An interview with
IVO BABUSKA

Conducted by Philip Davis
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Brown University

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This is an interview with Professor Ivo Babuska on June 20, 2004. Philip Davis interviewed Professor Babuska on the occasion of an International Congress on Numerical Methods and Partial Differential Equations, held at Brown University. Professor Babuska gave the opening lecture on the subject of generalized finite element methods.

<u>DAVIS</u>

It seems to me, we've been in the business a long time, you and I, and, well, my first impression here is of the remarkable growth of this field. When I first entered it as a young student, there were hardly a handful of people over the world who were doing, what you might call research into numerical methods. There were hardly any courses given in universities. The name "numerical analysis" hadn't been invented. Now look at it; there are several hundred people in this conference on one subset of numerical analysis. It strikes me as a remarkable growth. How does it strike you?

<u>BABUSKA</u>

In general, over very nearly fifty years, there was a single change, the computer. It changed everything. When I was young we studied the Goldstine-von Neumann[1] paper that implied it would be impossible to solve the system of linear equations of size one hundred.

<u>DAVIS</u>

This was approximately 1948?

[1] John v. Neumann and H. H. Goldstine, Numerical inverting of matrices of high order, Bull. Amer. Math. Soc., (1947), pp.1021-1099.

BABUSKA

Yes, yes. But of course now we are saying we can solve millions of equations. This means that two things have happened. Number one, in general, that mathematics from the qualitative side became to be more quantitative. Number two, because of that, in engineering, and in real life, numerical methods are used, and, therefore, research in these methods began to increase because of the need in the applied sciences. Of course, you can say at this Congress, where are the mathematicians? This is even more relevant in computational mechanics where congresses have 1500 to 2000 people. Now, in general, science has been developing and you can see this from the number of journals. After World War II, there were, you see, a couple of journals, but now we have hundreds of journals which also shows something in this direction.

DAVIS

Hundreds of monographs, also.

BABUSKA

Hundreds of monographs too.

DAVIS

Right.

BABUSKA

And this is an information explosion that makes things hard, in a way. People know only what is really published recently, and essentially all the old results are more or less forgotten because nobody's going farther than a couple years in references in the journals.

DAVIS

This of course is characteristic of research and mathematics quite generally, I mean, the memory of young people is very short. You mentioned something that is interesting, you said, that say, in computational mechanics, and perhaps in computational sciences, in general, biometrics and so on, the number of attendees in conferences is far greater than [the number] at this conference which you characterize as being mathematical. How would you describe the difference of attitude between the people that are now meeting here and the people in computational mechanics, or in other computational sciences?

BABUSKA

Here the people in mathematics are presenting the results of mathematical type with some kind of proofs and theories, and so they really concentrate on the mathematical part much more than on the computational part. In the engineering and computational sciences, their lectures and their research is presented as results that are related to mechanics, biomechanics, whatever you wish. The theory, if presented, is heuristic and without any proofs, but you see both sides. You show computer results and you show how it works. Essentially we see here a dichotomy. For example, finite elements were used, say, in the 50s, and the theory of finite elements essentially started in the beginning of the 70s, but already in the 60s there were commercial codes for finite elements. Mathematicians didn't pay attention at the beginning, but roughly speaking began in the 70s. So, therefore, a lot of, say, new numerical methods, etcetera, are coming from engineers because they are solving very complex problems which are much more complex than any mathematicians could analyze. This is what you see at big engineering conferences.

DAVIS

This reminds of some remarks that you made to me yesterday when we were sitting in the lobby. Sometimes there's a disregard of the relationship between what is computed and what is actually observed, a disregard on the part of the, let's say, the mathematical /numerical community. Could you elaborate that a little bit?

BABUSKA

Today, you see, in application fields the direction is (called) verification and validation. The reason is because we are computing things for which we cannot do the experimentation. And the problem is now how well, or how much, can we believe or trust in the results of the bigger computations. We have to see that we do not compute, in general, for fun. We compute to get some numbers on which some crucial decisions are made, crucial decisions that are related to our lives and safety, for example. Therefore, any kind of use of computation has two parts. One is a problem of the mathematical form that we are numerically solving. And here we have to take into consideration that the information we have always comes with some uncertainty or fuzziness, and therefore if you are using a fuzzy or uncertain input, you have an uncertain output. There is the problem, you see, whether you solve the right mathematical problem, and whether we solve the right mathematical problem is related to what is called validation. We have to discretize, say to solve some PDEs, or something of that kind, and now the question is how far the numerical solution is from the exact solution of the mathematical problem, and, of course, whether the codes we are using are without bugs. This is called verification. So, roughly speaking, validation is whether you are solving the right problem, and verification is whether you are solving this problem correctly.

