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COMPANY CONFIDENTIAL

PROJECT STRETCH

FILE MEMORANDUM #54

SUBJECT: Shape and Character Recognition  
BY: J. C. Logue  
DATE: January 4, 1957

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Introduction:

This memo describes a method of shape and character recognition in which the information contained in the character is represented by two parametric functions with time as the independent parameter. The parametric functions which represent the character are generated by a simple line scan process. It will be shown how it is possible to overcome the trouble that normally arises from broken lines in characters. The method of recognition to be described makes it possible to have complete freedom of trouble in recognizing characters such as R, O, 4, etc., in which a central area is either punched out or blacked in. It is relatively easy, with the method to be described, to make the recognition of the character be independent of its size or angular orientation on the page. It will be shown how it is possible to recognize miscellaneous shapes such as profiles of people, triangles, and other odd shapes. In addition to the above, it is relatively simple to check to see if the complete character has been scanned.

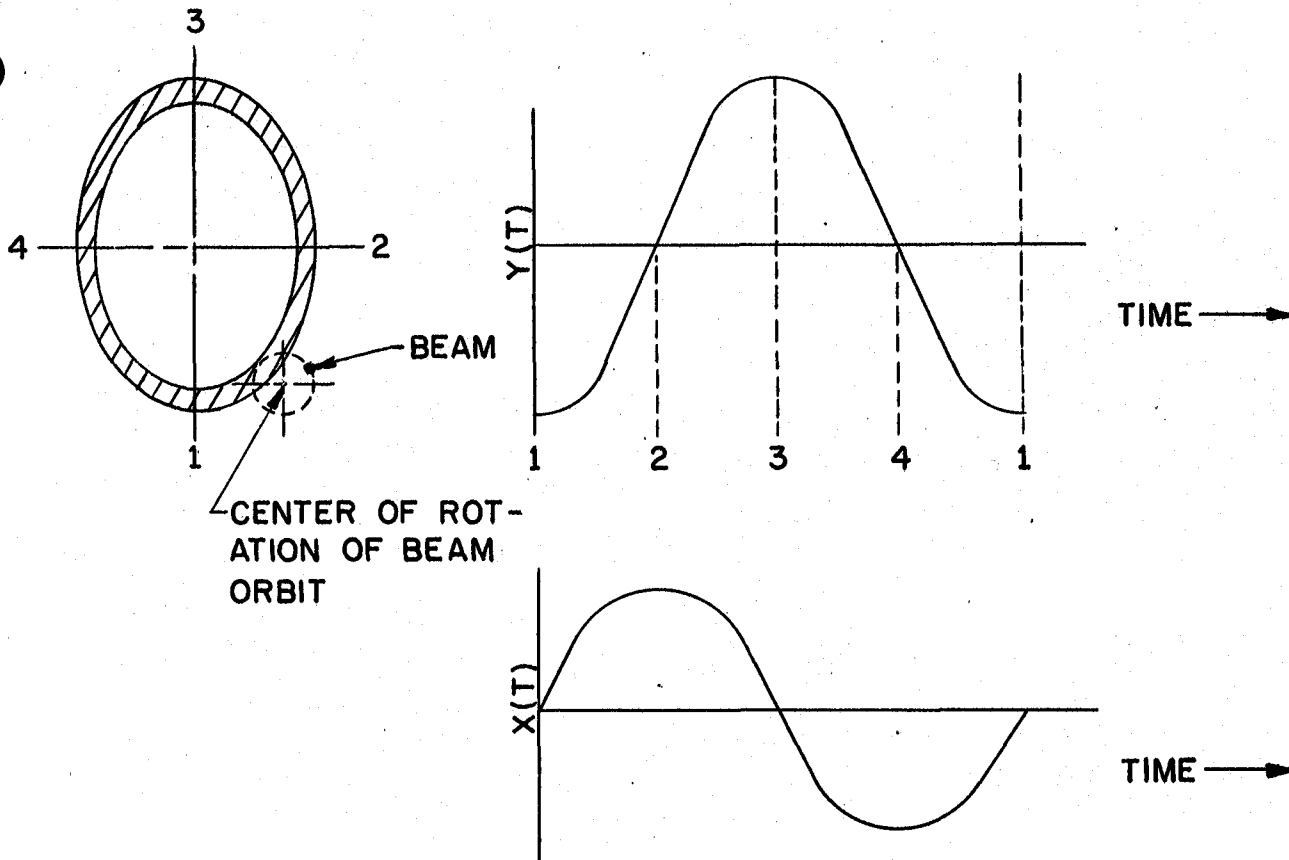
Parametric Representation:

The heart of the proposed system is the use of parametric functions to describe the character. There are several advantages in the use of parametric functions. One of the most fundamental is the fact with which everyone is familiar, that it is very easy to represent a multivalued function by two single valued parametric equations. If the multivalued function is continuous, then the parametric equations are also continuous. The multivalued function in the method to be described is the character being scanned. It will be seen that the character is scanned in such a way that the dark-light edge of the character is followed in a counter-clockwise manner. The dark area of the character is always to the left of the scanning direction.

A second advantage to be obtained from the use of parametric representation of the character is the fact that one then has all of the information about the character in analogue form. In this form, the information can be manipulated very rapidly. It will be shown that the center of gravity of the perimeter of the character can be determined so simply that one can hardly call it a calculation. It is rather a basic fact that a calculation which is to be performed over and over again at a high speed and with a relatively low degree of accuracy can be done most economically by analogue means.

A third advantage to be found in parametric representation is related to the second but can be discussed separately. It will be shown that it is relatively easy to utilize properties of the character that remain invariant with respect to a rotation of the character on the page. This may or may not be an important advantage since it is probably not too frequent that a machine is called upon to read information such as: **LI-9**

In order to describe how the parametric functions are generated, it is desirable to look at a representative character such as the number zero. It is shown enlarged in Figure 1.



Signals Produced By Scanning A Zero

Figure 1.

It is seen in Figure 1 that the character is scanned by the optical projection of the image of a beam from a flying spot scanner. In order to generate signals in the photomultiplier viewing the reflected light from the character, the beam of the flying spot scanner is caused to rotate in a circular orbit. The radius of this orbit is equal to or less than the width of the dark line defining the character. The phase of the signal from the photomultiplier, with respect to the vertical and horizontal components of the signals generating the circular orbit, will be dependent on the direction of the line being scanned. A more detailed description of the system is described in Appendix I. The frequency of rotation of the beam in its orbit could be about 1 megacycle per second. Its upper frequency limit is determined by the persistence of the phosphor on the face of the flying spot scanner. The lower frequency limit is determined by the lowest speed with which the character is to be traversed. By using a 1 megacycle per second orbit frequency, it should be possible to scan the character at a 10 Kc rate. Since the character will be traversed 50 to 100 times, it should be possible to identify about 100 to 200 characters per second. The functions that result from scanning the character will be described.

Referring to Figure 1 let us assume that at time  $t = 0$  the center of rotation of the beam is at point 1 on the figure. As time increases, the center of rotation of the beam will move at a uniform rate counterclockwise around the figure. The vertical and horizontal voltages produced on the deflection plates of the flying spot scanner as the center of the beam orbit traverses the figure are shown as  $y(t)$  and  $x(t)$ , respectively, for one complete cycle. It is seen in Figure 10 of Appendix I that the signals that generate the beam orbit are not present in the  $y(t)$  and  $x(t)$  signals. Likewise, the error correction signals are excluded from  $y(t)$  and  $x(t)$ .

It is seen in Figure 1 that the  $x(t)$  and  $y(t)$  signals are not rich in harmonics. This is true for any figure that does not have many sharp turns in its perimeter. Since  $x(t)$  and  $y(t)$  are continuous, then the two Fourier series that represent each of these functions will converge fairly rapidly.

