Sept 20, 57

Meeting at Spring Street, Numerical Analysis Seminar,

Ref: Householder p, 65-7, 78-79

Method of Modification.

Inverse: $\left(B - \sigma u v^T\right)^{-1} = B^{-1} + \overbrace{\frac{\sigma}{1-\sigma v^T B^{-1} u}}^{\text{a number}} \cdot \left(B^{-1} u v^T B^{-1}\right)$ ← not really a Triple prod.

matrix with known inverse; row matrix; col matrix

$$\left(B - \sigma\left(e_i e_j^T + e_j e_i^T\right)\right)^{-1}$$

$$B^{-1} + \alpha B^{-1} \underbrace{e_i e_j^T}_{1 \text{ in } ij \text{ pos}} B^{-1} + \beta B^{-1} e_i e_i^T B^{-1} + \gamma B^{-1} e_j e_j B^{-1} + \delta B^{-1} e_j e_i^T B^{-1}$$

equiv to Kron's method?

M.R. Hestenes: usually interested in a left inverse in numerical work

$$Ax = b$$

$$A^{-1} A = I$$

$u_1, u_2 \cdots u_n$ $\quad (u_i, v_j) = \delta_{ij}$

$v_1^{(n)}, v_2 \cdots v_n$

Start with a guess v_1 same as u_1, etc?

$$v_h^{(h)} = \frac{v^{(h-1)}}{\left(v_h^{(h-1)}, u_h\right)}$$

method of picking next

question of zero denom?

if $v_i^{(0)} = u_i^T$

Then $\left(v_h^{(h-1)}, u_h\right) = \dfrac{\det\left(U_h^T U_h\right)}{\prod_{k=1}^{h-1}\left(v_k^{k-1}, u_k\right)}$ conjecture only ?

Recipe: $v_i^{(h)} = v_i^{(h-1)} - \left(v_i^{(h-1)}, u_h\right) v_h^{(h)}$

$i \neq h$ ↑ from previous

a Test of calc.

$(A^{-1})^{-1} = A$

but this is no guarantee of goodness of inverse.

takes $\begin{cases} \text{div} & n \text{ steps} \\ \text{mpy} & 2n^3 - n^2 \\ \text{add} & \sim \text{"} \end{cases}$

works beautifully for matrix which is orthogonal.

Collatz: Eigenvalues - applying functional analysis - deriv. of operators

set in form $T\phi = \Theta$

↑ fn of λ use Newton itteration - on operators.

$$\| T(f+k) - Tf - Hk \| \leq \|k\| \, \varepsilon(\|k\|)$$

$f+k$ vectors $\varepsilon(\delta) \to 0$ as $\delta \to 0$

Then H is deriv (wrt f ?)

Bauer: Itteration processes - Bernoulli, Graf.

Power method to find largest eigenvalue

$$X_{i+1} = A X_i$$

use larger matrix $A^{(m)}$ all m-rowed minors of A. ?

Theorem $\left(A^{(m)}\right)^i = \left(A^i\right)^{(m)}$ (ith power)

The eigen values are the products of A taken m at a time?

never been tried on machine?

Ostrowski:

$$X_{i+1} = A X_i$$

a linearly convergent method
can be made cubically convergent

$$X_{i+1} = \underbrace{(A - \lambda_i I)}^{-1} X_i$$ for almost singular.

but this ill-conditioned

Ref: Dave Young very important work
(1954 Transactions)
"strop. Gauss-Seidel"

Elliptic D.E.'s
solns.

Difference
{ 5 pt stars
{ 9 pt stars.

$$Ax = b$$

$$X_{i+1} = \underbrace{E(\delta)}[X_i + b]$$

has eigenvalues modulus less than 1

E can be related to A with zero diag. elements,

must be able to put A in this form $\begin{pmatrix} I_p & U \\ L & I_{n-p} \end{pmatrix}$
0

L + U rectangular
I identity