

Author Listing of Conference Notes

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Rabinowitz	C-60	Meetings of the Mathematics Group
Hildebrandt	C-99	Runge-Kutta Method Applied to a Simplified Problem

Project Whirlwind
Servomechanisms Laboratory
Massachusetts Institute of Technology
Cambridge, Massachusetts

SUBJECT: MEETINGS OF THE MATHEMATICS GROUP

The Runge-Kutta Method for Solving Ordinary Differential Equations and its Variations.

Place: Room 250

Date: September 7, 1948

The Runge-Kutta Method

The formulae used at present for the solution of ordinary differential equations and termed the Runge-Kutta formulae are but one special set from an infinite-squared number of such formulae. These are derived from the following equations:

Given $y' = f(x, y)$ with $y = y_0$ at $x = x_0$

Form $K_1 = hf(x_0, y_0)$

$$K_2 = hf(x_0 + \alpha_1 h, y_0 + \beta_1 K_1)$$

$$K_3 = hf(x_0 + \alpha_2 h, y_0 + \beta_2 K_1 + \gamma_1 K_2)$$

$$K_4 = hf(x_0 + \alpha_3 h, y_0 + \beta_3 K_1 + \delta_2 K_2 + \delta_1 K_3)$$

$$K = \Delta y = R_1 K_1 + R_2 K_2 + R_3 K_3 + R_4 K_4$$

By expanding these terms in Taylor series and equating coefficients so that K is correct up to h^4 , we arrive at a system of 12 equations with ten unknowns. Thus we have two arbitrary parameters. For reasons of symmetry and simplicity the following choice was made:

$$R_1 = R_4 = \frac{1}{6} \quad R_2 = R_3 = \frac{1}{3} \quad \alpha_1 = \alpha_2 = \beta_1 = \delta_1 = \frac{1}{2}$$

$$\alpha_3 = \delta_1 = 1 \quad \beta_2 = \beta_3 = \delta_2 = 0$$

Yielding the well known formulae, which are also valid for n equations:

$$K_1^i = hf_1(x_0, y_{10}, \dots, y_{n0})$$

$$K_2^1 = hf_1 \left(x_0 + \frac{h}{2}, y_{10} + \frac{K_1^1}{2}, \dots, y_{n0} + \frac{K_1^n}{2} \right)$$

$$K_3^1 = hf_1 \left(x_0 + \frac{h}{2}, y_{10} + \frac{K_2^1}{2}, \dots, y_{n0} + \frac{K_2^n}{2} \right)$$

$$K_4^1 = hf_1 \left(x_0 + h, y_{10} + K_3^1, \dots, y_{n0} + K_3^n \right)$$

$$K_1^1 = \Delta Y_1 = \frac{1}{6} (K_1^1 + 2K_2^1 + 2K_3^1 + K_4^1)$$

This method is correct up to h^4 . It can be applied to any higher order differential equation since such an equation can be reduced to a system of equations. For given $y^{(m)} = f(x, y, y', \dots, y^{(m-1)})$, the substitution

$$z_1 = y^{(m-1)}, z_2 = y^{(m-2)}, \dots, z_n = y$$

yields the following system of first order equations

$$z_1' = f(x, z_1, \dots, z_n)$$

$$z_2' = z_1$$

$$\vdots$$

$$z_n' = z_{n-1}$$

There are no rigid formulae for error estimation but a practical way to estimate the error after two steps is to repeat the process using intervals of size $2h$. Then the error in h^5 would be approximately equal to $\frac{\Delta \bar{y}(2h) - \Delta y(h) - \Delta y(2h)}{15}$. Where $\Delta y(h)$ is result after the first

single step, $\Delta y(2h)$ after the second single step, and $\Delta \bar{y}(2h)$ the result after one double step. (For more complete discussion of error, the reader is referred to E147.)

