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BASIC COMPUTER MEMO # 8

SUBJECT: General Background for the Reliability Theory

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INTRODUCTION

Reliability of a computer can be considered a performance characteristic, for the customer may wish to buy it in addition to the conventional performance characteristics. The customer is interested in the time available to him between system halts because of errors. The maintenance designers are interested in the rate at which errors occur so that sufficient maintenance techniques can be provided for the customer engineer so that the desired availability is achieved.

Here background material will be presented to help the reader understand how the component aging characteristics and circuit configuration combine into circuit failure characteristic. Circuit failure characteristics can then be used to derive machine failure characteristics. For additional background references 3 and 9 are recommended.

Reliability is the probability of error-free operation for a specified period of time. For convenience, the plot of the reliability for each possible time period is made and is called the reliability curve. The mean of this curve is the mean time between errors.

CONCLUSIONS:

Reliability analysis is basically a statistical problem. The output of a circuit type has statistical variations because the components making up the circuit have parameters with a range of values. In addition these parameters change with time which give rise to further variations in the output of a circuit type. The effect of these factors determine the circuit reliability and thus ultimately the maintenance required by the machine system. The tools to affect the solution of this problem are available.

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Part I

Manufacturers have, at times, included in their specifications of an electrical component reference to the life of the component. First of all, at best, this represents the use of the component in a 'typical' application selected by the manufacturer. Typical may mean one selected application or a composite one, neither of which will probably fit the circuit into which the component is ultimately placed. This is especially true where component ratings are halved as here at IBM. On the other hand this life data may indicate the guaranteed life of the component during which time the manufacturer will replace the component which has exceeded its specifications. This usually depends more on the risk that the manufacturer will take than the actual life. The fact that a component has exceeded its specifications still does not mean that it has reached its end of life, for compensating deviations in the other components in the circuit can occur such that the circuit still gives a proper output. For the above reasons the use of individual component failure data has resulted in pessimistic predictions of machine reliability.

Most articles on reliability have discussions on the resultant reliability when components are combined to form a larger system. (Reference 1 and 2). Therefore, only a brief sketch will be presented here. The probability that a series combination of components will not fail is the product of the individual probabilities of not failing. The probability that a series combination of two components will fail is the sum of the individual probabilities of failure minus the probability that both have failed. Mathematically, these are stated as follows, using p_c and q_c as the probabilities of failure and successful operation for the combination and p_n and q_n for the individual elements. Keeping in mind, that $p_c + q_c = 1$ and $p_n + q_n = 1$.

For two elements in series:

$$q_c = (q_1)(q_2) = (1-p_1)(1-p_2) = 1-p_1-p_2+p_1p_2 = 1-(p_1+p_2-p_1p_2) = 1-p_{12} = 1-p_c$$

For three elements in series:

$$\begin{aligned} q_c = (q_1)(q_2)(q_3) = (1-p_1)(1-p_2)(1-p_3) &= 1-p_1-p_2+p_1p_2-p_3+p_1p_3+p_2p_3-p_1p_2p_3 \\ &= 1-(p_3+(p_1+p_2-p_1p_2)-p_3(p_1+p_2-p_1p_2) \\ &= 1-(p_3+p_{12}-p_{12}p_3) \\ &= 1-p_{123} = 1-p_c \end{aligned}$$

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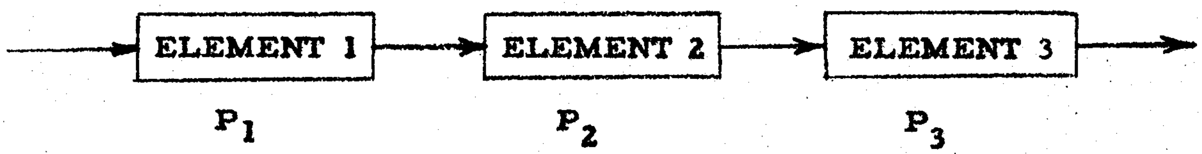
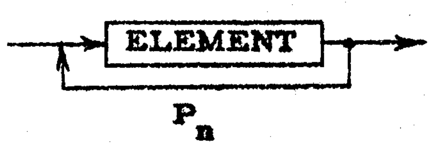
A diagram of this combination is shown below:



The signal must progress through the elements, therefore p_1 could be thought of as associated with time interval 1, p_2 with time interval 2, and so on. The reliability of the combination is the probability of successful operation at the output of element 3 at the end of time interval 3.

Thus, let the diagram be redrawn as follows:



Here p_1 is associated with the first time through the element, or time interval 1, and p_2 with the second time through, and so on. The reliability of this element as a function of time is the probability of successful operation at the output of the element at the end of each time interval. A combination of these two concepts gives the resultant reliability of a series combination as a function of time.

The difficulty with this approach is that once the first replacement is made in the series combination, the calculation must be restarted, for the reliability of the replacement may be different that the reliability of the part it replaced. There is a special case, which occurs when the remaining life of a component is independent of its previous history, that is, $p_1 = p_2 = \ldots = p_n$. This is exponential failure, which is the basis of the exponential law of reliability.

