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THE USE OF PARALLELISM IN NUMERICAL CALCULATIONS

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ABSTRACT: It is quite likely that future computing machines will permit simultaneous, independent arithmetic and logical operations. The problem of devising numerical techniques to take advantage of this increased parallelism is posed for consideration. A simple illustrative example is detailed and several specific problems suggested.

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Recent advances in computer technology present the possibility of constructing machines capable of performing several operations independently and simultaneously. One can thus envision a computer with some number of simultaneously addressable cells, each capable of operating arithmetically on the contents of any other storage cell.

For a significant class of data processing problems this presents immediate and obvious advantages. In the area of scientific calculations, however, the situation is otherwise. Numerical Analysis began with pen and paper methods for solving problems in the calculus. The transfer of computing emphasis, in ensuing years, to mechanical, electro-mechanical, and more recently to sequential digital computers has left the sequential nature, at least, of these methods remarkably unaffected. Thus it is the purpose of this note to suggest for consideration the problem of devising numerical techniques which make use of parallelism.

For purposes of illustration only, consider the problem of forming the polynomial

$$P(x) = \sum_{k=0}^{n} a_k x^k,$$

where $a_0, a_1, \ldots, a_n$ and x are given real numbers. This can be done, by a well-known trick, in roughly the time required for n multiplications. In a machine such as is contemplated here one could accomplish this result in N multiplication times where N is the smallest positive integer such that

$$n \leq 2^N,$$

that is,

$$N = 1 + [\log_2 n],$$

where $[A]$ denotes the greatest integer less than the number A. This can be done by forming successively,

x, x^2 (multiplication by x)

x, x^2, x^3, x^4 (simultaneous multiplication by x^2)

$x, x^2, x^3, x^4, x^5, x^6, x^7, x^8$ (simultaneous multiplication by x^4) etc., until x, x^2, x^3, ..., x^{2^N} are obtained. The calculation is completed by forming the inner product (surely an available operation) of $(1, x, x^2, \ldots, x^{2^N})$ with $(a_0, a_1, \ldots, a_n, 0, \ldots, 0)$. The magnitudes to be compared are n and $\log_2 n$. Clearly n need not be very large for the difference to be appreciable.

We conclude by stating some of the many questions raised by these considerations:

1. Should a new emphasis be placed on uniform approximation formulas for solving ordinary differential equations (e.g., the celebrated Picard Iteration Process)?

2. What methods for solving partial differential equations lend themselves immediately to the use of parallelism? In particular, does the Monte-Carlo method?

3. To what extent can matrix problems be speeded up and significance improved by performing true vector operations?