The Nystrom Variation

In the case of a second order equation, or a system of such equations, a variation was introduced by Nystrom. He objected that in reducing such an equation to a set of two first order equations, as much weight was given to the first derivative as to the function. He accordingly set up the following set of formulae:

Given $y'' = f(x, y, y')$ with $y = y_0, y' = y'_0$ at $x = x_0$

$$\text{Form } K_1 = f(x_0, y_0, y'_0)$$

$$K_2 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}y'_0 + \frac{h^2}{8}K_1, y'_0 + \frac{hK_1}{2}\right)$$

$$K_3 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}y'_0 + \frac{h^2}{8}K_1, y'_0 + \frac{hK_2}{2}\right)$$

$$K_4 = f\left(x_0 + h, y_0 + hy'_0 + \frac{h^2}{2}K_3, y'_0 + hK_3\right)$$

$$\Delta Y = hY'_0 + \frac{h^2}{6}(K_1 + K_2 + K_3)$$

$$\Delta Y' = \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

This set of formulae, besides being superior due to theoretical considerations, yields a great saving in computation in the case where Y' is absent from the function. In that case, as can be readily seen, $K_2 = K_3$.

The Case of m^{th} Order Equations for $m > 4$

Upon reducing an m^{th} order equation to a system of m first order equations and solving by the Runge-Kutta method, it was found that y was given in terms of a Taylor expansion up to h^4 . It was suggested that accuracy would be achieved up to h^m by using the Runge-Kutta method in modified form, and adding on terms of the Taylor series from data which was available. By this variation, the symmetry and simplicity would be sacrificed at the expense of gaining higher accuracy. In a machine this higher accuracy would pay off in the reduced number of steps it would take to cover an interval. If $\frac{1}{4}n$ were the number of steps needed using the regular Runge-Kutta method, $n^{\frac{m}{4}}$ steps would be needed using this variation.

This subject will be discussed more fully in a future Engineering note.

Signed: Philip Rabinowitz
Philip Rabinowitz

Project Whirlwind
Servomechanisms Laboratory
Massachusetts Institute of Technology
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SUBJECT: RUNGE-KUTTA METHOD APPLIED TO A SIMPLIFIED PROBLEM

To: Applications Study Group

From: T. W. Hildebrandt

Date: March 6, 1949

1. The problem, expressed in the notation of C-98, is the following:

$$\frac{dy_1}{dt} = y_1 + y_2 = f_1(y_1, y_2)$$

$$\frac{dy_2}{dt} = y_1 - y_2 = f_2(y_1, y_2)$$

Initial values $y_{10} = 0$

$$y_{20} = 1.$$

This problem is similar to the ballistic problem in the fact that t does not appear explicitly in f_1 and f_2 .

Since it is possible to find a closed solution, i.e. a solution expressed in terms of tabulated functions, for this problem by elementary methods, numerical methods would not ordinarily be used for its solution. However, we may find this an advantage in a demonstration problem, for it enables us to check our calculated results. In obtaining the closed solution, we differentiate each equation with respect to t , and, by substitution from the original equations, we obtain two second order linear differential equations, one of which contains only y_1 and its derivatives, and the other only y_2 and its derivatives. The initial conditions on

$\frac{dy_1}{dt}$, $\frac{dy_2}{dt}$ are obtained by substitution of the given initial conditions into original equations. We then have:

$$\frac{d^2 y_1}{dt^2} - 2y_1 = 0$$

and

$$\frac{d^2 y_2}{dt^2} - 2y_2 = 0$$

$$\frac{dy_{10}}{dt} = 1$$

$$\frac{dy_{20}}{dt} = -1$$

$$y_{10} = 0$$

$$y_{20} = 1$$

from which we easily obtain the solutions

$$y_1 = \frac{\sqrt{2}}{2} \sinh \sqrt{2}t, \quad y_2 = \cosh \sqrt{2}t - \frac{\sqrt{2}}{2} \sinh \sqrt{2}t.$$

We note that y_2 has a minimum value at about the time $t = .62$.