The functions  $x(t)$  and  $y(t)$  will have the form:

$$x(t) = x_0 + x_{c1} \cos \omega t + x_{c2} \cos 2\omega t + \dots + x_{cn} \cos n\omega t + \dots \\ + x_{s1} \sin \omega t + x_{s2} \sin 2\omega t + \dots + x_{sn} \sin n\omega t + \dots \quad (1a)$$

$$y(t) = y_0 + y_{c1} \cos \omega t + y_{c2} \cos 2\omega t + \dots + y_{cn} \cos n\omega t + \dots \\ + y_{s1} \sin \omega t + y_{s2} \sin 2\omega t + \dots + y_{sn} \sin n\omega t + \dots \quad (1b)$$

It is shown in Appendix II that  $x_0 = \bar{X}$  and  $y_0 = \bar{Y}$  where  $(\bar{X}, \bar{Y})$  are the coordinates of the centroid of the perimeter. The terms

$$x_{rn}(t) = x_{cn} \cos n\omega t + x_{sn} \sin n\omega t \quad (2a)$$

$$y_{rn}(t) = y_{cn} \cos n\omega t + y_{sn} \sin n\omega t \quad (2b)$$

can be thought of as representing a generating ellipse of order  $n$ . The terms  $x_{rn}(t)$  and  $y_{rn}(t)$  can be rewritten as:

$$x_{rn}(t) = (x_{cn}^2 + x_{sn}^2)^{\frac{1}{2}} \sin(n\omega t + \alpha_n) \quad \text{where } \alpha_n = \tan^{-1} \frac{x_{cn}}{x_{sn}} \quad (3a)$$

$$y_{rn}(t) = (y_{cn}^2 + y_{sn}^2)^{\frac{1}{2}} \sin(n\omega t + \beta_n) \quad \text{where } \beta_n = \tan^{-1} \frac{y_{cn}}{y_{sn}} \quad (3b)$$

The term

$$\vec{r}_n(t) = x_{rn}(t) + i y_{rn}(t) \quad (4)$$

is a space vector. The sum

$$\begin{aligned} |\vec{R}(t)| &= \left| \sum_{n=1}^{\infty} \vec{r}_n(t) \right| = \left[ \sum_{n=1}^{\infty} x_{rn}^2(t) + y_{rn}^2(t) \right]^{\frac{1}{2}} \\ &= \left[ \sum_{n=1}^{\infty} (x_{cn}^2 + x_{sn}^2) \sin^2(n\omega t + \alpha_n) + y_{cn}^2 + y_{sn}^2 \sin^2(n\omega t + \beta_n) \right]^{\frac{1}{2}} \quad (5) \end{aligned}$$

is the radius vector from the centroid of the perimeter of the figure to the perimeter. It is the vector sum of the radius vectors of all the generating ellipses. The locus of the end of the  $R(t)$  vector traces out the figure being scanned.

When a character is rotated, the scalar function  $R(t)$  will experience a translation in time; therefore, the voltage which represents  $R(t)$  will contain the same harmonic content regardless of the spacial orientation of the figure from which it is generated. A linear increase in size of the figure will multiply the magnitude of each harmonic by a scale factor. This assumes that the time constant of the vertical and horizontal integrators described in Appendix I are adjusted so that the fundamental frequency of the character is the same regardless of the length of its perimeter.

The calculation of  $R(t)$  is very simple.  $R(t)$  is given as:

$$R(t) = \sqrt{x(t)^2 + y(t)^2} \quad (6)$$

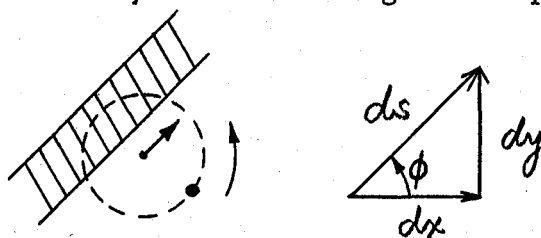
In view of the fact that  $R(t) \geq 0$ , it is probably unnecessary to make the square root calculation. If it were left in the form  $R(t)^2$ , it would contain much of the information about the character. The polar angle function  $\Theta(t)$  is given by

$$\Theta(t) = \int_0^t \left[ \frac{x(\lambda) \frac{dy(\lambda)}{d\lambda} - y(\lambda) \frac{dx(\lambda)}{d\lambda}}{x(\lambda)^2 + y(\lambda)^2} \right] d\lambda \quad (7)$$

$R(t)$  has uniqueness within a sign of  $\Theta(t)$ . This must be studied to determine if there is any possibility of ambiguity in the determination of a character or shape from a closed class of such characters or shapes. An example of such ambiguity will be found if we look at the character Z and the synthetic character  $\Sigma$ . The function  $y(t)$  is the same for both of these characters, whereas, the function  $x(t)$  of the first is the negative of the second. Ambiguities such as this can be resolved by looking at the phase of the fundamental of  $x(t)$ .

The function  $R(t)$  is somewhat more desirable to use than either  $y(t)$  or  $x(t)$  from the point of view that its harmonic content remains invariant with respect to a rotation of the figure being scanned. If one is interested in counting the number of straight sides to a figure, then  $x(t)$  and/or  $y(t)$  contain the information in a much more readily available form.

This is seen in the following way. Assume the beam is moving along a line in a north-easterly direction. Figure 2 depicts this situation.



Orbit Following a Diagonal Line

Figure 2.

The beam moves a distance  $ds$  in time  $dt$ ; therefore,  $ds = K dt$ . Where  $K$  is the speed of the beam, the distance  $dy$  is proportional to the change in voltage on the vertical deflection plates. It will be assumed that this proportionality factor is unity.

$$\text{then} \quad dy = K dt \sin \phi \quad (8a)$$

$$dx = K dt \cos \phi \quad (8b)$$

$$\text{hence} \quad \frac{dy}{dt} = K \sin \phi \quad (9a)$$

$$\frac{dx}{dt} = K \cos \phi \quad (9b)$$

The signals  $dx/dt$  and  $dy/dt$  appear at the output of the horizontal and vertical modulators, respectively. These are shown in Figure 10 of Appendix I. It is seen that  $dx/dt$  and  $dy/dt$  are constant if the beam is following a straight line. This makes it very simple to recognize straight sided figures such as polygons. Since it is possible to vary the speed of traversal of the figure so as to make the fundamental frequency of  $x(t)$  and  $y(t)$  constant, then the machine can count the number of times  $\frac{dx}{dt}$  or  $\frac{dy}{dt}$ , but not both, change within a complete cycle of  $x(t)$  or  $y(t)$ .

It is possible, for example, to determine which triangle of a number of polygonic figures drawn on a sheet of paper has a right angle, has an area of  $Z$  square inches, and has a side tilted at an angle of  $\phi$  degrees.

We have seen above how the machine would determine whether it is scanning a three-sided figure. To determine the right angle, the signals  $dy/dt$  and  $dx/dt$  could be converted to digital form and the angles calculated digitally, or the information could be processed in analogue form. If the machine was used to look for certain angles quite frequently, then it would probably be better to make this computation in analogue form.

The area can be determined quite simply. It is given in Eq. 11 of Appendix II and reproduced here as:

$$A = \frac{1}{T} \int_0^T x(t) \frac{dy(t)}{dt} dt \quad (10)$$

This requires just a multiplier and an integrator since  $x(t)$  and  $dy(t)/dt$  are both available. To satisfy the condition that a side of the triangle have a direction  $\phi$ , it is necessary to look for a particular magnitude and sign of  $dx/dt$  and sign of  $dy/dt$  or vice versa. The location of the figure on the page would be determined by the average value of the  $x(t)$  and  $y(t)$  signals. As shown in Appendix II, these average values give the  $x$  and  $y$  coordinates of the center of gravity of the perimeter of the figure.

