Example of Use of Hamming Error Correcting Code

Given an 8 bit number with 4 checking bits

$X_0X_1X_2X_3X_4X_5X_6X_7$ $\qquad$ $C_0C_1C_2C_4$

The four check bits are parity checks selected in such a way that an error in any bit (including the check bits) gives a unique pattern.

For 8 bits the check bits are chosen to give odd parity for the following combinations:

C_0 for bits	X_0	X_1	X_2	X_3	X_4	X_5	X_6	X_7
C_1 for bits		X_1		X_3		X_5		X_7
C_2 for bits			X_2	X_3			X_6	X_7
C_4 for bits					X_4	X_5	X_6	X_7

For Example take the word: 10110100

The check bits are

$C_0(10110100) = 1$
$C_1(0110) = 1$
$C_2(1100) = 1$
$C_4(0100) = 0$

Suppose that bit 2 is in error so that the number is read 10010100 with checks bits 1110 , the parity yields:

$C_0'(10010100) = 0$
$C_1'(0110) = 1$
$C_2'(0100) = 0$
$C_4'(0100) = 0$

on comparing these against the check bits, the fact that the C_0's differ indicates that there is an odd number of bit-inversions in the word. If there is only one, the pattern of C's changed gives the number of the bit which is to be corrected is as follows:

Check bits differing				
C_4	C_2	C_1	C_0	Bit Wrong
0	0	0	0	no error
0	0	1	1	X_1
0	1	0	1	X_2
0	1	1	1	X_3
1	0	0	1	X_4
1	0	1	1	X_5
1	1	0	1	X_6
1	1	1	1	X_7
0	0	0	1	C_0
0	0	1	0	C_1
0	1	0	0	C_2
1	0	0	0	C_4
0	1	1	0	} Double Errors
1	0	1	0	
1	1	0	0	
1	1	1	0	

The double errors are detected but cannot be corrected.

The same scheme used for the 64 bit words in STRETCH requires 8 check bits.