2. For the computation, we pick an interval $\Delta t = h = 0.1$. The equations for the K's of the Runge-Kutta process take the following forms (the index g , as in C-98, indicates the number of steps of the computation process which have been completed. Thus for $g = 0$, we have the initial values of y_1 & y_2 for $g = 1$, the values obtained from the first complete step of the process, etc. Four K's are calculated for each variable, for each step of the process. The first index on the K's identifies which of the four is mentioned, the second index on the K's identifies the variable to which it refers.):

For y_1

$$K_{11} = f_1(y_{1g}, y_{2g}) = y_{1g} + y_{2g}$$

$$K_{21} = f_1\left(y_{1g} + \frac{h}{2}K_{11}, y_{2g} + \frac{h}{2}K_{12}\right) \\ = (y_{1g} + 0.05K_{11}) + (y_{2g} + 0.05K_{12})$$

$$K_{31} = f_1\left(y_{1g} + \frac{h}{2}K_{21}, y_{2g} + \frac{h}{2}K_{22}\right) \\ = (y_{1g} + 0.05K_{21}) + (y_{2g} + 0.05K_{22})$$

$$K_{41} = f_1(y_{1g} + hK_{31}, y_{2g} + hK_{32}) \\ = (y_{1g} + 0.1K_{31}) + (y_{2g} + 0.1K_{32})$$

For y_2

$$K_{12} = f_2(y_{1g}, y_{2g}) = y_{1g} - y_{2g}$$

$$K_{22} = f_2\left(y_{1g} + \frac{h}{2}K_{11}, y_{2g} + \frac{h}{2}K_{12}\right) \\ = (y_{1g} + 0.05K_{11}) - (y_{2g} + 0.05K_{12})$$

$$K_{32} = f_2\left(y_{1g} + \frac{h}{2}K_{21}, y_{2g} + \frac{h}{2}K_{22}\right) \\ = (y_{1g} + 0.05K_{21}) - (y_{2g} + 0.05K_{22})$$

$$K_{42} = f_2(y_{1g} + hK_{31}, y_{2g} + hK_{32}) \\ = (y_{1g} + 0.1K_{31}) - (y_{2g} + 0.1K_{32})$$

Having found the four K's for each variable, we obtain the quantities

$$\Delta y_1 = \frac{h}{3} \left(\frac{K_{11} + K_{41}}{2} + K_{21} + K_{31} \right) = \frac{0.1}{3} \left(\frac{K_{11} + K_{41}}{2} + K_{21} + K_{31} \right)$$

$$\Delta y_2 = \frac{0.1}{3} \left(\frac{K_{12} + K_{42}}{2} + K_{22} + K_{32} \right)$$

and use them to find the new values for the variables:

$$y_{1,g+1} = y_{1g} + \Delta y_1$$

$$y_{2,g+1} = y_{2g} + \Delta y_2$$

These will be used as the initial values for the next step of the computation.

3. The computation is set up in a tabular form on the next page, each row in the table corresponding to one complete step of the computation. The columns of the table are explained on that page.