#### Checking for a Complete Scan:

It would be desirable to have the machine check to see if it has completely scanned an individual character or an array such as described above, when it thinks that it is finished. Another condition that might demand a check occurs when the machine cannot make the decision as to whether it has just scanned a B or a D. This confusion could arise if the cusp on the right of the B were poorly defined. Other characters that might cause trouble are the O and the Q, if the tail on the Q which is external to the circular portion were missing; the quotation mark " and the apostrophe ' ; the comma , and the semicolon ; etc. In all of these cases, the machine could be designed to automatically check to see if it has missed anything within an area defined by the type of scanning it has been doing.

The way in which it can be made to check for additional information that has not been scanned is very simple. When the machine is in doubt, it goes into a check cycle and starts a raster scan. Since the flying spot scanner has previously scanned a portion or all of the character in question, it has stored an electrical charge on its face that bears a one to one correspondence with the portion of the character previously scanned.

If for any reason it is undesirable to use the flying spot scanner for storing the electrical image of what has been scanned, then a slave tube may be used. In any event, when the raster scan is performed, either there is coincidence between the electrical signal stored on the face of the flying spot scanner and the signal from the photomultiplier or there is no coincidence. If there are regions where a signal representing a dark area is obtained from the photomultiplier and no corresponding signal is obtained from the face of the cathode ray tube, then it is clear that there is an unscanned area. The location of the unscanned area with respect to the center of gravity of the scanned area should determine in practically every case what the character is. The point to be made is that the degree of sophistication that can be built into a machine that operates on the principle described here is very high.

#### Starting the Scan of a Page:

To start the scan of a page, the sheet is placed on a dark scanning stage and the beam is deflected all the way over to the left edge of the page. Once it contacts the left edge, it moves vertically up the edge until it comes to the top edge, at which time the beam, while rotating in its orbit, performs a diagonal raster scan. It will be caused to move into the page from the upper left hand corner. Upon contacting a character, it proceeds to scan it. Since the beam followed the left edge of the paper, the machine can either align the paper or the flying spot scanner so that the left edge of the paper is vertical with respect to X and Y deflection plates.

#### Scanning Printed Characters:

Let us assume the beam has scanned the first character and has either identified it or has gone into a check cycle and has raster scanned the character. In any case, the location of the center of gravity of its perimeter has been established. A predetermined value of voltage  $\Delta x$  and  $-\Delta y$  is added to the location of the centroid of the perimeter of the first character and the center of rotation of the orbit of the beam is moved horizontally with positive and negative excursions at the same time as it is urged upward. Upon contacting a light-dark transition, the line scan takes over and the perimeter of the second character is scanned. If the signal on the vertical plates increases to a predetermined value and no character signal has been picked up, then the beam is indexed one more position to the right and the process repeated until a character is reached or the right edge of the paper is engaged.



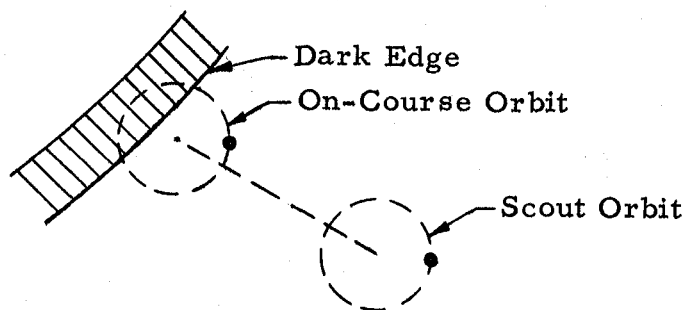
If a full line cannot be scanned by the flying spot scanner, than at a pre-determined value of  $x$  the beam will lock onto a character, and the scanning stage will be moved to the left. The relatively slow rate of change of the  $x$  signal due to motion of the scanning stage will not cause the beam to lose the character. When a full line has been scanned, the machine now knows the maximum height of the character and where the left edge is so that it can immediately proceed to the next line.

#### Jumping Over Breaks in Characters:

In view of the fact that the printing of characters is not perfect, they very often will have breaks. In order for a line-following system to produce a true parametric representation of a character, it must be capable of ignoring breaks. One of the ways in which this system can be made to do this will now be described.

Let us assume that instead of one beam orbit we have two beam orbits. The first will be called the on-course orbit. This is the beam orbit whose center lies on the light-dark edge. The second will be called the scout orbit. It will be located on a line that passes through the center of the on-course orbit and is perpendicular to the direction in which the beam is moving. Naturally, it will be located to the right of the path of the on-course orbit.

Figure 3 shows the on-course orbit and the scout orbit. In order to cause the beam to describe these two orbits, the



Position Of On-Course and Scout Orbits

Figure 3.

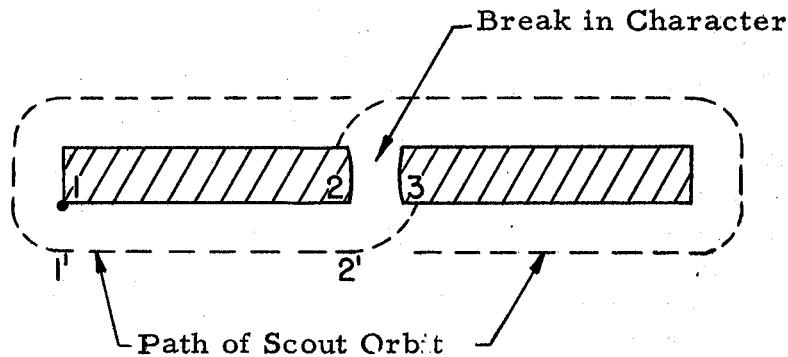
position of the center of the orbit is periodically changed at about a 100 kilocycle per second rate. The location of the center of the scout orbit is obtained by adding a signal  $- \gamma dx/dt$  to the signal  $y(t)$  and a signal  $\gamma dy/dt$  to the signal  $x(t)$ .

It is shown in Figure 10 of Appendix I that these signals are added to the signals that go to the deflection plates, but are excluded from  $y(t)$  and  $x(t)$ . The signals  $y(t)$  and  $x(t)$  therefore do not exhibit any discontinuities.

The vertical and horizontal modulators and error modulators shown in Figure 10 of Appendix I are operative both during the time that the on-course orbit is present and during the time when the scout orbit is present. It can be seen in Appendix I that during the scout orbit time there will be no net effect on either the vertical and horizontal modulators or the vertical and horizontal error modulators.

Another detector called a scout orbit detector is sensitized during the scout orbit time. It detects a lack of signal from the photomultiplier. If the beam, while in the scout position, passes over a dark region of sufficient area, the scout orbit detector will put out a signal. This signal will cause the scout-orbit position to be retained for a predetermined time,  $t_1$ . After a short interval of time  $t_2$ , where  $t_2 < t_1$ ,  $\delta$  will be allowed to decay to zero. At this time, the beam will be allowed to alternate between its scout orbit position and its on-course position. Since  $\delta$  is zero, these will be one and the same. Then  $\delta$  will assume its normal value. This will cause the scout orbit to be re-established to the right of the on-course position.

To see how the above action will jump over breaks, let us consider the type of break shown in Figure 4. Initially the on-course orbit is at point 1.

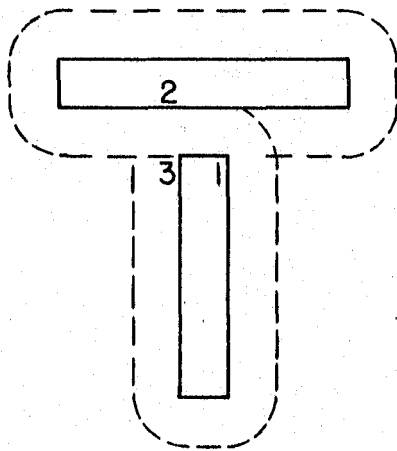


Method By Which Scout Orbit Detects Breaks in Characters

Figure 4.