DAVIS

In connection with validation, I would like to get your reaction to what is going on now with cosmology. One has string theory which apparently people say there's no way of making an experimental verification. At the same time, the amount of new mathematics that has come up from string theory is tremendous. So how does this fit in with the general picture of validation?

BABUSKA

Okay. I cannot say anything reasonable because I know the field of engineering and other fields but not cosmology. In general, the problem of validation problem is also a philosophical one. The problem is now whether under what condition we could believe it and this is related, for example, through the philosophy of Karl Popper, and essentially the problem of whether in the definition of Karl Popper it is a science or not. I don't want to go into the philosophy, but in validation we are only, say, comparing with the experiments some part of the problem which we are using for the predictions. [If] you can never, in a way, or almost never, validate by experiment, these predications, then it's called a post audit. For example, if you compute, say, the disposal of atomic waste, you cannot check whether in a hundred thousand years something will happen. If you are around in a hundred thousand years, you would "post audit" and see whether these predictions were reasonable or not. We have to make some decisions always based on uncertainties. Of course, no decisions are completely, you see, safe or completely correct because Nature is more complicated than that. Not only that, we have for these decisions only some information with some kind of uncertainties. And now the trend is beginning to be determined because of the computers that we are now bringing slowly in uncertainty, and deterministic computations which were a tradition, etcetera, are still used, but now are beginning to be augmented by the uncertainty in information. In mathematics, it is assumed that a mathematical problem is coming from somewhere and usually mathematicians are not interested where the mathematical problem is coming from. They are interested to solve the mathematical problem in some sense, and they are solving it as a pure mathematical problem or also in numerical ways. But, in fact, any numerical result is used for, at the end, some crucial decisions, and this has to be reasonable. I hope you see it is now slowly happening that mathematicians also understand it. The major problem is the following one, different fields have different languages. Mathematicians are not able to read engineering papers and engineers are not able to read mathematical papers, and communication between these two groups is not so simple because of educational vagaries. Unfortunately, in way, in nearly all places, mathematicians' education is such that they are not getting any basics of physics, any basics of mechanics, in applications.

DAVIS

You would say that engineers and mathematicians are divided by language. But my observation is this, that even within mathematics itself the different fields are divided by language, and this in my philosophy contradicts the assertion that mathematics is a unified whole, is a unity, I think it is not a unity, as practiced. Maybe, ideally it is unity.

BABUSKA

Yes, but I believe, in mathematics, matters are related by common background. People believe that when they are, you see, discussing something with someone else, that the person has the same background. I see in mathematics what is common, even if the only common element is logic. In engineering it’s a little different. I would characterize the difference between a mathematician and engineer in such a way that mathematicians think in the counter examples, and engineers think in examples, which has a lot of importance. Because engineers, in applications, are designing methods or using sophisticated methods, and if it works in a couple of places then (they assume) it works always. So engineers are not, say, used to thinking under what assumptions (the method) is valid. On the other hand, mathematicians have something they would like to create, to see whether (a method) works, and so create counter-examples to see what is going on. Many times, I am discussing with an engineer, a little bit, you see, not very reasonable counter-examples. The engineer would say fine this is true, but who cares about such things.

DAVIS

These remarks about counter-examples reminds me of a passage in one of Poincaré's books, in which he makes this point also, and he says he doesn't like this whole tendency among mathematicians to come up with counter-examples. He was speaking of things like Weierstrass' continuous non-differentiable functions, and things of this sort. Well we talked about philosophy a little bit, I'd like to change the direction a bit and dig into some more personal things. Can you remember when you first realized your interest and talent in mathematics, how old were you and under what circumstances, and so on?