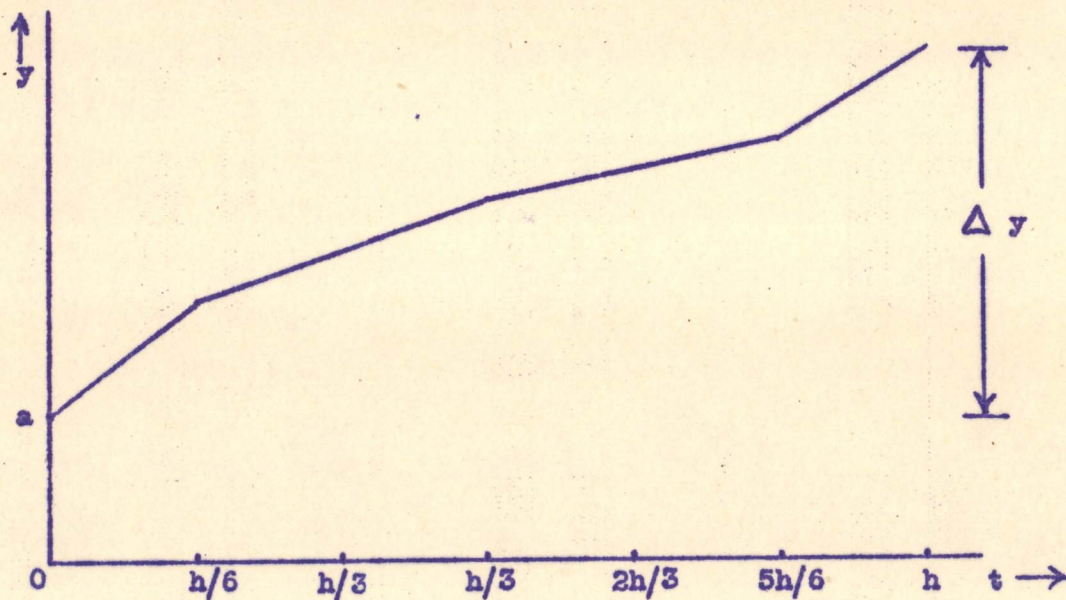
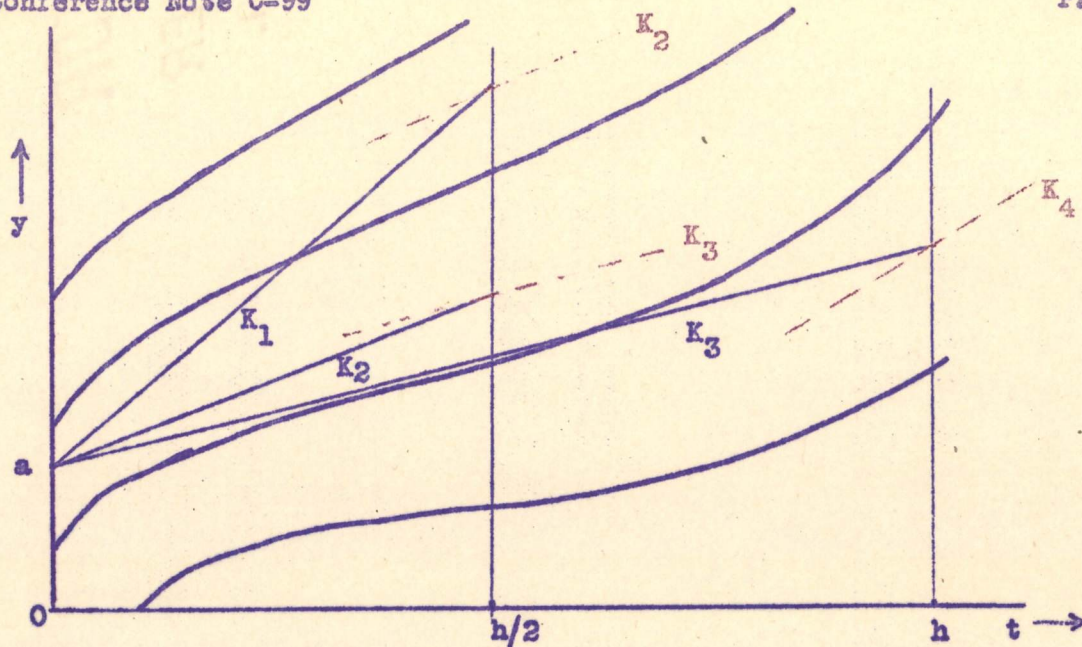
Following through the computation in the first row, starting with the initial values of y_1 and y_2 , will be instructive. We first form $K_{11} = y_{10} + y_{20} = 1$, and $K_{12} = y_{10} - y_{20} = -1$, in columns 4 & 5. Then find new $y_1 = y_{10} + 0.05 K_{11} = 0.05$ and $y_2 = y_{20} + 0.05 K_{12} = 0.95$, entered in columns-6 and 7, and use these values to compute K_{21} and K_{22} . We proceed in this manner until we have computed K_{41} and K_{42} , and then form Δy_1 and Δy_2 from the four K 's computed for each. Adding Δy_1 and Δy_2 to y_{10} and y_{20} , respectively, we find y_{11} and y_{21} , which are the values for $t = 0.1$.