The on-course orbit will move at a uniform rate to point 2. The scout orbit will move from 1' to 2'. Upon reaching point 2, the output of the horizontal modulator will start to diminish and the output of the vertical modulator will start to build up. Since the output signal of the vertical modulator is proportional to  $dy/dt$  and the signal from the horizontal modulator is proportional to  $dx/dt$ , it is seen that the path of the scout orbit will rotate about point 2. When the scout orbit detector senses the dark area at point 3, the on-course orbit is inhibited and the beam remains at point 3. The modulators and error modulators take over and the beam starts moving to the right. At this time, the value of  $y(t)$  is correct, but  $x(t)$  has the value it had back at point 2. As the output of the horizontal modulator increases at point 3, the signal  $-\gamma dx/dt$  increases and the orbit remains at point 3 because the vertical error correction modulator compensates for  $-\gamma dx/dt$ . As  $\gamma$  is caused to decay to zero, the signal from the vertical error correction modulator goes to zero also. If  $\gamma$  goes to zero faster than  $x(t)$  can increase from its value at point 2 to its value at point 3, then the horizontal error correction modulator will make up the difference. When  $\gamma$  is zero, the beam can be cycled from its on-course orbit to its scout orbit without any effect other than to increase the speed of motion of the center of the orbit. When  $\gamma$  is re-established at its normal value, the on-course orbit proceeds at its normal rate and the scout orbit is re-established.

Another type of break which this system can detect and correct for is shown in Figure 5. In this case, it is seen that points 1 and 2 could be troublesome. The scout orbit pivots about point 1 until it engages the top of the "T", at which time the process previously described takes place. At point 2 the scout beam engages the lower arm of the "T" and again proceeds from 2 to 3. It should be pointed out that by jumping across breaks in the manner described, there are no discontinuities nor rapid fluctuations in  $x(t)$  or  $y(t)$ .



Scanning a Stenciled T

It might be possible without adding much more complexity to the system to read characters composed of a series of dots. In this case, the scout orbit should probably be made somewhat larger than the on-course orbit. In addition, the time constants of the integrators should be made fairly large.

### Recognition of People:

Two methods present themselves for the identification of people. The first to be described makes use of contour lines projected onto the face of the individual by means of planes of light. The second method directly determines the shape and location of identifiable features such as eyebrows, eyes, nostrils, etc., of the individual being scanned.

To utilize contour lines projected onto the face of the individual, the person would sit or stand in a cylindrical enclosure which surrounds his head. The individual would face in a direction parallel to the axis of the cylinder. Naturally, there would be a section missing from the cylinder which would accommodate his neck. Narrow slits would be cut into the inside surface of the cylinder. These slits would be separated by a distance of about one quarter of an inch along the axis of the cylinder. The purpose of the slits would be to produce a plane of light with parallel light rays from a series of gas tubes located outside the cylinder. The result of the lights and the slits would be to produce thin planes of light perpendicular to the axis of the cylinder. These planes of light would be separated from each other by about one quarter of an inch. When a three dimensional object such as the face of an individual is inserted into this cylinder, contour lines would be projected onto its surface. These contour lines can be scanned by the system described and the results compared to information stored in memory.

While, in principle, this is a workable system, it has a number of shortcomings. A major difficulty is the fact that such a system could not identify an individual from his photograph. Another objection is that one feels intuitively that it does not utilize information efficiently and hence would require a very large memory in order to identify a limited number of people. There are other equally obvious objections to this method which lead the writer to pursue another approach.

It is surprising how little information is contained in a caricature of an individual and yet it is relatively easy to identify the subject. The reason for this seems to stem from the fact that a caricature exaggerates certain features of the individual. This would be difficult for a machine to do. A caricature is, however, a line drawing of the individual. This can be achieved by the use of extreme contrast and flat lighting. If extreme contrast and flat lighting is utilized in scanning the face of an individual, then fine detail will be eliminated. The flying spot scanner produces flat lighting if the photomultiplier is located near the area from which the beam of light from the flying spot scanner originates. It should be possible to achieve extreme contrast with the type of system described. If not, it is possible to use a television system between the individual being scanned and the shape recognition system. The video signal can be differentiated and displayed on a cathode ray tube. This will produce a line drawing of the individual being scanned. A Vidicon Tube can then be used in place of the flying spot scanner for the input of the shape recognition system. By means of such a system, it should be possible to exclude extraneous detail from the shape recognition system.

The actual determination of the individual would be accomplished by having the shape recognition system scan a feature such as an eyebrow. The machine would recognize the shape as an eyebrow because of its harmonic content and can in the same manner determine if it is a right or left eyebrow. Its relative location would be determined by its centroid. The machine would then know where to go to find an eye or the nose or the mouth or the hair line, etc. A feature such as an eyebrow might be divided up into ten or twelve classes, the same would be true for eyes, noses, etc. In addition to this type of information, the machine would know the absolute or normalized distance between the centroids of these features. In principle then, it should be possible to make an identification.

Naturally there are many problems to be solved before a workable model could be built which would identify people. The writer only wishes to convey some of his thoughts on the subject and to enlist the help of others who have wrestled with this problem.

Making an Identification:

So far the method of obtaining the parametric function has been described in detail with little attention given to the identification of a character. There are several ways in which the parametric functions may be used for the actual determination of the character. The most attractive method requires that the speed of transit around the character be constant for all characters. This is the same as saying that the fundamental frequency of the parametric function be the same for all characters. This can be achieved by insuring that the orbit initially traverses all characters too rapidly. One of the parametric functions that is obtained from a character, say  $y(t)$ , is fed into a narrow-band amplifier which feeds a discriminator. The output of the discriminator is made to control the amplitudes of the signals that drive both the vertical and horizontal integrators. Since the orbit is initially traversing the character too rapidly, the amplitudes of the signals that drive both the vertical and horizontal integrators are made to decrease. This slows down the orbit and lowers the fundamental frequency until it is "windowed" by the band-pass amplifier and discriminator. At this time the discriminator takes over and adjusts the amplitudes of the signals driving the integrators until the frequency of the fundamental is at the frequency to which the discriminator is tuned. The frequencies of the second, third, etc., harmonics are then known. There are several ways in which one can proceed from this point.

The machine can be designed so that the amplitude of the parametric function being identified is adjusted until the amplitude of its fundamental is unity. This makes the parametric function invariant with regard to size of the character. The amplitudes of all the harmonics can be measured by means of individual band pass amplifiers and detectors. From this point it is possible to convert to digital information in order to make the final determination.

It was previously mentioned that if only  $y(t)$  is analyzed in this way, then characters that are the mirror image (about a vertical axis) of another character cannot be distinguished from one another. The capital letters and numbers contain no vertical axis mirror image pairs. The number 2 and the letter S are the two with the closest vertical axis mirror image symmetry.

Another method involves determining the actual magnitude and phase of the harmonics. This can be done by multiplying the parametric function by the sine function obtained from the fundamental band pass amplifier and integrating the product. The same can be done for a cosine function derived from the sine function. This process would be repeated for the higher harmonics. If this were done for  $x(t)$  and  $y(t)$ , the information about the generating ellipses would be available in order to make a determination of the character being scanned.

The most attractive method from the point of view of flexibility involves storing the parametric information about the characters to be recognized on, say, a magnetic drum or a loop of magnetic tape. The parametric function obtained from the character would be normalized with respect to the amplitude, frequency, and phase of the fundamental. The parametric function obtained from the character would then be compared to what was stored in memory. The parametric function could, of course, be  $R(t)$  or  $R(t)^2$ , in which case the identification could be made regardless of the orientation of the character. A least square error fit could be made between the figure being scanned and the information stored in memory.

Such a system would have a high degree of flexibility because it could be taught to identify a completely new set of characters by typing or hand printing the characters, in order, on a page. The page would then be placed on the scanning stage and a button would be pressed which would cause the machine to store this new information in its memory. When the machine is asked to read a document produced from the same typewriter or printed by the same individual, it would compare the scanned characters with what it had stored in its memory. It could thus be taught to read various forms of type and hand printing.