BABUSKA

Okay. My history is a little bit complicated for any American [to understand], because we went through the occupation by Hitler, because I lived in Czechoslovakia. I was a son of a well-known architect and I lived in Prague. After elementary school, (which goes to eight years for us) through middle school, my father wanted me to leave, and learn a little bit more about building. Therefore -

DAVIS

That's your father?

BABUSKA

My father and my mother –

DAVIS

Your mother, also?

BABUSKA

You see, they're of an older generation. My father said it would be good if I would go to a vocational school and enter the building [trade]. I went during the German occupation. But my father and my mother were very wise. They said you have to have some mathematics because I had (that) interest too, and there was not enough mathematics there. The Germans, Hitler, in '39, closed all the universities. Czechoslovakia was a protectorate after the occupation. University professors were in a way on leave or whatever. So, I had already, as a small boy, a university professor in mathematics (and in discrete geometry) as a tutor. Therefore, at the end, in '45, when I was finishing school, I already knew a lot of mathematics. My tutor was a university professor at a technical university. So, I already had a mathematical education. But, it was interesting, because vocational school didn't provide, by definition, enough prerequisites for a university. So, I had to take an exam. I remember that one problem was to see if you could give the formula for the volume of the sphere, in three dimensions. And I gave the formula for a volume of a sphere in the N dimensions, which impressed my examiners.

DAVIS

Well you knew already the gamma function.

BABUSKA

I new a lot of things, yes I did. So I then –

DAVIS

Which was impressive –

BABUSKA

I studied civil engineering and I studied very quickly. An engineer's education is not as it is here, but in a different style. After finishing civil engineering [studies], I began working in engineering and I wrote my first Ph.D., in engineering, which was in the technology of welding. But I was also known, you see, for mathematics. In ’48, essentially, I was finishing at the university and there was, you see, a change, the communists came to power.

DAVIS

Was there a Russian occupation immediately in ’45?

BABUSKA

’45? It was not then, the Russians came, but then went away. There was from ’46 on, in a way, a democratic republic. But, you see, after the war, the communists had a lot of votes, and the intelligentsia was very leftist oriented. But in ’48 there had to be some elections, and the communists lost. In February of ’48, there was an insurrection where, you see, communists came completely to power.

DAVIS

I know that there was immigration from Prague to the United States around ’48. In fact one of our professors here at Brown was in that generation. I knew a number of Czechs that left. They were able to leave at some time, although, it was probably was not easy to leave.

BABUSKA

No, [there was] a lot of persecution, especially for the people associated with the West or who were [deemed] dangerous for the regime. For example, people who fought in the British army, and so on, they had a lot of difficulties.

DAVIS

Was this the time that Jan Masaryk [Minister of Foreign Affairs of pre-Communist Czechoslovakia and son of the first President and founder Tomáš Masaryk] died?

BABUSKA

Yes.

DAVIS:

Around that time?

BABUSKA

Yes, it was in ’48. He jumped out of a window, supposedly. In ’48, of course, lot of things happened, and I have to say I had a wrong origin, nobody understands what is the meaning of “origin” here. My father was not a blue-collar worker, but was bourgeoisie. The bourgeoisie were, you see, on the index.

DAVIS

He was an architect?

BABUSKA

Yes, and therefore –

DAVIS

Yeah, well he was middle class.

BABUSKA

Middle class, a bourgeoisie. But anyway, things turned out okay. [But at the time], I didn't know the people in mathematics under the leadership of Eduard Čech, the topologist of Stone-Čech fame [Stone-Čech compactification].

DAVIS

Stone-Čech?

BABUSKA

Yes, you see, he was influential person, and he and others felt we had a gap of ten years and we have to bring together the best young people which we could get in all of Czechoslovakia, [in order to] create the new generation of the scientists. And it was, I think, a great thing and I enjoyed it very much because it was a group of the twelve best people in mathematics in the whole of Czechoslovakia, these people as Eduard Čech, Vladimir Kosina(???), and others at universities, they were completely dedicated to us. It was a great thing for education and the group was, you see, not only studying together, they are skiing together. Our professors went to ski with us. Of course, you see, after skiing, in the evening, we had mathematical seminars and lectures. It was a really great thing.

DAVIS

Seems to me this is quite a variance with the old stories of how in Germany the professors were great professors and quite remote from the students. This was a change.