After replacements have been made on a machine the ages of the components will vary widely and the replacements averaged over a long period of time will occur at a constant rate which is the reciprocal of the mean life of the component. Under these circumstances, what is the mean time between replacements? It is the reciprocal of the replacement rate. (Reference 5 and 6). That is, if there are M failures or replacements necessary in a time period, T units long, the average or mean time between replacements will be T/M, since the replacement rate is M/T. For example, consider a die, where rolling a 'four' is considered a failure. On the average we know that a four will come up one out of six times. The failure rate is one failure each six rolls and mean time between failures is six rolls. The example could be extended to determine the probability of exactly X number of rolls from one four to the next. In one sixth of the cases a four will occur on the next roll, in 5/36ths of the cases a four will occur on the second roll after the last four and the probability of each larger number of rolls between fours is 5/6ths of the preceeding. The distribution of time between random events of a constant average rate of occurance is exponential in form.

PART II

A descriptive approach to component failure will now be developed (Reference 10). Let there be an environment parameter to which the component is subject. It will include both external environment and internally induced stresses such as temperature rise, load, inter-action between components, electrical stress, etc. Consider this parameter, which will be called influence, as a constant or as achieving its maximum in each small time interval such that it can be represented by the maximum value.

The component has an ability to withstand the influence, which will be called strength. A failure occurs whenever the influence exerted upon a component exceeds its strength. The component strength distribution changes as the components age. Figure 1 shows how this might occur as a function of time. This might be called an aging characteristic.

Marginal checking techniques attempt to either increase the influence or decrease the strength temporarily so that the components near the normal influence will fail and be detected. Failures are divided into two classes; catastrophic and drift. The aging characteristics for two sample components which illustrate these types of failures are shown in Figure 1.

Because we cannot measure the strength of the component without exerting an influence that would cause it to fail and thus weaken its strength to withstand later test influences, this approach cannot be used to predict the machine replacement rate or reliability.

PART III

Other parameters of the component can be measured. At any one time, the measured parameters of a component type of a particular age will usually spread about a mean value. Some parameter variations can be described as normally distributed. A collection of this data for various component ages will be called the component aging characteristic. This data can come from life tests.

The output distribution of the circuit type can be determined by applying the aging characteristics. (Reference 4). The output distribution will be a function of the circuit age. If the circuit output is not in the region of proper performance then it has made an error. Therefore, the reliability of a circuit is

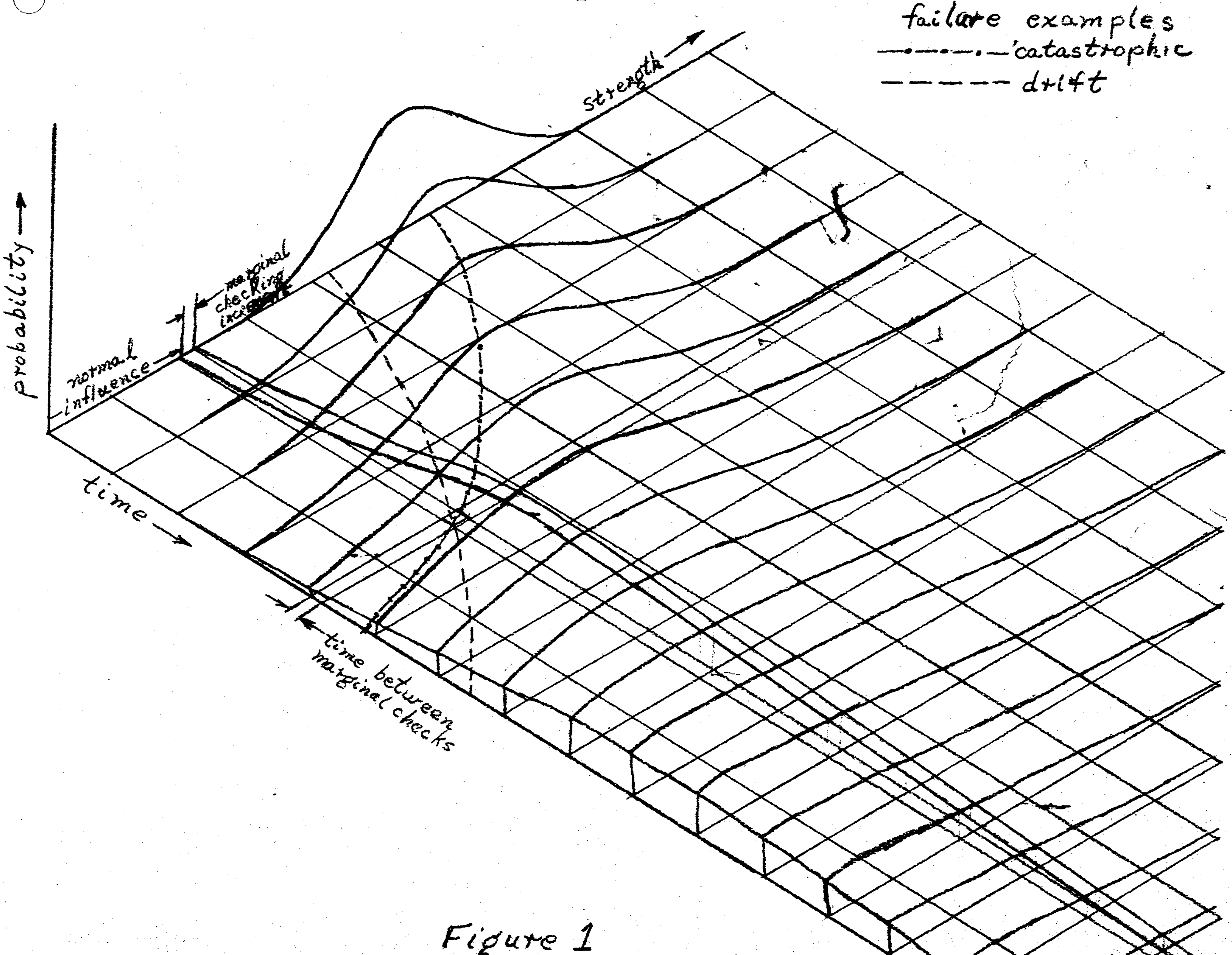

failure examples
catastrophic
drift
strength
probability
normal influence
marginal checking increment
time
time between marginal checks