Before it is possible to determine the best method of identifying a character, a careful study of the parametric functions of all characters must be undertaken. This would involve determining the Fourier coefficients for some representative letters of the alphabet and numbers. From this investigation it would be possible to determine the number of harmonics that it would be necessary to handle in order to make a positive identification of the character even though it is distorted. In addition to a Fourier analysis it would be desirable to make use of information about straight lines and curves which appear at the output of the modulators.

Subsequent to the writing of this memo Mr. W. W. Boyle programmed the 704 so that it could be used to calculate the amplitudes and phases of complex waves. The  $y(t)$  functions were obtained from a 2 and a Z by graphical means. The amplitudes and phases of the harmonics of these functions were determined by means of the 704 and are presented below in normalized form.

Harmonic	<u>Amplitude</u>		<u>Phase (Degrees)</u>	
	Z	2	Z	2
1	1.0000	1.0000	0.00	0.00
2	0.0485	0.0711	221.50	66.84
3	0.2647	0.1996	304.03	315.18
4	0.0458	0.0961	198.55	236.55
5	0.1012	0.1388	61.38	140.54
6	0.0173	0.0398	3.75	357.29
7	0.1040	0.0562	175.19	202.83
8	0.0170	0.0259	80.17	218.09
9	0.0689	0.0124	334.99	46.81
10	0.0299	0.0057	248.02	319.48

It can be seen that there is quite a bit of difference between the amplitudes of the harmonics of  $y(t)$  for a 2 and a Z. While it is still too early to say how well this system will work, the results at this time look encouraging.



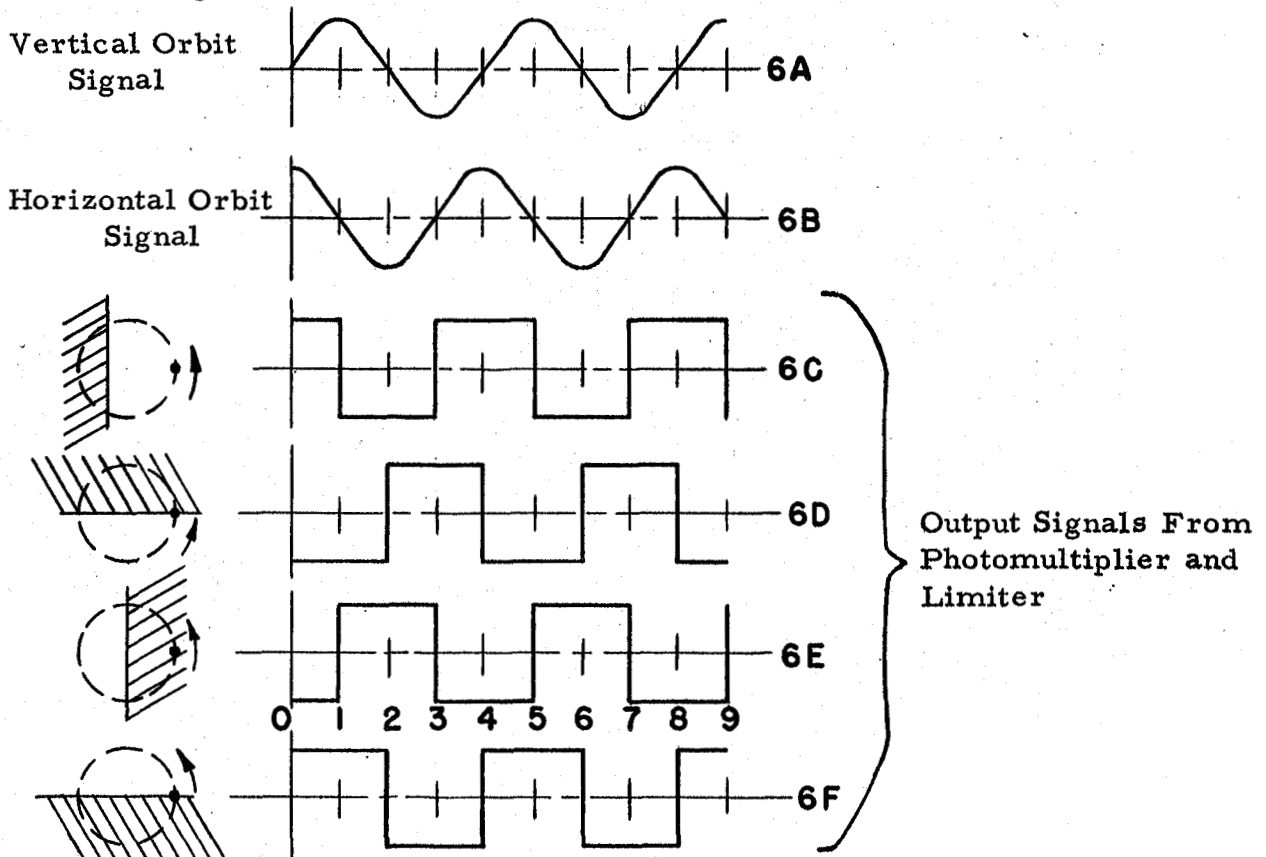
APPENDIX I

SYSTEM ORGANIZATION

Edge Following Circuits:

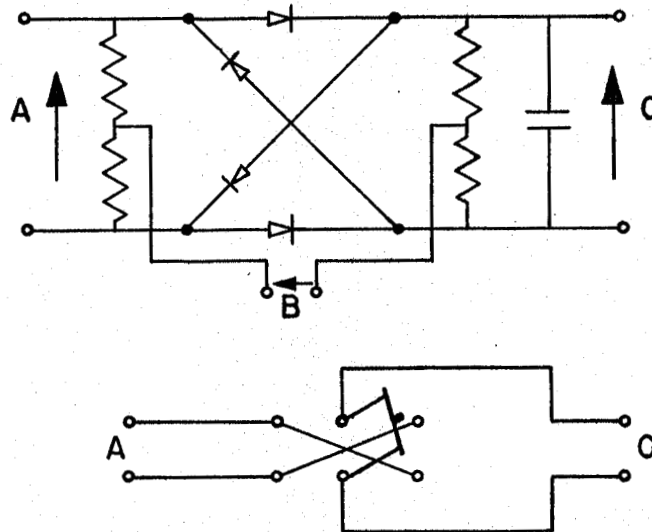
The edge following circuits will be described before the edge lock-on circuits since the latter make use of signals generated by the former. It has been shown in the body of this memo that the beam describes a circular orbit whose center of rotation, under ideal conditions, coincides with the dark-light edge being scanned. The beam is caused to pursue its circular orbit by means of two relatively high frequency sinusoidal signals. One is applied to the vertical plates of the flying spot scanner in the form of a sine wave. The other is applied to the horizontal plates after it is advanced 90° from the signal applied to the vertical plates. By virtue of these two signals the beam produces a circular sweep. Naturally, the radius of the circular orbit can be controlled by simultaneously adjusting the amplitude of the two signals.

We will now investigate the signals produced by the photomultiplier as the beam scans vertical and horizontal edges. As we shall see, the phase of the signal from the photomultiplier is important, not its amplitude. Amplitude variations are cancelled out by means of a limiter following the amplifier fed by the photomultiplier. The signals obtained from the limiter when the beam is located on various edges are shown in Figure 6.