BABUSKA

We are not Germans. In Czechoslovakia, it was, you see, the western part; there were none of these kinds of German types even before the war, World War II. But this was really very great and let me say a few more words which are interesting. I was trained in engineering and, as I said, engineers think in examples and not counter-examples, and so on. Therefore I had to be retrained, you see, in the study of mathematics at the university. I had in a way to be at least partially re-educated as a mathematician in order to do very well. They gave me to study some elementary set theory, with all intersectional theorems, and which are obvious after making some pictures and so on, and so on –

DAVIS

Some Venn diagrams or something?

BABUSKA

Yes. My professor said you have to make these examples, and I had to be ready with my homework with this university professor. He said, "Good, good Ivo, fine". After half a year or perhaps four months, he said, "Ivo, we have to have a discussion. Ninety percent of the homework you did is wrong. How is it possible", he said? "Look I will show you practically to each of your things a counter-example". These counter-examples were a little bit perverse, yes, but you see mathematics means exact logic, and therefore he said fine you have to learn elementary logic. Therefore I had to study elementary logic what is AB, etc. Then in a paper, [he asked me to] show every one of logical axioms that are being used. It was a little bit hard, yes. But after, you see, although I was not feeling any better, I understood completely what is mathematics and what is engineering.

DAVIS

How old were you at the time of this interchange with this professor?

BABUSKA

I was, about 21.

DAVIS

About 21? Of course this is not my philosophy of mathematics, I mean where mathematics is logic. But not to go into in this conversation, we're concentrating on your biography –

BABUSKA

But then of course I would say I then I was able to master both languages, yes, and until now I am in the both camps. On one side, in mathematics, I speak a mathematical language of functional analysis. On the other side, mechanics, I speak in engineering language, and until now I am, for example, involved in real experiments, physical experimentations [related to] all of the computations.

DAVIS

So, actually you are in what we would call applied mathematics from the start, right from the start?

BABUSKA

Yes.

DAVIS

This [happened] because of the vocational attitude of your parents?

Looking over the last half-century or so, what do you consider your main contributions to numerical methods? Probably many.

BABUSKA

Yes, there are many, but I would say three or four are the major ones. One is, you see, at the beginning of finite elements. It was a problem of the, so called, inf-sup conditions, which made some kind of opening of doors –

DAVIS

Inf-sup conditions?

BABUSKA

Yes, Inf-sup conditions for a lot of methods. Another one was, you see, a-posteriori estimation in finite elements which we sere not taking into account for about ten or twelve years. After twelve years, people discovered this, and there was an explosion. Now you see there are a hundreds of papers on a-posteriori estimation which, in a way, I initiated. Another one, which I initiated, was the h-p version [of the finite-element method] which this conference here is about. The terminology "h-p version" is my terminology and as we say [the basis for] a joke. When I took [the subject up] with engineers over a summer, they said, well, this is nonsense; this is not a p-version this is a perversion. But after ten years, this is used in engineering. As you saw today, all the lectures here are related to p-versions or the h-p versions.

DAVIS

How would you characterize the distinction between the a-priori estimates and a-posteriori estimates?

BABUSKA

A-priori estimates are the typical estimates in mathematics. You are assume, for example, that a solution belongs to some spaces or it has a second or higher derivative that is bounded, and therefore your estimate says that the error is some number times some power of a measure of the grid times [a bound on] the derivative of the exact solution. Or if you knew the integrations, any kind of the simple integrations where the error is h to the some power times you see the maximum of the derivative in the p-norm. Of course, these a-priori solutions are important to prove your convergence, but on the other hand it is not what we need [when we compute]. We need to have the error with some confidence and we, of course, don't know the derivatives. We don't know anything, but we have to utilize the information we are getting during the computations. This is the reason why I'm saying "a-posteriori". We are using the information for which we are able, you see, to recover from the computation. A-priori means that you don't have any kind of a computation, we assume something, and we say that with such and such assumptions we have convergence at such and such a [rate].

DAVIS

On the basis of the information that you get out as the computation goes forward, do you do things like changing the grid size and so on, and then make further error estimates? How does that work?