Figure 1

the probability that its output is in the region of proper performance. In passing, we note that some component parameters are more important than others in determining the output. In fact, if the same component type is used in two different circuit locations different importance may be attached to its parameters. In this approach, the circuit, not the component, is the smallest unit to which a reliability is attached. It is usually sufficient to apply this method to a 'new' circuit, for separate calculations must be made for each different combination of component ages.

PART IV

For example, let us assume some distributions of circuit outputs as functions of time and calculate the failure rates. For the first case, assume the mean of the outputs linearly approaches the boundary of the region of proper performance. Also, assume that the spread of the variance of the outputs does not change with time. Figure 2 shows the process as a function of time. The portion of outputs from circuits which produce an output less than the minimum acceptable have not been ploted. Notice here that the requirements on the output distribution as a function of time to give a Guassian failure characteristic is two fold: first, during the time that the circuit type fails the variance does not change and secondly, that the whole distribution shifts linearly towards the region of failure. This is one good arguement against electronic circuits exhibiting a Guassian or wear-out failure characteristic.

Figure 3 shows the associated failures, hazard, and reliability. These curves show the shape of the curves if the circuit were to exhibit a wear-out characteristic.

Reliability is the probability of successful operation as a function of time or R(t). Unreliability is the probability of unsuccessful operation as a function of time or F(t)=1-R(t). Now, F(x)-F(x-1) is the portion of unsuccessful operations or failures (normalized) in the time interval x-1 to x. The failure rate is the derivative of F(t) or F(t)=dF(t)/dt. This, though, is not the probability of failure that was associated with element at time x in Part I. For that case, we were interested in the portion of failures in the time interval x (ie, x-1 to x) for the portion of cases where the operation is successful up to time interval x which means that the portion of failures should be divided by the reliability at time x-1. Note that the probability of failure also is a function of the time interval chosen. A similar but different measure is usually used in reliability studies which is called Hazard. It is the failure rate at x divided by the reliability at the time x or Z(t) = (f(t)/(R(t).

For a second example, let us assume that the mean of the output distribution linearly approaches the boundary of the region of proper performance as before, but in this example assume that the variance increases linearly with time. Figure