The computation is carried out to the point where y_2 shows that the expected minimum has been passed. We notice that by the computation, the minimum occurs between $t = 0.6$ and 0.7 , which is in reasonable agreement with $t = 0.62$ for the minimum calculated from the closed solution above.

4. Graphical interpretation of the Runge-Kutta method.

Consider, in this case, a single differential equation of the form $\frac{dy}{dt} = f(y,t)$, $y(0) = a$. The function $f(y,t)$ associates with every point in the (y,t) plane a direction or slope $\frac{dy}{dt}$. In particular, if we start from any point in the plane and proceed in the direction indicated by $f(y,t)$ for that point, an infinitesimal distance along a straight line to a new point, and from that point proceed in the same manner, we generate a curve. Such a curve is called an integral curve, and the whole (y,t) plane is covered by integral curves corresponding to the function $f(y,t)$. In particular, one of these integral curves will be a solution for our differential equation. What the Runge-Kutta process does is to produce an approximation to the desired integral curve, by a series of straight line segments of finite length. Suppose, then, that we have a family of integral curves in the (y,t) plane. We know that the slope demanded by the differential equation at each point is the same as the slope of the integral curve passing through that point (since we cannot draw all of the members of the family, we interpolate to find the slope at points not on the curves we have drawn). The process is as follows:

(continued on page 5)



starting from the point $(0, a)$ we draw a line segment having the slope K_1 required at that point to some point on the line $t = \frac{h}{2}$. At that point we find a new slope K_2 , and using that slope, we again take a line segment from $(0, a)$, and find the new slope K_3 at the point where it intersects the line $t = \frac{h}{2}$. We again return to point $(0, a)$ and draw a line segment with slope K_3 to a point where it intersects the line $t = h$, and at that point find the slope K_4 .

The process of finding the weighted average of these slopes can also be indicated graphically. We have

$$\Delta y = \frac{h}{3} \left(\frac{K_1 + K_4}{2} + K_2 + K_3 \right) = \frac{h}{6} K_1 + \frac{h}{3} K_2 + \frac{h}{3} K_3 + \frac{h}{6} K_4$$

$$y_1 = y_0 + \Delta y$$

We see that to obtain y_1 , we start from $y_0 = a$ and draw a line segment of slope K_1 to a point on the line $t = \frac{h}{6}$, from that point a line segment of slope K_2 from $t = \frac{h}{6}$ to $t = \frac{h}{2}$, then a line segment of slope K_3 from $t = \frac{h}{2}$ to $t = \frac{5h}{6}$, and finally a line segment of slope K_4 from $t = \frac{5h}{6}$ to $t = h$. The last point is our desired point y_1 . Obviously, the order in which we take the segments is immaterial, as long as they are of the same length and direction.

5. The following exercise is suggested:

Find several points of the solution of the following differential equation:

$$\frac{dy}{dt} = \cos y; y_0 = 1, \text{ using the Runge-Kutta method with } h = 0.1.$$

Signed T. W. Hildebrandt
T. W. Hildebrandt

Approved W. Gordon Welchman
W. Gordon Welchman

TWH: lfu

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

SAGE & LINCOLN LABORATORY

A GUIDE FOR SPEAKERS

FEBRUARY 1956

DO-158

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LEXINGTON

MASSACHUSETTS

SAGE & LINCOLN LABORATORY

A GUIDE FOR SPEAKERS

This guide is designed to be of assistance to MIT and Lincoln representatives who may be asked to give unclassified talks on the SAGE System, with particular reference to an introductory briefing and follow-up question period in connection with a showing of the SAGE movie.