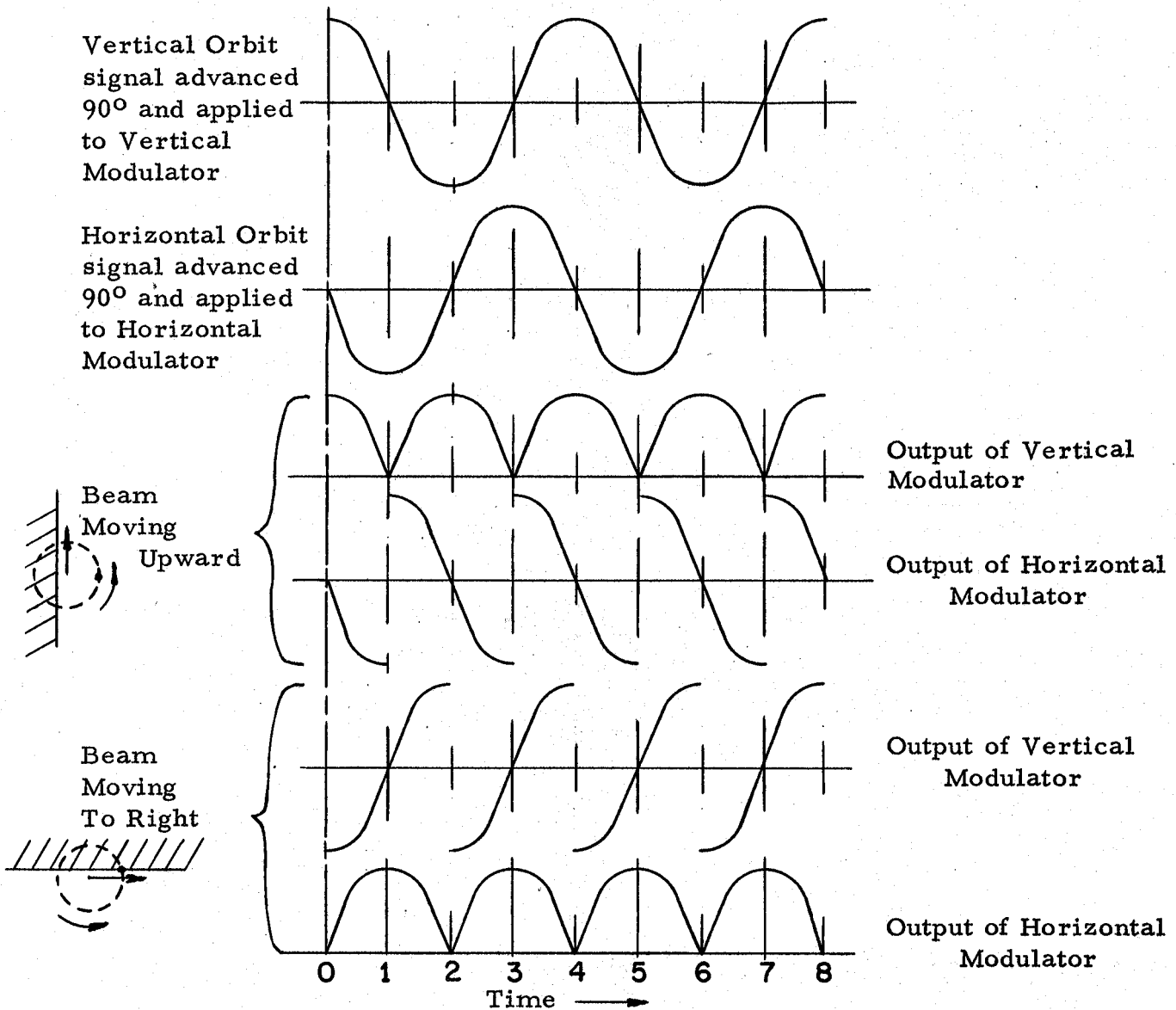
Orbit and Photomultiplier Signals  
Figure 6.

The vertical beam orbit signal and the horizontal beam orbit signal are both advanced  $90^\circ$  in phase and individually mixed, in a vertical and a horizontal bridge modulator respectively, with the signal from the limiter in the photomultiplier circuit. Since the horizontal beam orbit signal is  $90^\circ$  ahead of the vertical signal, this additional shift of  $90^\circ$  means that the signal which is applied to the horizontal modulator is the negative or inverse of the vertical orbit signal. The bridge modulators have the form shown in Figure 7.



Bridge Modulator  
and Its Equivalent, the Reversing Switch  
Figure 7.

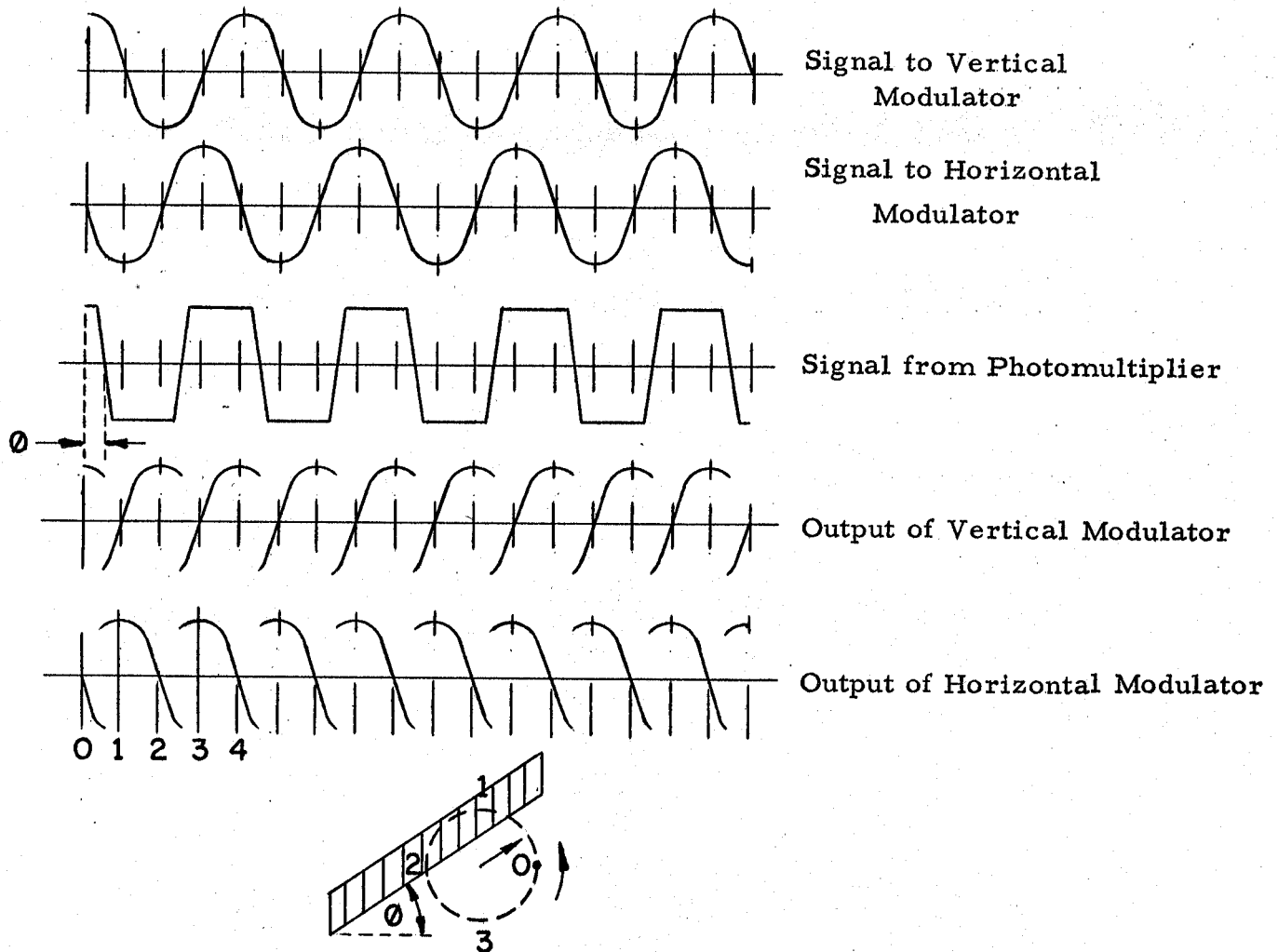
It can be seen that the bridge modulator performs the function of a reversing switch. The frequency of operation of the knife switch is dictated by the signal applied at terminals B in Figure 7. The resistors at input A may be replaced by a centertapped secondary winding of a transformer, if only the a-c components of an input signal are to be modulated by the signal at B. It will be assumed that the beam orbit signals are modified as described earlier and amplified before being applied to input A of the modulators. In this case, an input transformer with a centertapped secondary would be used in place of the resistors at input A. The wave-forms are shown for the various cases in Figure 8.