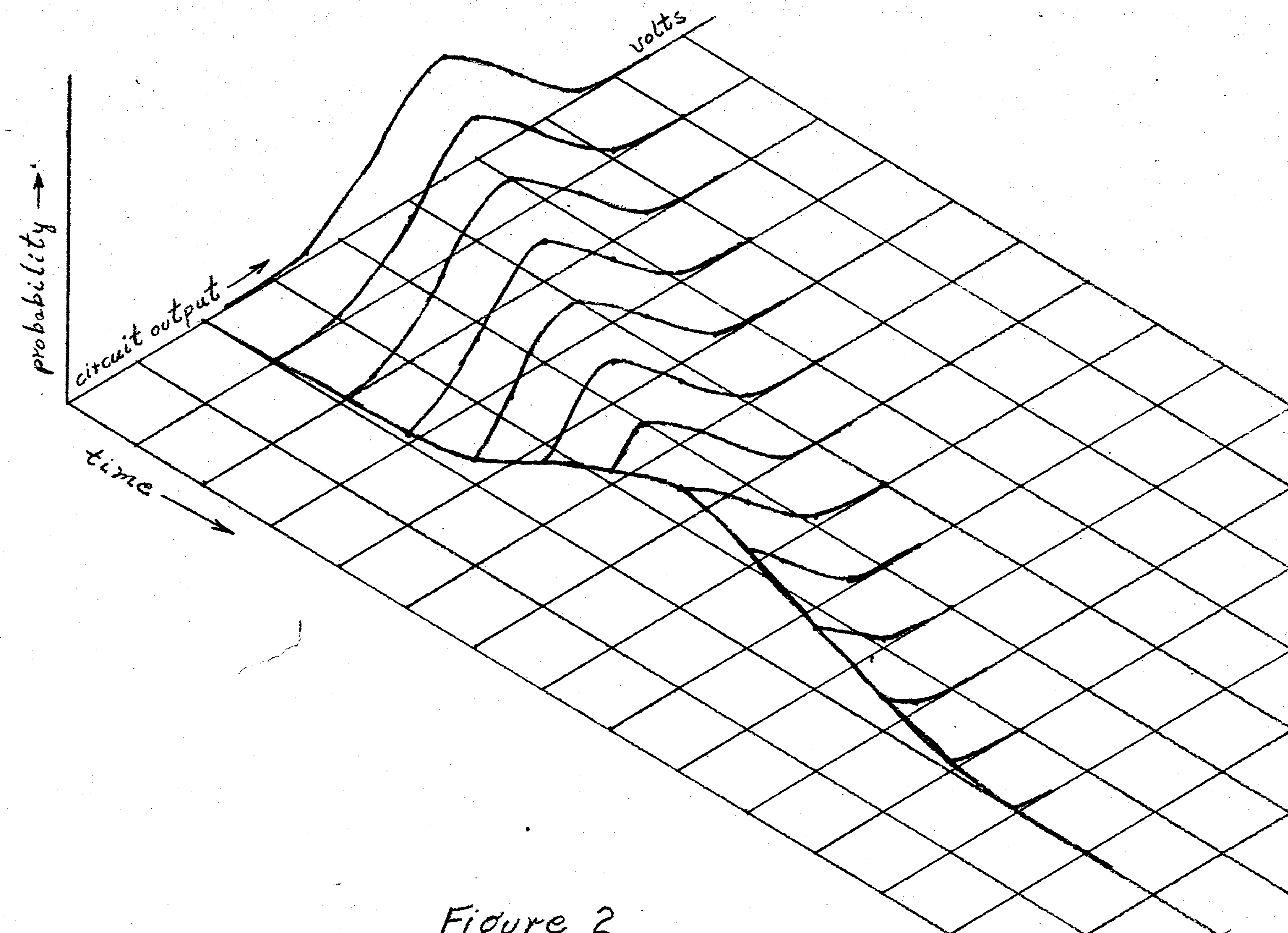


Figure 2

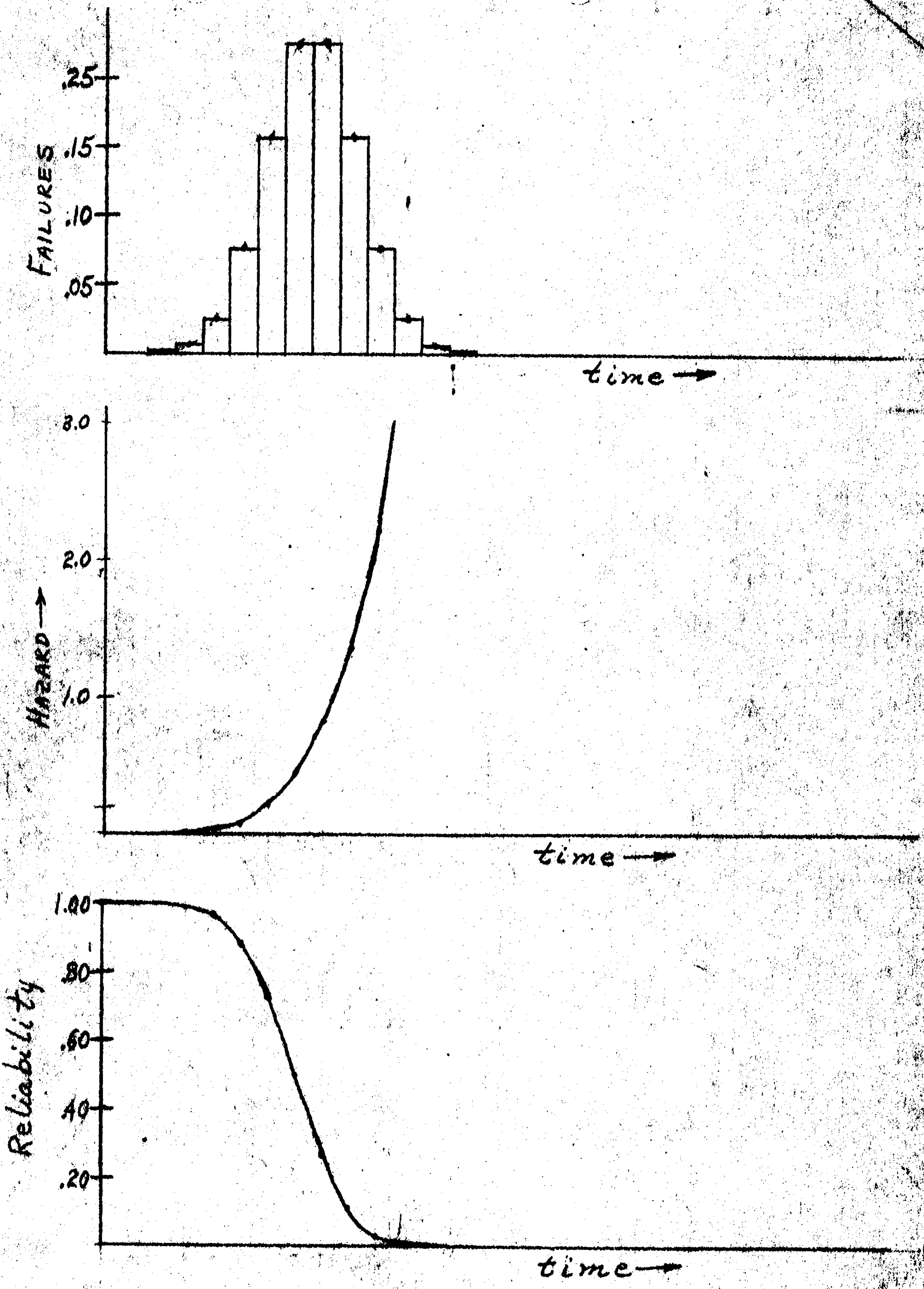

.25
.15
.10
.05
FAILURES
time →
3.0
2.0
1.0
HAZARD →
time →
1.00
.80
.60
.40
.20
Reliability
time →

FIGURE 3

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4 shows the process as a function of time. Figure 5 shows the associated failures, hazard, and reliability. The middle curve is of interest for it shows that the circuit doesn't become more unreliable. This is significant for in the case of wear-out type failure, all circuits of a certain age could be replaced to avoid the higher hazard of the older circuits. In this case, though, the hazard reaches a maximum and then decreases thus a wholesale replacement would replace circuits whose hazard is decreasing with those whose hazard is increasing.