The contents of the guide include:

1. A general statement concerning security regulations and classified information.
2. A news release prepared by Lincoln Laboratory and issued at the SAGE press conference, 16 January 1956.
3. Background information on Lincoln Laboratory.
4. Background information on the Air Force Cambridge Research Center.
5. The script of the SAGE movie.
6. Typical questions concerning SAGE, and answers that may be given within security limits.

COMPLIANCE WITH SECURITY REGULATIONS

Speakers will wish to comply with security regulations and, where doubt exists, to err on the side of security. In order that discussions of SAGE may be well received, speakers should not unnecessarily restrict their remarks. To avoid this circumstance, speakers will wish to be familiar with the SAGE declassification limits and be able to distinguish between classified and unclassified areas.

While the declassification limits may be gray bands of varying widths rather than sharp lines of demarcation, there are nevertheless several general statements that can serve as guideposts. These are:

1. The SAGE movie (not the classified version called Counterbalance) is unclassified. The pictures and sound script of the movie represent the declassified limits of SAGE. Thus, any feature of SAGE not included in the movie is probably classified and should not be mentioned or discussed. Also not included in the declassification are explanations or further clarifications of some of the mechanisms, systems or methods that make possible the results shown in the movie. An example of this latter might be the array of push-buttons on the panel of the display scope. This array can be seen in the movie and, in itself, is unclassified. The movie does not show the markings on the individual push-buttons nor are their specific functions explained. This information is therefore classified.
2. Information concerning any capabilities or capacities of SAGE in terms of numbers, precise functions, or any other specificities including geographical locations, time schedules, and possible operating problems is classified and is not to be stated, implied, or indicated in any way.
3. All the printed material contained in this Guide has been cleared by either the Office of Security Review of the Department of Defense or the Lincoln Project Office and may be quoted as desired.

NEWS RELEASE

DO-158

From Office of the Director
M.I.T. Lincoln Laboratory
Lexington 73, Massachusetts

For release in MORNING papers
of Wednesday, 18 January 1956

The first opportunity to learn about the recently developed electronic Semi-Automatic Ground Environment (SAGE) System for continental air defense and its potential was given the nation's press at a news conference on Monday (January 16) at M.I.T.'s Lincoln Laboratory in Lexington, Massachusetts.

Developed by Lincoln scientists and engineers under contract with the U.S. Air Force, SAGE revolutionizes the air defense methods and equipment involved in the defense of America. The SAGE System makes available to defense personnel comprehensive and detailed information about enemy air attacks as soon as the outer defense ring is breached. The contract is administered by the Air Research and Development Command through the Air Force Cambridge Research Center.

SAGE combines the abilities of the electronic computer to receive information, to memorize, to calculate and to record answers with the perceptive and display talents of radar to present an instantaneous graphic picture of the location, speed and direction of all planes within radar range.

With a knowledge of flight plans of friendly planes available in the computer, hostile planes can be identified immediately and the most effective defense action taken -- again on the basis of computer information and instruction.

M.I.T. Lincoln Laboratory was organized in 1951 at the joint request of the Army, Navy and Air Force following announcement of Soviet possession of the atomic bomb and long-range bombers.

Its primary purpose is to launch an all-out technological attack on some of the new problems of air defense. Although supported by the three armed services, Lincoln's prime contract is with the Air Force.

Director of Lincoln is Dr. Marshall G. Holloway, an MIT professor and eminent nuclear scientist who came to Lincoln last May to succeed Dr. A. G. Hill, who returned to his work as MIT Professor of Physics. Dr. Holloway came from Los Alamos Scientific Laboratory where he made important contributions to the development of atomic weapons during his twelve years of service.

Dr. George E. Valley, Jr., Associate Professor of Physics at MIT and a specialist in nuclear physics and cosmic radiation, is Associate Director of Lincoln. Dr. Valley was an early advocate of improved air defenses. In 1949 he was asked by the Air Force to form a committee to study and make recommendations on the existing air defense system.

Lincoln Laboratory is managed for MIT by faculty members with organizational liaison to the Institute through Admiral Edward L. Cochrane (Ret.), Vice-President for Industrial and Government Relations.

