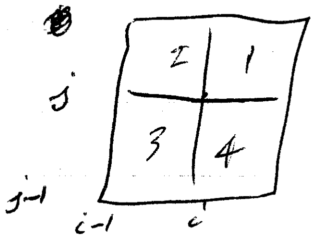
Sept 23, '57

a Simple Mesh Problem:
(calculation of gradient by cross method)

$$\frac{\Delta P}{\rho\Delta X} = \frac{1}{\frac{1}{4}\left[\rho_1+\rho_2+\rho_3+\rho_4\right]}\;\frac{(P_1-P_3)(Y_2-Y_4)-(P_2-P_4)(Y_1-Y_3)}{(X_1-X_3)(Y_2-Y_4)-(X_2-X_4)(Y_1-Y_3)}$$



$$P_1 = P_0 + \frac{\Delta P}{\Delta X}(X_1-X_0) + \frac{\Delta P}{\Delta y}(Y_1-Y_0)$$

$$P_3 = P_0 + \frac{\Delta P}{\Delta X}(X_3-X_0) + \frac{\Delta P}{\Delta y}(Y_3-Y_0)$$

$$P_1-P_3 = \frac{\Delta P}{\Delta X}(X_1-X_3) + \frac{\Delta P}{\Delta y}(Y_1-Y_3)$$

$$P_2-P_4 = \frac{\Delta P}{\Delta x}(X_2-X_4) + \frac{\Delta P}{\Delta y}(Y_2-Y_4)$$

$$\begin{vmatrix} X_1-X_3 & P_1-P_3 \\ X_2-X_4 & P_2-P_4 \end{vmatrix}$$

$$(P_2-P_4)(X_1-X_3)-(P_1-P_3)(X_2-X_4)$$

$$\left(\frac{\Delta P}{\Delta X}\right) = \frac{\begin{vmatrix} P_1-P_3 & Y_1-Y_3 \\ P_2-P_4 & Y_2-Y_4 \end{vmatrix}}{\begin{vmatrix} X_1-X_3 & Y_1-Y_3 \\ X_2-X_4 & Y_2-Y_4 \end{vmatrix}} = \frac{(P_1-P_3)(Y_2-Y_4)-(P_2-P_4)(Y_1-Y_3)}{(X_1-X_3)(Y_2-Y_4)-(X_2-X_4)(Y_1-Y_3)}$$

$$\left(\frac{\Delta P}{\rho\Delta X}\right)_{ij} = \frac{-1}{\bar\rho}\,\frac{\overset{p}{(P_{i,j}-P_{i-1,j-1})}\overset{b}{(Y_{i-1,j}-Y_{i,j-1})}-\overset{q}{(P_{i-1,j}-P_{i,j-1})}\overset{d}{(Y_{i,j}-Y_{i-1,j-1})}}{\underset{a}{(X_{i,j}-X_{i-1,j-1})}\underset{b}{(Y_{i-1,j}-Y_{i,j-1})}-\underset{c}{(X_{i-1,j}-X_{i,j-1})}\underset{d}{(Y_{i,j}-Y_{i-1,j-1})}}$$

$$\left(\frac{\Delta P}{\rho\Delta Y}\right)_{ij} = \frac{-1}{\bar\rho}\,\frac{\overset{q}{(P_{i-1,j}-P_{i,j-1})}\overset{a}{(X_{i,j}-X_{i-1,j-1})}-\overset{p}{(P_{i,j}-P_{i-1,j-1})}\overset{c}{(X_{i-1,j}-X_{i,j-1})}}{(\quad a\quad)(\quad b\quad)-(\quad c\quad)(\quad d\quad)}$$

$$\left(\frac{\Delta P}{\rho\Delta X}\right) = -\frac{1}{\rho}\,\frac{(pb-qd)}{D} = -\frac{A}{RD}$$

$$\left(\frac{\Delta P}{\rho\Delta Y}\right) = \frac{-1}{\bar\rho}\,\frac{(qa-pc)}{D} = -\frac{B}{RD}$$

$$D = ab - cd$$

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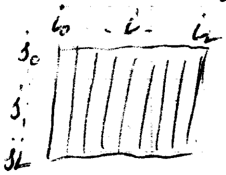
SIMPLE MESH Problem

EXAMPLE of PROBLEM using considerable arithmetic.

1. $a \overset{1}{=} X_{i,j} - X_{i-1,j-1}$
2. $b \overset{2}{=} Y_{i-1,j} - Y_{i,j-1}$
3. $c \overset{3}{=} X_{i-1,j} - X_{i,j-1}$
4. $d \overset{4}{=} Y_{i,j} - Y_{i-1,j-1}$
5. $p \overset{5}{=} P_{i,j} - P_{i-1,j-1}$
6. $q \overset{6}{=} P_{i-1,j} - P_{i,j-1}$
7. $R = \frac{1}{4}\left(R_{i,j} + R_{i-1,j} + R_{i-1,j-1} + R_{i,j-1}\right)$
8. $D = \overset{1}{a}\overset{2}{b} - \overset{3}{c}\overset{4}{d}$
9. $A = \overset{5}{p}\overset{2}{b} - \overset{6}{q}\overset{4}{d}$
10. $B = \overset{6}{q}\overset{1}{a} - \overset{5}{p}\overset{3}{c}$
11. $G_{ij} = \dfrac{-A}{RD}$
12. $H_{ij} = \dfrac{-B}{RD}$

most of arithmetic

original data tables.