Additional sets of assumptions could be examined, but Figure 5 suggests the exponential type of failure. The associated failure rate, hazard, and reliability for the exponential type failure are shown in Figure 6. Comparison of these curves with those of Figure 5 show much similarity. Figure 7 shows these same curves for an actual life test (Reference 7). Experience has shown that vacuum tube circuits failure curves can be fitted by an exponential type curve quite well. At this time, the aging studies has not established the form of failure curve that transistors circuits will exhibit.

PART V

From the failure data generated in the manner outlined in the proceeding part, we can calculate the replacements required as a function of time. Since the machine is made up of all new circuits at the beginning of its life, this first generation of circuit failures has a distribution like the individual circuit failure probability distribution. The second generation of circuits are placed in the machine as the first fail, therefore the distribution of the second generation failures is more complex. The distribution of these individual generations of replacements could be summed for each time interval to give the over-all replacements but this is not really required for the over-all replacements as a function of time can be arrived at by using a method worked out by Lotka. (Reference 8). The resultant curve of an assumed Guassian failure characteristic is shown in Figure 8. Note how the replacement rate becomes a constant after a period of time. The sum of the replacements for each circuit type will be the total replacement of the machine as a function of time. This is the figure that the maintenance designer needs to know to determine if the maintenance techniques are sufficient to achieve the desired machine availability.

Prior to the replacement rate becoming constant the following approximation could be used. In Part I it was pointed out that if M failures are expected in a period of time, T, the average time between failures will be T/M. This is assumed that a failure in one unit of time is as probable as another, then the probability of failure is M/T. This is the approximation, for with the changing number of failures, there in the time period, T_n, M_n, failures and in the next time period (of the same length), T_{n+1}, there are M_{n+1} failures, the probability