While Lincoln has been successful in rapid enrollment of top-level people, there is still an unmet need for highly qualified scientists and engineers. Of special interest to outstanding technical graduates is the opportunity to combine professional work at Lincoln with a program of concurrent graduate study at MIT.

The Western Electric Company, International Business Machines Corporation, Bell Telephone Laboratories, Burroughs Corporation Research Center and the RAND Corporation have been associated with Lincoln in the SAGE program.

Within the broad scope of air defense problems Lincoln's program is constantly changing. One of its first assignments was assistance in the design and development of equipment for the Distant Early Warning (DEW) line of radars and communication circuits across the northern part of the continent. The largest current project is SAGE.

Another Lincoln project has been the development of the "ionospheric" and "tropospheric" scatter systems of long-range radio communications. These systems provide ultra-high frequency, beyond-the-horizon radio communication of extreme reliability. Other areas of work include radar development and Airborne Early Warning (AEW) systems in planes.

Close relationships at the policy level between Lincoln management and the armed services are maintained by a Joint Services Advisory Committee with top level representation from each service. Lt. General D. L. Putt, USAF, is chairman of this committee which includes as other members Colonel J. K. Johnson, USAF; Brig. General J. P. Daley, USA; Brig. General F. F. Uhrhane, USA; Rear Admiral R. Bennett II, USN; and Captain E. H. Eckelman, Jr., USN.

Air Defense of America involves a variety of high performing defensive weapons; such as, interceptor planes, guided missiles and anti-aircraft guns. The most effective use of these weapons is one of the basic problems of air defense. Study of this problem by Lincoln and Air Force scientists and engineers has produced the SAGE System.

The SAGE System starts with a radar ring -- on land, on Navy picket ships at sea, on Texas Towers and on Airborne Early Warning planes. These radars are linked by telephone lines or ultra-high frequency radio directly to a high speed digital computer. Information about aircraft anywhere within the radar area is relayed continuously and automatically to the computer. Other data supplied to the computer is from Ground Observer Corps, height-finding stations, flight plans and weather stations.

The computer digests all information as fast as it is received and translates it into an over-all picture of the air situation. These automatic T. V. -like pictures show the air-battle as it develops and provide the basis for the necessary human judgments.

The computer automatically calculates the best application of defense weapons. It guides interceptor planes and long-range missiles to targets automatically by radio connections. As the air battle moves, information is transferred spontaneously to an adjacent computer.

To test the SAGE System, Lincoln and the Air Force built in eastern Massachusetts an experimental test network known as the Cape Cod System. Radars were erected at strategic locations and linked to a computer in MIT's Barta Building.

At Hanscom Air Base the Air Force Cambridge Research Center set up a special test support wing to operate planes used in evaluating the System. Additional test flight facilities were provided by the Naval Air Development Unit at South Weymouth, Massachusetts.

A Direction Center -- the heart of each SAGE operational unit -- was established next to the computer. Here the operational features of the System were tested. The functions of detecting aircraft, identifying them, plotting and predicting their courses are done electrically and automatically. Once weapons are committed the System directs them to their targets with a minimum of human intervention.

A staff of nearly 700 university and industrial scientists and engineers from 46 states have enlisted at Lincoln. These professional men and women profit from the advantages of program management by scientists and the freedom and objectivity of an established scientific community with superior academic standing. About 1200 people provide technical and clerical support to these scientists.

Support and close working liaison between Lincoln and the Air Force is provided by resident units of the Air Research and Development Command and the Air Defense Command.

The Air Force Cambridge Research Center under Maj. General Raymond C. Maude and located at Hanscom Field in Bedford provides Lincoln with technical liaison, contract administration and Air Force logistical support including that required for system-wide testing. AFCRC is one of nine major research centers scattered throughout the United States and operated by the Air Research and Development Command. Lt. Colonel R. S. LaMontagne is in charge of the Lincoln Project Office and Captain H. E. Spangler is Contract Administration Officer.