Input and Output Signals of Vertical and Horizontal Modulators

Figure 8.

In Figure 8 the outputs of the modulators are shown as a complex waveform. Actually, it is the average value of output signal of the modulator that is present at the output terminals of the modulator. If the beam is on a horizontal light-dark edge, the horizontal modulator delivers a d-c signal of unit magnitude. The output signal is only dependent on the relative reflectance rather than the absolute reflectance of the light and dark areas being scanned because the signal from the photomultiplier is amplified and clipped top and bottom. The average output of the vertical modulator is zero when the orbit is following a horizontal edge. The reader can see from Figure 6 that the fundamental component of the signal from the limiter is retarded by  $\phi$  degrees when the dark-light edge is rotated in the counter-clockwise direction through an angle  $\phi$ . Figure 9 shows the waveforms obtained from scanning an edge lying at an angle of  $\phi$  degrees from the horizontal.



Waveforms Associated With Scanning a Diagonal Edge

Figure 9.

It can be shown that the average value of the output of the vertical and horizontal modulators are proportional to  $\sin \phi$  and  $\cos \phi$  respectively.

The outputs of the vertical and horizontal modulators are connected to vertical and horizontal integrators respectively as shown in Figure 10. Assume the center of the beam orbit is located on a horizontal light-dark edge with the dark area above the edge. Then the unit positive signal from the horizontal modulator will cause the output signal of the horizontal integrator to increase in the positive direction. The rate of increase of this signal is inversely proportional to the time constant of the integrator. As seen in Figure 10, the output signals  $y(t)$  and  $x(t)$  of the vertical and horizontal integrators are connected to the vertical and horizontal plates of the flying spot scanner. As described in the body of this memo,  $y(t)$  and  $x(t)$  are the parametric functions for the character being scanned.



Since the x and y position of the beam is proportional to the integral of the outputs of the horizontal and vertical modulators, the outputs of the modulators are proportional to  $\frac{dx(t)}{dt}$  and  $\frac{dy(t)}{dt}$ . It was stated previously that

$$\frac{dx}{dt} = K \cos \phi \qquad \frac{dy}{dt} = K \sin \phi$$

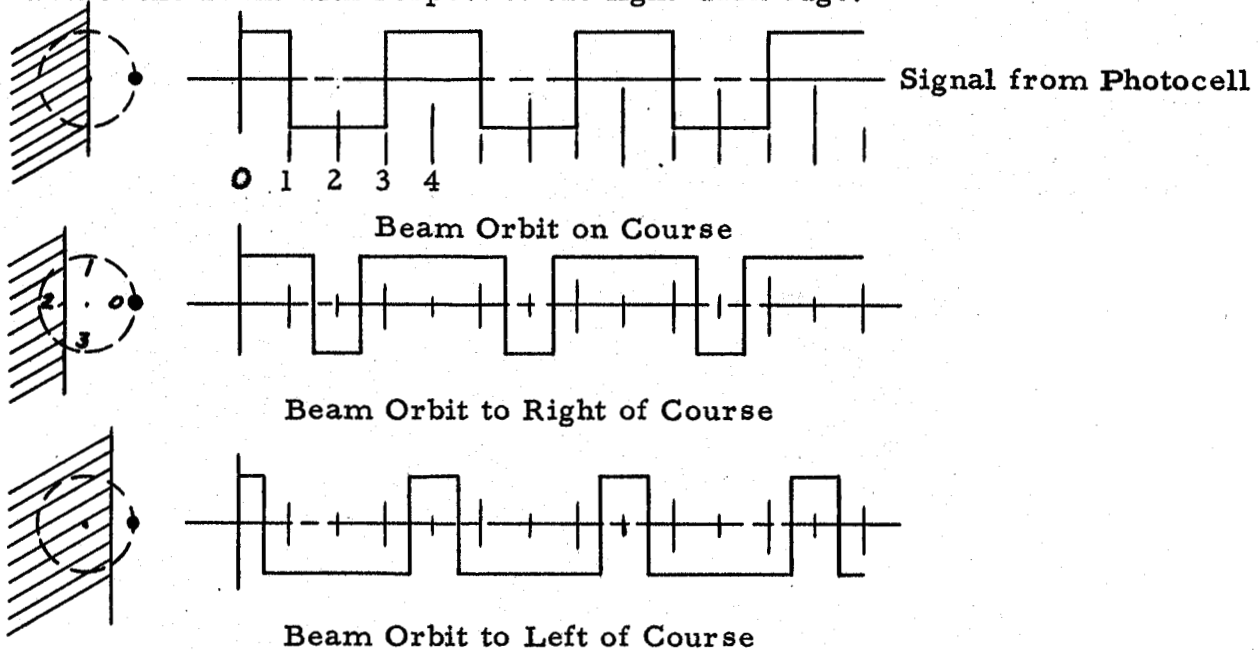
$$\text{but } \frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{K^2 \cos^2 \phi + K^2 \sin^2 \phi} = K$$

The rate of traversal of the character is therefore constant.

It has been shown that the edge following circuits cause the beam orbit to be urged in a counter-clockwise direction around a dark area at a uniform rate. The system lacks the ability to lock onto the light-dark edge. A method for doing this will now be explained.

Edge Lock-On Circuits:

It can be seen in Figure 11 that the symmetry of the signal from the photomultiplier is dependent on the location of the center of rotation of the beam with respect to the light-dark edge.



On Course and Off Course Photocell Signals  
Figure 11.

When the beam orbit is to the right of course the signal from the photomultiplier is on longer than it is off. When the beam orbit is to the left of course the opposite is true. This characteristic can be utilized by means of the bridge modulator. By applying the signal from the photomultiplier to terminals B of Figure 7, the dwell of the reversing switch action is determined by the location of the beam orbit center with respect to the edge. If the beam is following a vertical edge, as shown in Figure 11, then the vertical modulator will produce a positive output signal with either unit magnitude or somewhat less than unit magnitude. The horizontal modulator will have zero output. The output of the vertical modulator is applied to the A input terminals of a horizontal error correction bridge modulator. The B input is supplied by the photomultiplier signal.

To understand how the error correction modulator supplies a correction signal, certain polarity conventions will be defined and a truth table developed. In Figure 7 the arrows at input A and output B signify that the upper terminals of these pairs are positive with respect to the lower terminals. The arrow on input B has a somewhat different meaning. In this case, the arrow indicates that the left hand terminal is positive with respect to the right hand terminal for more than 50% of the cycle. A zero means that the signal from the photomultiplier is symmetrical. Using this convention the following truth table describes the operation of the horizontal error correction modulator.

	<u>Signal from vertical modulator at input A</u>	<u>Signal from photomulti- plier at input B</u>	<u>Output at C</u>
1	↑	→	↓
2	0	→	0
3	↓	→	↑
4	↑	0	0
5	0	0	0
6	↓	0	0
7	↑	←	↑
8	0	←	0
9	↓	←	↓

The condition shown in Figure 11 is represented by condition 1 in the above table. The orbit is moving up a vertical edge. Hence, the signal from the photomultiplier is present for a longer period than it is not. The result is a negative output from the horizontal error correction modulator. The orbit must be displaced to the left (negative x direction) to place it on course. This will occur if the output signal of the horizontal error correction modulator is applied to the horizontal plates of the flying spot scanner as shown in Figure 10. It becomes evident that a vertical error correction modulator is also needed as shown in Figure 10.