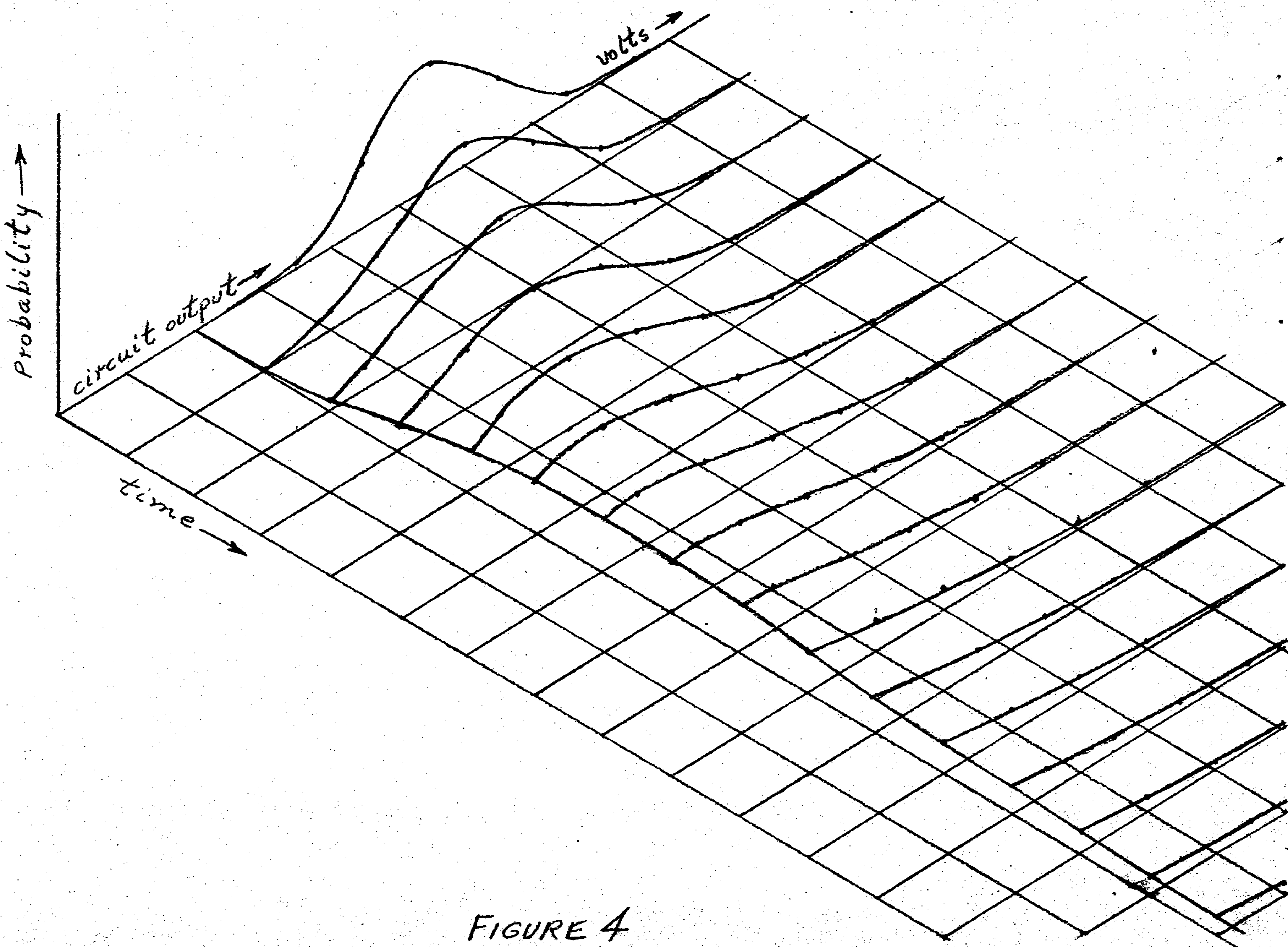


FIGURE 4

