(1)

COMPANY CONFIDENTIAL

MEMORANDUM December 9, 1955

TO: B. Housman

SUBJECT: A Method of Decreasing the Multiply Time in the AN/FSQ-7

The following outlines a scheme where the probability of saving in multiply time is 33% or greater by using only one additional shift path. The 33% value is obtained by assuming equal probabilities for the occurance of ones and zeros. Since many of the multipliers as used in the SAGE system will not require the full word size to describe them, the average multiply time will be decreased further.

Table 1 (from S. W. Dunwell's Project Stretch File Memorandum #2, 11/1/55) lists some probabilities of occurance of various size zero groups (1) and the probability that a zero will occupy such a group (2). (The discrepency between Table 1 and the table by S. W. Dunwell is because he multiplied (2) by 48 to get the number of zeros in his 48 bit word)

TABLE 1

Zero Group	Probability of Occurance	Probability that a zero will occupy such a group
x101x	1/8	1 x 1/8
x1001x	1/16	2 x 1/16
x10001x	1/32	3 x 1/32
x100001x	1/64	4 x 1/64
x1000001x	1/128	5 x 1/128
x10000001x	1/256	6 x 1/256
	etc.	

Mr. Dunwell calculates that if equipment is built into the multiplier such that any of the six groups can be jumped, i.e. 6 additional shift paths, a saving in multiply time of 47% results.

In order to calculate the probable increase in speed for some cheaper combinations of shift paths, Table 2 was constructed. Table 2 lists the number of shifts required for the various combinations of shift paths. 1 represents the present system. 1 or 2 represents a system that can make a shift of 2 or 1. 1 or 2 or 3 and 1 or 2 or 4 are further extensions of the system.

TABLE 2

No. of Zeros in Sequence	Number of Shifts Required			
	1	1 or 2	1 or 2 or 3	1 or 2 or 4
1	1	0	0	0
2	2	1	0	1
3	3	1	1	0
4	4	2	1	1
5	5	2	1	1
6	6	3	2	2
7	7	3	2	1
8	8	4	2	2
9	9	4	3	2
10	10	5	3	3
11	11	5	3	2
12	12	6	4	3
13	13	6	4	3
14	14	7	4	4
15	15	7	5	3
16	16	8	5	4

In the ratio of the shifts saved to the present number of shifts required are combined with the probabilities of the zeros occupying such a group, the following probabilities of decrease in the multiply time results.

TABLE 3

No. of Zeros in Sequence	Probability for Decrease in Shifts		
	1 or 2	1 or 2 or 3	1 or 2 or 4
1	1 x 1/8	1 x 1/8	1 x 1/8
2	1/2 x 2/16	2/16	1/2 x 2/16
3	2/3 x 3/32	2/3 x 3/32	3/32
4	2/4 x 4/64	3/4 x 4/64	3/4 x 4/64
5	3/5 x 5/128	4/5 x 5/128	4/5 x 5/128
6	3/6 x 6/256	4/6 x 6/256	4/6 x 6/256
	etc.		
TOTAL	.333	.425	.400

An additional saving of time occurs during shift instructions. If the assumption is made that the probability of a shift of any one of the possible places is equal to the probability of a shift to any one of the other possible places, the saving in time by the various schemes is given in Table 4.

TABLE 4

Type of Shift	1	1 or 2	1 or 2 or 3	1 or 2 or 4
Amount of time required (Normalized to Present System)	1.0	.53	.375	.324

The additional shift paths required in Table 3 and the necessary control equipment can be built from present basic circuits*.

The author recommends a study be made to determine the desirableness of one or more additional shift paths.

W. A. Hunt

WAH:SG

* The author plans to investigate the logic changes necessary for the shift of 1 or 2, but to answer the most obvious question resulting in the delay caused by the carry chain, a delay system would be included in the shift selection. This delay system would probably need only two to four delays in it.

A shift of 1 or 2 would be easy to sense for in multiply by setting the shift selection with the one or zero as it is transmitted into the next to the last bit position.