It is seen that the edge lock-on circuits are needed to urge the beam onto the light-dark edge. Once the orbit starts intercepting an edge, signals are developed by the vertical and horizontal modulators which drive the center of the orbit onto the edge. There is no horizontal correction if the orbit is on a horizontal edge. The same is true of a vertical edge.

#### Scout Beam Circuits:

In the body of this memo it was shown how the scout beam was used to jump over breaks and to enable the system to identify stenciled characters. The location of the scout beam orbit is given as  $x(t) + \gamma \frac{dy(t)}{dt}$ ,  $y(t) - \gamma \frac{dx(t)}{dt}$  The term  $\gamma$  is a time dependent

factor that decreases to zero shortly after a break is discovered, and thereafter instantaneously returns to its normal value. Circuits are shown in Figure 10, in block diagram form that will perform this function.



APPENDIX IIInvariants:

This appendix will deal with quantities that remain invariant when the character being scanned is either translated, rotated, or magnified.

Translation:

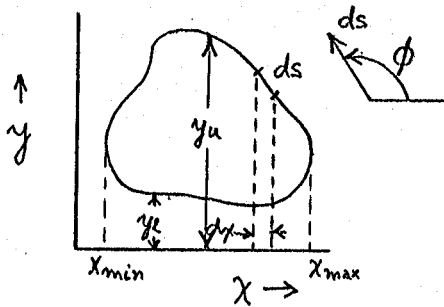
A translation of the figure being scanned causes only  $x_0$  and  $y_0$  to change in the parametric functions given in equations 1a and 1b. Since it is the alternating components of these equations that convey information about the shape of the character, then a simple translation of the character will not affect the determination of the character.

Rotation:

There are several properties of characters that remain invariant with respect to rotation. They are: the length of the perimeter of the character, the area of the character, and the location of any axis about which the figure is rotated. This last appears trivial but is, as we shall see, quite important.

It is clear that the length of the perimeter remains invariant with respect to rotation. This knowledge is probably not too useful. Differences in perimeter length might be useful in identifying characters that appear similar in other respects. An example is a 2 and a Z. In some type fonts there is quite a difference between the lengths of their perimeters. When the fundamental frequency of the parametric functions is adjusted to a predetermined value, the value of the gain of the variable gain amplifier that feeds the vertical and horizontal modulators gives a measure of the perimeter length.

The area of the character, naturally remains invariant with regard to a rotation. This might be used in character recognition if "areas" of characters were compared after the parametric functions were normalized to unit amplitude of the fundamental of say  $y(t)$ . It could be quite useful if a machine, based on the proposed method, were used to measure such things as the area of leather, the grading of fruit according to size and color, etc. The area of the object being scanned can be derived in terms of the parametric functions as shown on the following page.



$$dA = y dx$$

$$A = \int_{x_{\max}}^{x_{\min}} y_u dx + \int_{x_{\min}}^{x_{\max}} y_l dx$$

From Eq. (8b)  $dx = K \cos \phi dt$

hence:

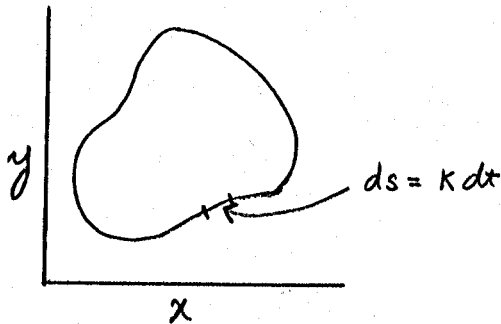
$$A = - \int_{x_{\min}}^{x_{\max}} y_u K \cos \phi dt - \int_{x_{\max}}^{x_{\min}} y_l K \cos \phi dt$$

$$A = - \left[ \int_0^{t_1} y(t) \frac{dx(t)}{dt} dt + \int_{t_1}^T y(t) \frac{dx(t)}{dt} dt \right]$$

$$A = - \int_0^T y(t) \frac{dx(t)}{dt} dt \quad (11)$$

Let us take the apparently trivial case of the location of the axis, about which the character is rotated. If the parametric functions are  $R(t)$  and  $\Theta(t)$ , this implies an origin for these polar functions. Such an origin cannot be arbitrarily chosen. If it is, identical characters may have vastly different  $R(t)$  and  $\Theta(t)$  functions. Two origins that immediately come to mind are the centroid of the perimeter and the centroid of the area. If either of these are chosen as the origin for the  $R(t)$  and  $\Theta(t)$  functions, then these functions will be identical for identical characters. A linear magnification of the character will cause  $R(t)$  to be multiplied by the linear magnification factor. The choice of which centroid to use as the origin is dependent on which is the easiest to locate.

The co-ordinates  $(\bar{X}, \bar{Y})$  of the centroid of the perimeter of the character will now be derived from the parametric functions  $x(t)$  and  $y(t)$ .



$$dM_y = x ds \quad (12 a)$$

$$dM_x = y ds \quad (12 b)$$

$$\bar{X} S = M_y \quad (13 a)$$

$$\bar{Y} S = M_x \quad (13 b)$$

$$M_y = \int_0^S x ds \quad M_x = \int_0^S y ds \quad (14 a, b)$$

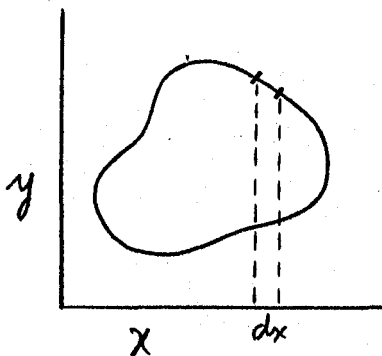
$$\text{Therefore } \bar{X} = \frac{\int_0^S x ds}{S} = \frac{k \int_0^T x(t) dt}{S} = \frac{1}{T} \int_0^T x(t) dt \quad (15 a)$$

$$\text{and } \bar{Y} = \frac{1}{T} \int_0^T y(t) dt \quad (15 b)$$

$$\text{hence } \bar{X} = x_0, \quad \bar{Y} = y_0 \quad (16)$$

From Eq. 16 it is seen that  $\bar{X}$  and  $\bar{Y}$  are the average values of  $x(t)$  and  $y(t)$  respectively. A rotation of the character through an angle  $\phi$  about any axis will produce a translation of its centroid and a rotation of the character through an angle  $\phi$  about the centroid. We have seen that a translation will not influence the determination of the character. The rotation of the character about its polar origin will produce a translation in time of  $R(t)$  and  $\Theta(t)$ . The magnitudes and relative phases of the harmonic components of these functions will remain unchanged. If  $R(t)$  is adjusted, so as to make the magnitude of the fundamental component of  $R(t)$  unity, then  $R(t)$  will also remain invariant with respect to a change in size of the character.

The calculation of the centroid of the area is much more difficult than the calculation of the centroid of the perimeter. Because of this difficulty, it is probably impractical to make use of its location, even though the distance between the two centroids gives some information about the characters. The location of the centroid of the area is derived below:



from Eq. 11 we have:

$$dA = - y(t) \frac{dx(t)}{dt} dt$$

$$\therefore dmy = x dA = - x(t) y(t) \frac{dx(t)}{dt} dt \quad (17)$$

$$\text{and} \quad \bar{x}_A A = my \quad (18)$$

$$\text{hence} \quad \bar{x}_A = \frac{\int_0^T x(t) y(t) \frac{dx(t)}{dt} dt}{\int_0^T y(t) \frac{dx(t)}{dt} dt} \quad (19a)$$

$$\bar{y}_A = - \frac{\int_0^T x(t) y(t) \frac{dy(t)}{dt} dt}{\int_0^T y(t) \frac{dx(t)}{dt} dt} \quad (19b)$$

#### Magnification:

A change in size or magnification of the character is automatically compensated for when the frequency and amplitude of the fundamental component of say  $y(t)$  is normalized. A non-uniform magnification which produces tall and thin, or short and fat characters causes trouble as does non-uniform magnification with a rotation if determination were made on harmonic content alone.