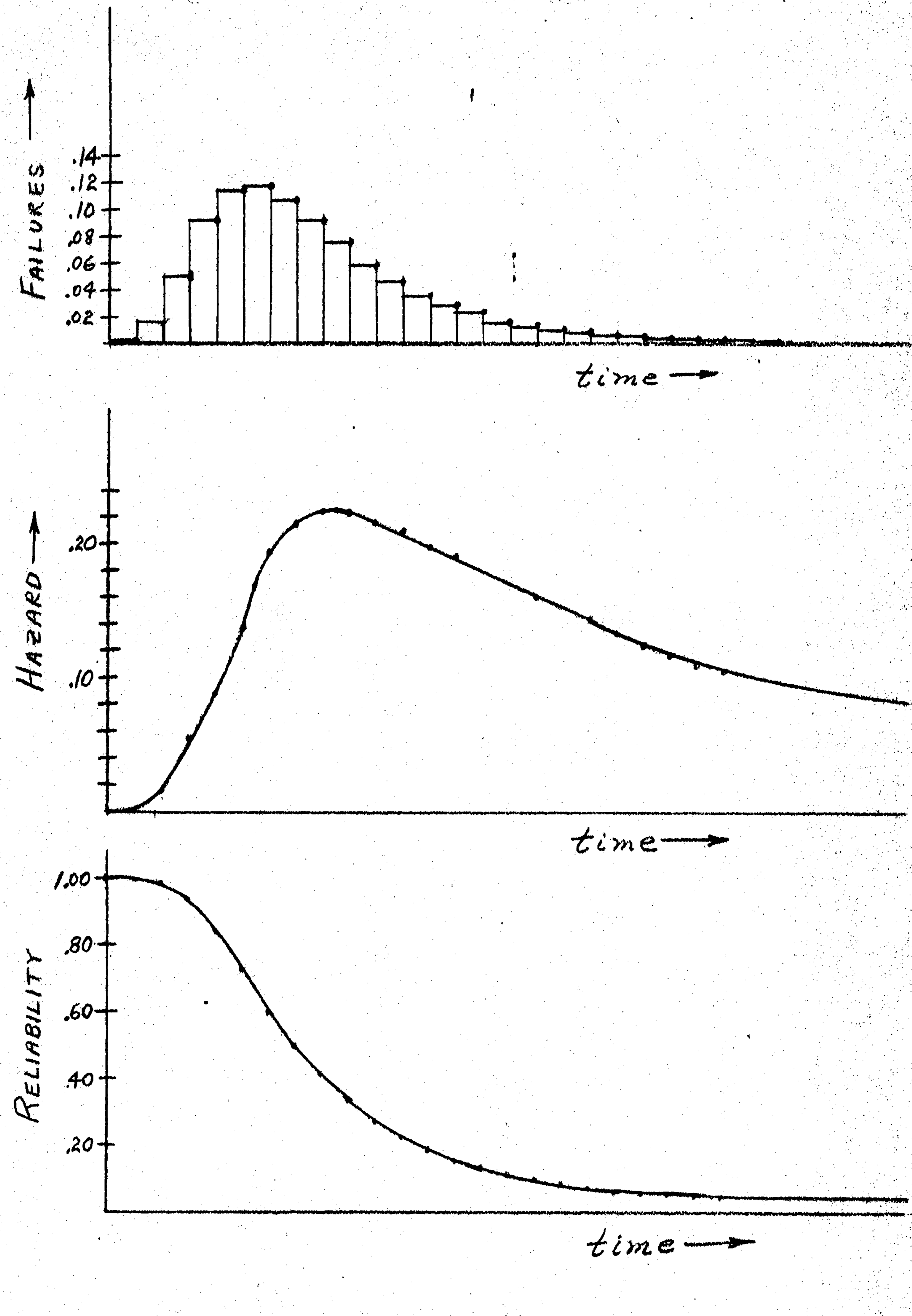
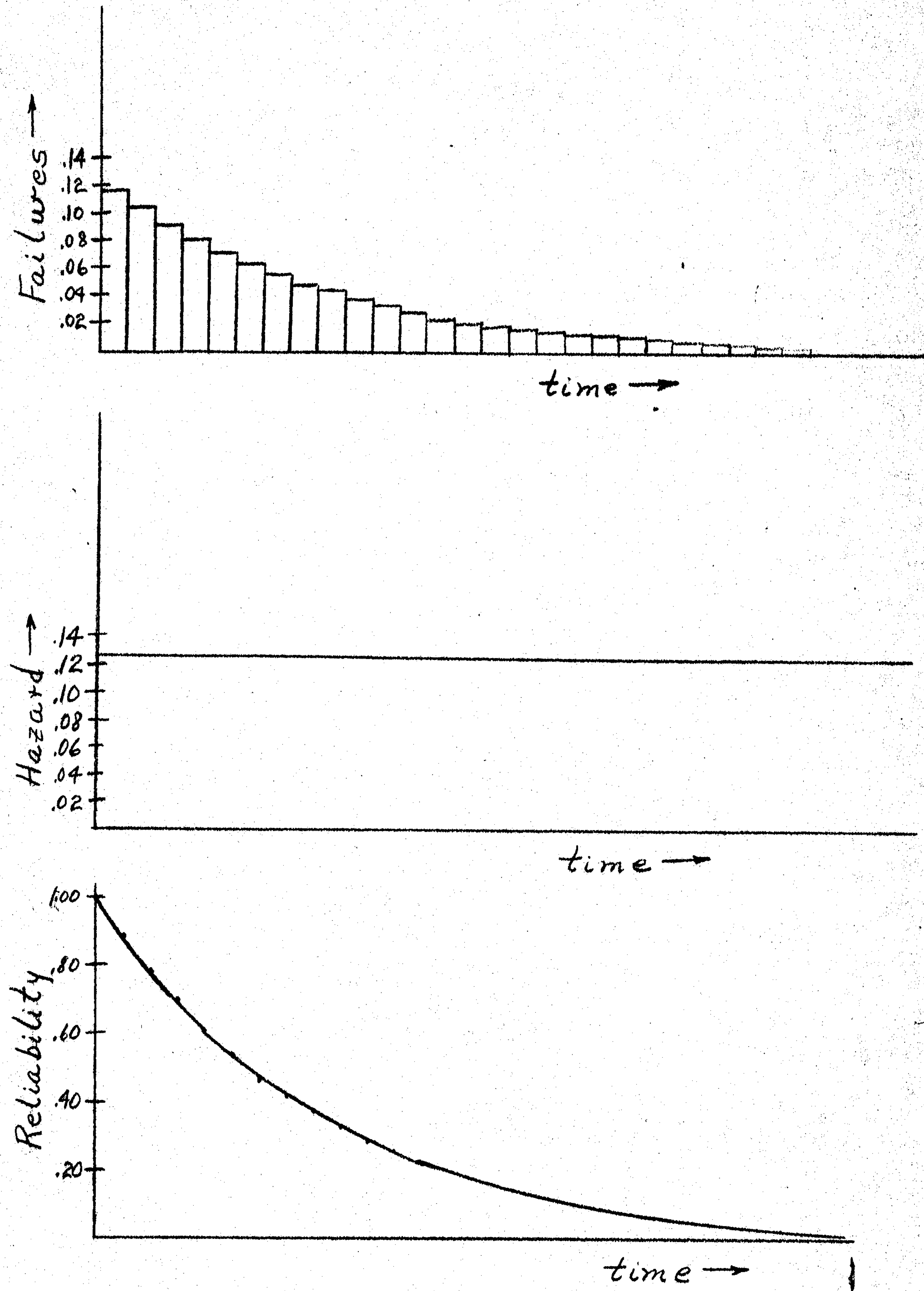