BABUSKA

It is possible to do this other ways, but usually it's not the case. Typically, when I started with that, we were interested in solving PDEs, using energy norm estimates. Here are some of these computations, we have some computed solutions, we then utilize the fact that there is some orthogonality and we know essentially that in a formal way that the error in the energy norm and error in the negative norms are related with one to one correspondence. The question is how do you compute this negative norm, and you have to utilize some kind of essential facts of finite elements, maybe some kind of orthogonality and it's possible to do this. So, now, as I said, there are hundreds of the papers. It is very expensive when you are making a mesh and then you are dividing it in half and a quarter, etcetera. Of course, we do have to change the meshes during the computations anyway. We have to do mesh generation because we have to make some progress. We would like to make adaptive meshes that use error computation to provide the most efficient way. Thus my work with a-posteriori estimation was also related to adaptive meshing, which today everybody speaking of and is using.

DAVIS

Well I have to say that I had a finger in this many years ago where we had adaptive methods for integration of functions of one or two variables. How about some unsolved problems for the future, what do you see there?

BABUSKA

Yes, I believe today, and it is also clear that tomorrow, the major problems and emphasis will go to selection and the modeling of the physical sciences. We are able today to compute very complicated problems. The problem is that we have to have the confidence that we are solving the right problems. This is related to the problem of validation I already mentioned before. I believe we will see trends to solving new, very complicated problems in various fields, from medicine to engineering to physics because we would like to avoid, or we have to avoid, various experiments. We have to do validation by experiment, but we are interested in the predictions and we cannot usually make exactly the experiments. Making decisions, then doing experiments later, we, of course, do see that [happening]. If you make decisions and something is happening in some experiment, and we show that something was wrong, but if we made the decisions? Then what? So the question is how, you see, how much we can trust our prediction.

DAVIS

Because decisions lead to action?

BABUSKA

Exactly. And we are computing because [we want to make] decisions and not only for fun, we are computing to take actions.

DAVIS

Let me ask you something a little bit psychological. As you have worked on the things that you have studied over the years and done research, where do your ideas come from? You have a new idea, where does it come from? Are you introspective at all [on this]?

BABUSKA

I believe it is very essential, I believe, in applied mathematics that people have intuitions, which are from mathematics and from physical intuition, because it is in a way how engineers are thinking. We see that people are computing in ways that use very effective methods. Only much later, do we prove that, you see, they are in a way completely correct. And I would say my friends the high engineers are criticizing mathematics. They are saying, look we are mathematicians, we are designing methods that are working. We are using these results in real-life things, and then you are proving that what we are doing is right. You believe that you did something, you see, which should make you very happy.

DAVIS

They [mathematicians] think that they've accomplished something tremendous?

BABUSKA

This rubber-stamping does not impress me. You have to do something new and work together. Impulses or stimuli should not come only from the mathematics, as von Neumann said in his paper, this basic paper, "I am a Mathematician", something like that.

DAVIS

It's called "A Mathematician"[2], I think, and he expresses the feeling that mathematics can get too ingrown.

BABUSKA

Yes, yes.

DAVIS

I know that feeling.

BABUSKA

You see, mathematicians, as I said before, do not know other fields, and therefore their intuition is based, although everybody has different intuitions no doubt, in mathematics. Therefore they

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[2]John von Neumann, *The Mathematician,* in *Works of the Mind* Vol. I no. 1, University of Chicago Press, Chicago, 1947, 180-196.

don't see in a way what kind of new programs are around them. They are generalizing some method this way or that way, and each is important. However, they don't see that all these tremendously complicated programs around us, not all of which are still [completely] formulated but in which there is new mathematics. I believe I have a little bit the ability to see this, and I was successful really in doing work for which I am completely accepted by engineers. I did something that is important for them too.

DAVIS

So you can attribute this to your own training.

BABUSKA

Certainly.

DAVIS

Is it correct that over the years you have done considerable teaching?

