COMPANY CONFIDENTIAL

PROJECT STRETCH

STRETCH MEMO NO. 41

SUBJECT: A Note on Automatic Square Rooting

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A Note on Automatic Square Rooting

One of the classic methods of square rooting is to develop the radical digit by digit (as in division) by the successive subtraction of the odd integers from the radicand. The count of the number of subtractions is the first digit of the radical. The last odd number subtracted is increased to the next even digit, the radicand and radical are shifted left one position and the process is applied again using the last trial and emitting the odd integers into the next position to the right of the trial radical.

In a decimal machine there are some disadvantages of applying this method automatically. They are that the trial radical is twice the value of the developed radical, and the necessity of having to subtract the odd integers up to 19 requires the ability to add one to the previously developed position of the trial radicand.

These difficulties can be overcome by premultiplying the radicand by 1/2 and subtracting 1/2 of the odd integers. This gives a trial radical the same as the radical and only requires the subtraction of up to 9.5 instead of 19.

For example: $\sqrt{.5} = .7071....$

Method I

Radicand	Trial Radical	Radical
.500000	.010000	.000000
.490000	.030000	.100000
.460000	.050000	.200000
.410000	.070000	.300000
.340000	.090000	.400000
.250000	.110000	.500000
.140000	.130000	.600000
.010000	.150000	.700000
← no go		
.100000	.141000	.700000
← no go		
1.000000	.140100	.700000
.859900	.140300	.701000
.719600	.140500	.702000
.579100	.140700	.703000
.438400	.140900	.704000
.297500	.141100	.705000

Method I (contd.)

Radicand	Trial Radical	Radical
.156400	.141300	.706000
.015100	.141500	.707000
← no go		
.151000	.141410	.707000
.009590	.141430	.707100
no go		etc.

Method II

	Radicand	Radical
	.250000	.050000
Shift	2.500000	.050000
	2.450000	.150000
	2.300000	.250000
	2.050000	.350000
	1.700000	.450000
	1.250000	.550000
	.700000	.650000
	.050000	.750000
	← no go	
	.500000	.705000
	← no go	
	5.000000	.700500
	4.299500	.701500
	3.598000	.702500
	2.895500	.703500
	2.192000	.704500
	1.487500	.705500
	.782000	.706500
	.075500	.707500
	← no go	
	.755000	.707050
	.047950	.707150
	← no go	
	.479500	.707105
		etc.

In a binary machine the same principles apply with the advantage that the differences of Methods I and II disappear in binary shifts.

$$\sqrt{.5} = .7071....$$

or in binary $\sqrt{.10000} = .101101010....$

Radicand		Radical
.100 000 000		.010 000 000
.010 000 000		.110 000 000
←	no go	
.100 000 000		.101 000 000
←	no go	
1.000 000 000		.100 100 000
.011 100 000		.101 100 000
←	no go	
.111 000 000		.101 010 000
.001 110 000		.101 110 000
←	no go	
.011 100 000		.101 101 000
←	no go	
.111 000 000		.101 100 100
.001 011 100		.101 101 100
←	no go	
.010 111 000		.101 101 010
←	no go	
.101 110 000		.101 101 001
.000 000 111		.101 101 011
←	no go	
.000 001 110		.101 101 010
		etc.

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