FAILURES
.14
.12
.10
.08
.06
.04
.02
time
HAZARD
.20
.10
time
RELIABILITY
1.00
.80
.60
.40
.20
time

FIGURE 5



Failures →
.14
.12
.10
.08
.06
.04
.02
time →
Hazard →
.14
.12
.10
.08
.06
.04
.02
time →
Reliability
1.00
.80
.60
.40
.20
time →

FIGURE 6

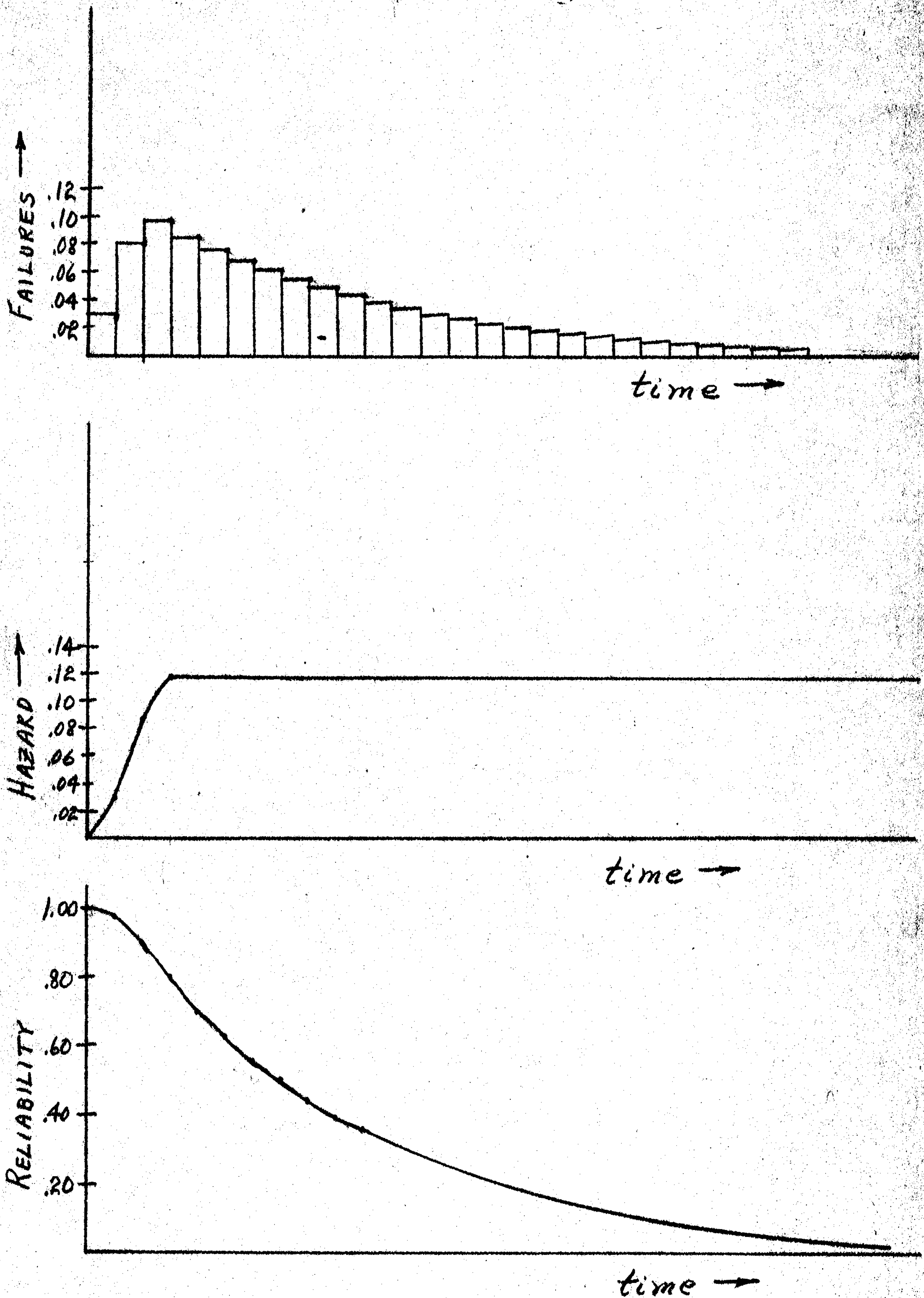

FAILURES →
.12
.10
.08
.06
.04
.02
time →
HAZARD →
.14
.12
.10
.08
.06
.04
.02
time →
RELIABILITY
1.00
.80
.60
.40
.20
time →

FIGURE 7

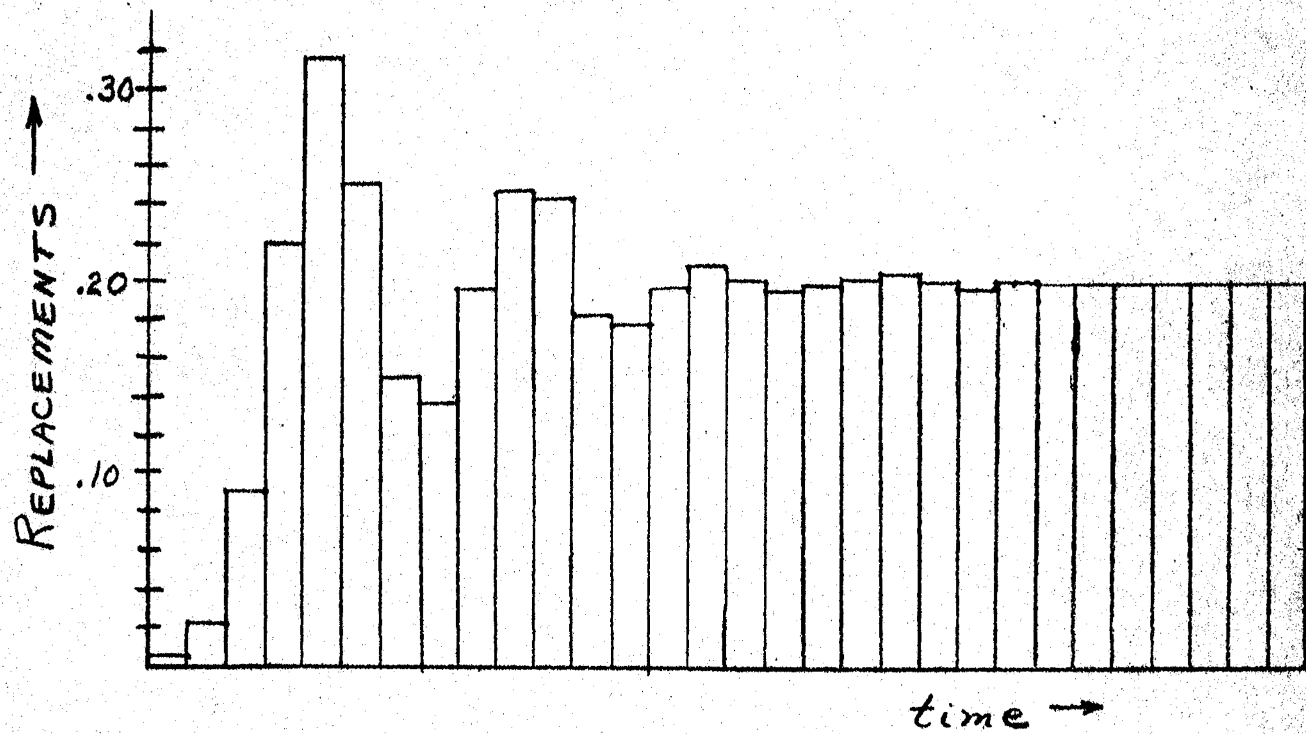


FIGURE 8

of failure must change from M_n/T to M_{n+1}/T. Therefore, the probability of failure in one unit of time is different from the probability in the next unit of time. If M_n and M_{n+1} are nearly equal the use of M_n/T as the probability failure for all time units in T_n will result in a small error. Thus the reliability, mean time between errors and other figures can be estimated.

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