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Theoretical Possibilities of Automatic Computers

Theorem Proving, etc

by

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I. Introduction

In considering the future role and improved design of computers there is need for a grasp of the possibilities of computers without being limited by the presently practical possibilities nor by the immediate usefulness of the conceivable applications. Despite the amount of speculation concerning such matters, I have seen none which state explicitly a few of the basic facts concerning what automatic computers actually do and what they may do.

Mathematicians have contributed to this confusion by misunderstanding the role and nature of a proof. Symbolic logicians have contributed to the same confusion by the tacit assumption that a proof lies in a concatenation of symbols. In my estimate there could scarcely be a more silly solgan than "mathematics is deductive science."

II. What Computers Do

A computer accepts in its input a sequence of symbols which it translates into another set more appropriate for its structure. The computer then associates with these symbols another set which it translates into output. The computer thus appears as a proposition-proving device. The input and the machine design constitute the hypotheses and logic, the manipulations are the steps in the proof, and the output contains the conclusions (with identification). If the computer makes no errors (i.e. failures) then the output is a rigid consequence of the input and logic. The connection of this output with other structures is a matter of importance but does not change our statement of the role of the computer.

For example, one may set up a difference equation scheme to approximate the solution of a differential equation. The introduction of this scheme by appropriate symbols into the computer and the subsequent manipulations are followed by a table of values. The table, provided no computer errors are made, contains the exact consequences of the assumptions and logic. That the values may be widely different from those of the actual solution of the differential equation due to round off, mesh size, etc., does not alter the absolute precision of the conclusions given by the computer resulting from assumptions and logic. Errors in the code form part of the assumptions. Liberalizing the interpretation you might say errors in the machine could form part of the assumptions also.

The analogy with the manipulations made in proving theorems should be now discernible. Hypotheses are often stated explicitly, the logic, as with the machine is partially submerged in the mathematician's errors in code correspond to errors in logic, machine errors correspond to errors in application of the logic.

Now we come to another fundamental point. What is the meaning of a symbol in the machine? I say that the symbol itself has no meaning. What the symbol represents is reflected only in the manner in which the machine treats it. Thus a series of magnetic spots on a tape represents whatever a translation dictionary says it represents. It might represent the social security number of an individual and thereby the individual, it might represent an instruction to the machine, a number in fixed or floating form, it might represent one of Shakespeare's plays, or it might represent one of x^3, ln x, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$, or L where L is the Laplace transform operator. It is clear that the meaning of the symbol is not in the symbol itself.

It has been a drawback to the optimal use of high-speed computers that it is frequently assumed that the symbols or words "are" either numbers or orders. Partially for this reason the so-called "infinite" processes of analysis are assumed beyond the pole of computers. Obviously this is not the case since infinite series, derivatives, integrals, etc., may be represented in a computer if they may be represented in print.

III. Symbols, Meanings, and Information

Symbolic representations form the most powerful tools of civilization. As a result they have been both defied (In the beginning was the word, the word was God, etc.), and abused. You "know" a person if you know his name; for example.

Despite the large numbers of languates, books, pictures, records, speeches, blue-prints, statues, charts, samples, and what have you, I believe we may agree that on this earth at any time there is only a finite set of distinct symbols. All communications regarding infinite sets, limits and so on must be made with a finite set of symbols. Thus it is physically impossible to record a distinct symbol for each integer.

By a symbol, I include combinations and arrangements of symbols. Thus an entire treatise is a symbol as well as a single letter in the treatise. The symbol itself is subject to interpretation perhaps by humans, perhaps by machines but an abstract symbol customarily has <u>none</u> of the properties of the phenomena it represents. (A sample rymbol, for example, presumably represents the larger whole of which it is a part by displaying the same properties.) Thus we may claim that the abstract symbol has no meaning or that it conveys no information apart from an interpreter. Incidentally, the present-day information theory is misnamed, it would better be called <u>signal</u> theory.

From the standpoint of a theoretically large enough computer, it is then theoretically possible to translate <u>all other</u> symbols by an adequate dictionary into the memory of the machine. Thus, again, the computer might be used to prove theorems in the ordinary sense provided proper manipulation with symbols can be said to constitute a proof.

IV. The Nature of Proofs

Reduction of human errors in making proofs of theorems has been accomplished by use of established rules of manipulation of symbols. This, together with the invariance of certain mathematical theorems in time have led many mathematicians and symbolic logicians to assume that human beings possibly can be ignored in relation to a proof. This I see no reason to believe and I state the following Relativistic

Principle: To a given individual a proposition has been proved if, and only if, he is convinced that the proposition is true. His position on the truth of the proposition may change from time to time.

Thus, mathematical theorems are established as true essentially by affirmative vote of those who are convinced of its truth. There is no way conceivable of avoiding the fact that symbols recorded in order cannot constitute a proof if only simply due to the fact that there is no way of absolutely guaranteeing the accuracy of recording the symbols. However, it is possible that improvements in accuracy result from the mechanization of proofs such as exemplified in the arithmetization of analysis.

V. Immediate Improvements in Computer Use

Since we have indicated that computers are capable of carrying out any symbolic proof, the question remains whether or not some of this sort of activity might not be useful now. I believe the answer is affirmative and I will illustrate some directions of approach. The reason for computing is the same as that of proving theorems: to obtain information for some form of action. If, however, the computer output is so tremendous that the results cannot be used or interpreted for any kind of action then the value of the activity is negative insofar as results are concerned. Hence, we need to use modes of output optimized with regard to ~~xx~~ subsequent usage. Apart from visual aids there are certain modes of presenting functions, foreexample, which would be better than the tables of values of the function at certain mech points <u>for some uses</u>.

Thus a function may be represented by a linear combination of a basic set of functions (finite or infinite) by simply recording the coefficients in order (the coefficients could be functions of a parameter). We emphasize that there is no a priori reason for excluding infinite series from machine computation. For some uses a finite linear combination of functions would be far superior to a table of values.

There is no need for output to be interpreted as numbers. The answer might be that a proposition is false or that an operation is not feasible.

Machine (exact) differentiation of functions should be workable for large classes of functions, thereby permitting the derivation of Taylor Series from a differential equation.

Proof of theorems by induction is a possibility I am now investigating for feasibility. It is clear that this type of process should be readily mechanizable.

Finally, while it is possible in principle to prove any theorem using present computers, the practice will be greatly aided when we have studied application and improved designs accordingly.