BABUSKA

Yes, I did teach in mathematics, yes. At [the University of] Maryland and so on, I did teach numerical mathematics. But I had the conviction always that students in mathematics don't have enough intuition. You see, they don't have enough background in fields other than mathematics. And this is sometimes frustrating, because mathematicians, the professorship, don't have this background, and, therefore, many times their generalization is for a generalization itself. Are they addressing the question do the generalizations still have connections [to real-world problems]? How great are the connections to their use in reality? Are some generalizations only abstractions, or are they applicable in some scientific field? I discovered various well-known paradoxes. My paradoxes are quoted in various ways. One of these paradoxes is, for example, if you take [the bending of a] simply supported plate, which is one of the major problems in engineering, and suppose [the plate is] a polygon. If you are taking more and more sides [of the polygon], it converges to a circle, but the solution [for the plate bending problem] doesn't converge to the solution on the circle. Now you see, the problem [is that] any kind of real plate would be polygon because of how you are making it. The question is now what is happening and how to avoid [the paradox]. There are a couple these paradoxes which are I am known for. It shows that we have to be a little bit aware of things from the last century. Were they formulated in an intuitive way? These mathematical paradoxes show that some kind of intuitions, because mathematics is not identical with reality, could be contested. You have you understand it, and if you understand it, then everything is okay. So it is necessary to see in the both sides and not only to take mathematical problems.

<u>DAVIS</u>

I know that there's a tendency among mathematicians, certainly among pure mathematicians, to do generalization for its own sake.

<u>BABUSKA</u>

Yes.

<u>DAVIS</u>

Would you say that in a conference such as the one we are now attending that there is tendencies along that line?

<u>BABUSKA</u>

I would say partially.

(END OF TAPE)

DAVIS (Continuation of the interview with Professor Babuska.)

You were talking about the tendency to do generalization for its own sake and whether or not, in a conference such as this, one detects that kind of a tendency.

BABUSKA

As I said, it is not so much [of a problem] because people [here] are addressing some numerical treatment. On the other hand, it may be a little bit here because, you see, that [many] people here don't have education in the applied sciences and most of them do not work in a team on some kind of engineering problems. And this is very essential. Very likely most of the people here are going to only to mathematical conferences and not to engineering ones, and vice versa. This is a usual gap. Of course, neither extreme is correct, I believe. You see the problem with generalization is that many times it is very essential. Generalization is important because it could have, and it will have, some impact sooner or later. But there are the two kinds of generalizations, some generalization is only formal generalization, and other generalizations are, I would say, I don't know, very natural, healthy generalizations. But it is very essential that in mathematics we will do generalization. That we will not only, you see, be completely utilitarian.

DAVIS

I recall that a few years ago, maybe ten years ago, maybe fifteen years ago, there was an outcry from the French engineering community that the school of Bourbaki[3] had ruined mathematical

[3] A group of mainly French 20th Century mathematicians (named for the French General Charles Denis Bourbaki) who wrote a series of books presenting an exposition of modern advanced mathematics

education in France. You are familiar with what's going on in many countries; do you see a different quality of mathematics in different countries?

BABUSKA

I would say that Bourbaki, even in France, is in a way not popular anymore. One of the leaders in applied mathematics in France was Lions, Jacque-Louis Lions, who died a couple of years ago.

DAVIS

Jacque-Louis Lions, yes.

BABUSKA

He had a very tremendous influence on applied mathematics there, and he was not a Bourbaki-ist. I believe now the influence of Bourbaki went very much down. It is my personal belief that on one side Bourbaki did a very good thing, on the other side they did, you see, also do some damage. Intuition, in a way, was suppressed by technique. Let me tell one story about this Bourbaki stuff. When I was at the university, I was there as a young guy, there were some lectures about the proof of some continuities. And, of course, the proof which went something like, given an epsilon there is a delta given by some complicated formula and then by steps one two three [we are done]. When I saw it for the first time, I said, I could never do that, I would need a miracle. And then of course soon I found that it in a way, it was flawed –

DAVIS

Yes they worked backwards –

BABUSKA

Yes, of course, yes. Many times it is technique. In number theory, you see, there was a competition among my professors, to find the shortest proof of some number theoretical result.

There was some number series involved. Landau won. He said by lemma 72, using theorem 455, and taking into account lemma 154, the results follows.

DAVIS

Well, Edmund Landau wrote his books on number theory just in this way. He wrote an elementary calculus book and there is not a single picture in it, but what you say is correct. Talking about national characteristics, when the Soviet Union became, the scientists in the Soviet Union became more open and so on, and there was more inter-relationships between Russian and the western Europe and America, I got the feeling that the Russian mathematics had a different quality somehow, that they were interested in different kinds of things then what we were interested in. Did this strike you in any way?

