAUTSCHI: I very much appreciate that you selected me to be interviewed, and I've enjoyed it. I've always enjoyed meeting with you because it's always been a pleasure.

¹⁴ *Chicago Manual of Style*