

Example of matrix 3×3

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1(45-48) - 4(18-24) + 7(12-15) = (-3) - 4(-6) + 7(-3) \\ = -3 + 24 - 21 = 0$$

$$\begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix} \quad 45 + 84 + 96 - 105 - 48 - 72 = 225 - 225 = 0$$

$$\begin{vmatrix} (45-48) & (18-24) & (12-15) \\ (36-42) & (9-21) & (6-12) \\ (32-35) & (8-14) & (5-8) \end{vmatrix} = \begin{vmatrix} -3 & -6 & -3 \\ -6 & -12 & -6 \\ -3 & -6 & -3 \end{vmatrix} \quad \text{singular}$$

non singular case

$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 1 & 4 \end{vmatrix} = 1(8-1) - 3(8-3) + 4(2-6) = 7 - 3(5) - 16 \\ = 7 - 15 - 16 = -24 \\ 8 + 8 + 9 - 24 - 1 - 24 = 25 - 49 = -24$$

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix}$$

$$\frac{7}{8} + 3 - \frac{15}{8}$$

$$\frac{1}{24} \begin{vmatrix} (8-1) & -(8-3) & (2-6) \\ (12-4) & (4-12) & -(1-9) \\ (3-8) & -(1-8) & (2-6) \end{vmatrix} = \frac{1}{24} \begin{vmatrix} 7 & -5 & -4 \\ -8 & -8 & +8 \\ -5 & +7 & -4 \end{vmatrix} = \begin{vmatrix} \frac{7}{8} & -\frac{5}{8} & -\frac{1}{2} \\ -1 & -1 & 1 \\ -\frac{5}{8} & \frac{7}{8} & -\frac{1}{2} \end{vmatrix}$$

$$\begin{vmatrix} \frac{7}{8} & -\frac{5}{8} & -\frac{1}{2} \\ 1 & -1 & 1 \\ -\frac{5}{8} & \frac{7}{8} & -\frac{1}{2} \end{vmatrix} \begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 1 & 4 \end{vmatrix} = \begin{vmatrix} \frac{7}{8} + \frac{15}{8} & -2 \\ 1 & -3 & 4 \\ -\frac{5}{8} & \frac{21}{8} & -2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2$$

$$\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$$

$$\begin{vmatrix} 4 & 2 \\ 3 & 1 \end{vmatrix} = \begin{vmatrix} +2 & -1 \\ -\frac{3}{2} & +\frac{1}{2} \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \cdot \begin{vmatrix} +2 & -1 \\ -\frac{3}{2} & +\frac{1}{2} \end{vmatrix} = \begin{vmatrix} 2 & -3 \\ 6 & -3+2 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}$$



multiply back:

$$\begin{array}{c|ccc|ccc} 1 & 2 & 3 & \frac{7}{8} & -\frac{5}{8} & 2\frac{1}{2} \\ 3 & 2 & 1 & -1 & -1 & +1 \\ 4 & 1 & 4 & -\frac{5}{8} & +\frac{7}{8} & -\frac{1}{2} \end{array} = \begin{array}{c|ccc|ccc} \frac{7}{8} & -\frac{5}{8} & 2\frac{1}{8} & -\frac{5}{8} & -2 & +\frac{21}{8} \\ \frac{21}{8} & -\frac{5}{8} & -\frac{15}{8} & -\frac{15}{8} & -2 & +\frac{7}{8} \\ \frac{7}{2} & +1 & -\frac{5}{2} & & & \end{array}$$

$$\frac{1}{-24} \begin{array}{c|ccc|ccc} 1 & 2 & 3 & 7 & -5 & -4 \\ 3 & 2 & 1 & -8 & -8 & +8 \\ 4 & 1 & 4 & -5 & +7 & -4 \end{array}$$

$$= \begin{array}{c|ccc|ccc} 7 & -5 & -4 & & & \\ -8 & -8 & +8 & & & \\ -5 & +7 & -4 & & & \end{array}$$

2x2 CRR

$$\begin{array}{|c|c|} \hline a_{11} & a_{12} \\ \hline a_{21} & a_{22} \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a_{11} & a_{21} \\ \hline a_{12} & a_{22} \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a_{22} & -a_{12} \\ \hline -a_{21} & a_{11} \\ \hline \end{array}$$

$$\begin{array}{c|ccc|ccc} \frac{1}{-24} & (7-16-15) & (-5-16+21) & (-4+6-12) & -24 & 0 & 0 \\ & (21-16-5) & (-15-16+7) & (-12+16-4) & 0 & -24 & 0 \\ & (28-8-20) & (-20-8+28) & (-16+8-16) & 0 & 0 & -24 \end{array} = \frac{1}{-24} \begin{array}{|c|c|c|} \hline -24 & 0 & 0 \\ \hline 0 & -24 & 0 \\ \hline 0 & 0 & -24 \\ \hline \end{array}$$

$$= \begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a_{11} & a_{12} \\ \hline a_{21} & a_{22} \\ \hline \end{array} \begin{array}{|c|c|} \hline a_{22}-a_{12} \\ \hline -a_{21} & a_{11} \\ \hline \end{array} = \begin{array}{|c|c|} \hline a_{11}a_{22} - a_{12}a_{21} & -a_{12}a_{22} + a_{11}a_{11} \\ \hline a_{21}a_{22} - a_{21}a_{22} & -a_{12}a_{21} + a_{11}a_{22} \\ \hline \end{array}$$

$$= \begin{array}{|c|c|} \hline a_{11}a_{22} - a_{12}a_{21} & 0 \\ \hline 0 & a_{11}a_{22} - a_{12}a_{21} \\ \hline \end{array} = |a| \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

determinant:

$$\begin{array}{c|ccc|ccc} 1 & 2 & 3 & 7 & -5 & -4 \\ & & & -8 & & \\ & & & -5 & & \end{array} = \frac{1}{-24} (7-16-15)$$