$X_{i,j}$ $\quad i = i_0 \cdots i_2$

$Y_{i,j}$ $\quad j = j_0 \cdots j_2$

$R_{i,j}$ $\quad i_L$ not necess. $= j_L$

$P_{i,j}$ (assume separate) Tables or in seq.

intermediate quantities — not page

a, b, c, d, p, q, R

D, A, B

answers,

$G_{i,j}$) assume

$H_{i,j}$

Set indices : $\begin{cases} x : b_x = X_0 + i(j_L) + j - 1 \\ y : b_x - j_L \end{cases}$ | i.e. $(i, j-1)$ $(i-1, j-1)$

Note : 1. Indexed operations are fairly simple

2. most of arithmetic is done from Temporary locations

Indexing Mesh Problem

assuming (1) X_{ij}, Y_{ij} are stored together $\cdots | X_{ij} | Y_{ij} | X_{i,j+1} | Y_{i,j+1} | \cdots$

(2) R_{ij} stored separately $\cdots | R_{ij} | R_{i,j+1} | \cdots$

above are $\left.\begin{array}{l} i : 0 \text{ to } i_L \\ j : 0 \text{ to } j_L \end{array}\right\}$ meshes

(3) P_{ij} are stored separately $\cdots | P_{ij} | P_{i,j+1} | \cdots$

however there are only two columns which ~~alternate~~

i.e. $i : 0$ or 1 where

$i=0$ use $\overset{a}{|}\ \overset{b}{|}$ $\quad$ $i=1$ use $\overset{b}{|}\ \overset{a}{|}$ $\quad$ $i=2$ use $\overset{a}{|}\ \overset{b}{|}$ $\cdots$

$j : 0$ to j_L

(4) G_{ij}, H_{ij} are stored together as $X + Y$ are,

for Base at i,j:

$-j_L$	0
$-j_L - 1$	-1

$i-1, j$	i, j
$i-1, j-1$	$i, j-1$

~~$= b_0 + 2j_L i + 2j$~~

address of X

i,j: $\quad b_0 + 2\{j_L \cdot i + j\} \quad = (b_0) \quad + 2j_L \cdot i + 2j$ $\qquad \Delta$ for $j = 2$

$i-1,j$: $\quad b_0 + 2\{j_L(i-1) + j\} \quad = (b_0 - 2j_L) \quad + 2j_L \cdot i + 2j$ $\qquad \Delta$ for $i = 2j_L$

$i-1,j-1$: $\quad b_0 + 2\{j_L(i-1) + (j-1)\} \quad = (b_0 - 2j_L - 2) \quad + 2j_L \cdot i + 2j$

$i,j-1$: $\quad b_0 + 2\{j_L \cdot i + (j-1)\} \quad = (b_0 - 2) \quad + 2j_L \cdot i + 2j$

carried in addr. (The ones with j_L must be stor-addressed at beginning of problem.)

(Y's are same as X's $+ 1$)

If the tables are stored normally so that V_{i,j_L} precede $V_{i+1,0}$

Then one can use only <u>one</u> index k to sweep thru, where $k = 2(j_L \cdot i + j)$

$x, y \quad (G, H)$

$X_{i,j} : b_0 + k$

$X_{i-1,j} : (b_0 - 2j_L) + k$

$X_{i-1,j-1} : (b_0 - 2j_L - 2) + k$

$X_{i,j-1} : (b_0 - 2) + k$

y's same ~~as~~ $x + 1$

k starts at 0

$\Delta = 2$

$L = 2j_L \cdot i_L + 2j_L$

R

$R_{i,j} : r_0 + l$

$R_{i-1,j} : (r_0 - j_L) + l$

$R_{i-1,j-1} : (r_0 - j_L - 1) + l$

$R_{i,j-1} : (r_0 - 1) + l$

l starts at 0

$\Delta = 1$

$L = j_L i_L + j_L$

P

$P_{i,j} : (p_0) + j \mp m$

$P_{i-1,j} : (p_0 - j_L) + j \pm m$

$P_{i-1,j-1} : (p_0 - j_L - 1) + j \pm m$

$P_{i,j-1} : (p_0 - 1) + j \mp m$

$m = j_L$ on interchange

or change base addr.
by store address in ops
which ~~use~~ it

j starts at 0

$\Delta = 1$

$L = j_L$

- Initialize k, l, j
- calculation
- Incr. j, k, l
- Test j
- interchange P
- Test k