BABUSKA

Well, you see, I cannot say in mathematics in general. Of course they had, in older traditions, mathematical traditions related to applications. Kolmogorov, Sobolev, and others really had some kind of feeling for or related education in applications. You could say that because of that very essential thing, the Soviets were able to do things such as Sputnik and all these technology related things without computers. Here in the U.S. [scientists were] relying on computers. In the Soviet Union, they had to work on a piece of paper.

DAVIS

By hand –

BABUSKA

So there was more thinking, yes. Here there was, I think, more computing and less thinking. Maybe you know the memoir of von Kármán. There he said that unlike in the Soviet Union, in the U.S. during this time, there was not enough thinking, it was more doing than the thinking.

DAVIS

You're talking about Theodore von Kármán?

BABUSKA

Yes –

DAVIS

Theodore von Kármán, the aerodynamicist?

BABUSKA

Yes, he wrote a very nice book[4]. Where, for example, among others, he wrote various things about [David] Hilbert. [Felix] Klein argued with Hilbert on mathematics and wanted to have some relations with [engineering]

DAVIS

Felix Klein?

BABUSKA

There was some [engineering] conference in Hanover, and Klein said to Hilbert please go there and make some statements in this direction. Hilbert came, and, a little bit in this spirit, he said, "I

[4]Theodore von Kármán with Lee Edson, *The Wind and Beyond - Theodore von Kármán Pioneer in Aviation and Pathfinder in Space* (Little Brown, 1967).

am here and I would like to say something about the relation between mathematics and engineering. The relation is - no relation!"

DAVIS

Let me go back to your remark and have you intensify it a little bit about the relation of the tension between computing and thinking. Do you believe that the existence of such powerful computers that we have now has reduced the amount of genuine thinking that goes on?

BABUSKA

Yes, once more, you see, you have to see this in the complexity of what community you are speaking of. People in practice are using commercial codes, and there are a lot of students today who are trained to put the input into the computer and get the output. I believe, in generalities of course, that because there are many more of such things [like commercial codes] around, that it is easier for there to be much less thinking and more relying on the computer. If something is coming out of the computer, especially if in color pictures, it has to be correct.

DAVIS

It is correct. The computer knows.

BABUSKA

The computer knows and it is in color pictures. And it is very important that it be in color picture because if it is in black and white maybe it is not completely correct, but if it is color it is correct.

DAVIS

[Laughter] Well, I've noticed that the articles that are printed have more and more color pictures and I find sometimes I'm confused by them, that the black and white do more –

BABUSKA

It is a very psychological impact I suppose.

DAVIS

Yeah, but I suppose that you can get trained to this at the age of six or something, then you demand it.

BABUSKA

No, no, it is not demanding, but it is common now and you need it to sell better.

DAVIS

Ah, so now let's talk about that a little bit. What is the relationship between, you might say, commercialization and numerical methods? For example, we have many packages now and we have many combinations of packages, such as Mathematica and MATLAB, and so on and so on. These packages embody, I believe, algorithms that are commercially secret, that is, you cannot get into them.

BABUSKA

Mostly yes.

DAVIS

Yes. Do you have any comments on this commercialization?

BABUSKA

The problem is not with Mathematica. Engineers are using codes that are related to some more or less practical situations, for example, finite elements for this and that. Of course, the problem is that there is a tremendous investment to make such a program. It costs a lot a lot of money, and so, of course the problem is where to get the money. Usually, in these programs, there are a lot of fine points, for example to speed it up. But this is how life is. Even if you would get the source code, for example, you would not be able to read and to see exactly what the program is doing because it is a hundred thousand or million lines.

DAVIS

Absolutely.

BABUSKA

Therefore, you see, the question is not if it is any good but if somebody would use it or misuse it, etcetera. Therefore they are not giving the secrets of the code. But this is life and it will go still farther in this direction.

DAVIS

It seems to me that this additional complexity with which you can now handle by computer is linked to a an increase in confidentiality –

BABUSKA

Yes –

DAVIS

Company confidentiality and there's no way around this.

BABUSKA

Yes, I believe yes.

DAVIS

Well, I think that I am getting a little bit tired and so I would like to thank you for a very revealing interview. But thank you very much again.