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May 23, 1958

PROJECT 7000

FILE MEMO

SUBJECT: Results of a Small Experiment using Two Types of Noisy Mode Floating Point

Introduction

In recent discussions with Messrs, Brooks and Blaauw concerning the best way of doing Noisy Mode Floating Point, the following questions have arisen:

(1) Does the concept of the Noisy Mode really give us the answers we desire concerning numberical significance?

(2) Should there be a bit reversal on every arithmetic operation as presently proposed, or only on subtractions involving significant figure loss?

(3) Are there systematic cancellations of the noise generated in one instruction by subsequent instructions?

(4) Should a large scale computing effort be launched to study the whole question?

The following simple cases were tried by hand to see what could be learned quickly, and to help decide what needs to be done next.

Test Problem Used: One term of the ill-conditioned matrix

$$A = \begin{vmatrix} 1 & 1/2 & 1/3 \\ 1/4 & 1/5 & 1/6 \\ 1/7 & 1/8 & 1/9 \end{vmatrix}$$

The 1,1 element of A^{-1} is given by:

$$b_{11} = C_1/(a_{11} C_1 - \overset{a_{12}}{a_{21}} C_2 + \overset{a_{13}}{a_{31}} C_3)$$

where

$C_1 = A_{22} A_{33} - A_{23} A_{32} = 1/45 - 1/48$

$C_2 = A_{12} A_{33} - A_{13} A_{32} = 1/18 - 1/25$ 24

$C_3 = A_{12} A_{23} - A_{23} A_{13} = 1/12 - 1/15$

Doesn't affect [illegible]

The ill-conditioned nature of the matrix is apparent. The time value of b_{11} = 4 2/3 (100.101010··· in binary)

Cases Evaluated: (All in floating point) **Results**

Case I. Solved in decimal using 4 decimals: b_{11} = 4.726 error = .060
Case II. Solved in decimal using 5 decimals: b_{11} = 4.6734 error = .0067

Difference between 4 and 5 decimal case = .053

Case III. Solved using 5 decimals with Noisy Mode Type I. (The last decimal is increased by 5 if it is 0 to 5, reduced by 5 if it is 6 to 9. This is done for every arithmetic operation.) b_{11} = 4.7504

Difference between Type I Noise and No Noise = 00.07

Case IV. Solved using 5 decimals with Noisy Mode Type II. (The last decimal is inverted as above only on subtractions which result in leading figure loss.) b_{11} = 4.7282

Difference between Type II Noise and No Noise = 0.055

Case V. Solved in binary using 9 bits b_{11} = 101.101011 error = 1.00
Case VI. Solved in binary using 10 bits b_{11} = 100.1001111 error =.0000110

Difference between 9 and 10 bits = 1.00

Case VII. Solved in binary using 10 bits with Noisy Mode Type I. (10th bit was reversed on every arithmetic operation, b_{11} = 101.011110001

Difference between Type I Noise and No Noise = 0.111

Case VIII. Solved in binary using 10 bits with Noisy Mode Type II. (10th bit reversed on subtraction with leading figure loss, b_{11} = 100.1111011

Difference between Type II Noise and No Noise = 0.011

Conclusions: (within the accuracy of the experiments only.)

1. There are large statistical fluctuations in calculations with as few as 10 bits.

2. The Noisy Mode does seem to give the right order of magnitude indication of imprecision.

3. Type I seems to give results a little closer to those obtained using one less bit than Type II.

4. The Type II results show that the figure-loss subtractions are indeed the dominating ones. The noise on multiplys, etc., contribute much less.

5. No evidence of any systematic cancellation of noise was apparent in either method.

6. Further study using interpretive floating point on the 704 should be made. The evaluation of a larger matrix with various word lengths and methods of inserting noise is an obvious next step.

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