The 4620th Air Defense Wing under the command of Colonel J. D. Lee maintains liaison with Lincoln for the Air Defense Command and provides Lincoln with operational requirements.

Liaison Officers from the Army include Colonel W. L. McNamee (CONARC) and Major O. K. Gardner (Signal Corps), and from the Navy, Captain E. Tatom and Commander W. C. Hilgedick are the Representatives.

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LINCOLN BACKGROUND

BACKGROUND MATERIAL ON M. I. T. LINCOLN LABORATORY

M. I. T. Lincoln Laboratory was organized in 1951 in Cambridge, Massachusetts, by the Massachusetts Institute of Technology at the joint request of the U. S. Army, Navy, and Air Force for the primary purpose of mounting an all-out technological attack on some of the problems of continental air defense. The Laboratory is a tri-partite organization jointly supported by the three Armed Services. The prime contract is with the Air Force through the Air Research and Development Command.

The decision to establish Lincoln Laboratory resulted from and followed closely the report and recommendations of a study of the effectiveness of U. S. defense against air attack, by a special Air Force appointed study group known as the Air Defense Systems Engineering Committee (ADSEC).

Soviet possession of nuclear weapons and the ability to deliver them to the North American continent rendered the U. S. vulnerable to large-scale air attack for the first time in history. ADSEC's reassessment of U. S. air defense potential indicated an urgent need for more effective defense.

The ADSEC proposal was given further study by a group formed under the name "Project Charles" headed by Dr. F. Wheeler Loomis, on leave from the University of Illinois, where he is now chairman of the Physics Department. Dr. Loomis became the first director of Lincoln Laboratory and was succeeded after two years by Dr. Albert G. Hill, M. I. T. Professor of physics. Dr. Hill resigned as director last spring.

Several cogent reasons influenced the decision to enlist the cooperation and support of a private university in the creation of necessary laboratory facilities rather than to attempt the establishment of a new laboratory within the military structure in times of nominal peace. The performance record of such laboratories during

World War II was impressive. Distinct advantages would accrue from direction of the laboratory and its programs by scientists. The freedom and objectivity of an established scientific community with academic standing would enhance the effectiveness of the effort. The possibility of providing top level scientists with greater tangible and intangible satisfactions would be increased.

The initial nucleus of Lincoln personnel was composed of scientists and technicians already engaged in related work. From the M. I. T. Research Laboratory of Electronics came groups working on long-range radio communication, radar, and solid state physics; from the M. I. T. Digital Computer Laboratory came the Whirlwind I computer groups working on real time control applications of computers, random access magnetic storage, and computer design; from various departments of M. I. T. came specialists in mathematics, physics, servomechanisms, and mechanical engineering; from outside M. I. T. were brought scientists experienced in pulse-coded data transmission and other pertinent technological and scientific areas.

To meet its responsibilities in researching and developing better methods and better equipment for the air defense of the United States, Lincoln Laboratory has been faced with continuous growth problems -- the need for people and the need for space. Originally housed in small, scattered quarters in Cambridge, the Laboratory moved in 1954 to a series of fine laboratory buildings with 300,000 feet of floor space in Lexington, Massachusetts, specially constructed by the Air Force for Air Defense research and development work.

Lincoln's personnel complement has likewise expanded. A staff of nearly 700 university and industrial scientists and engineers from 46 states have enlisted at Lincoln. These highly trained men and women receive technical and clerical support from 1200 non-staff people.

While Lincoln has had a high degree of success in enrolling the right kinds and numbers of people to prosecute its mission, there is still an urgent unmet need for highly qualified scientists and engineers. Of special interest to outstanding technical graduates is the opportunity to combine professional work as Staff Associates at Lincoln with a program of concurrent graduate study at M. I. T.

Organization and Management

Lincoln Laboratory is managed by members of M. I. T.'s faculty. Liaison to the Institute is maintained through Admiral Edward L. Cochrane (Ret.) Vice-President for Industrial and Government Relations. Close relationships with the armed services at both the policy and working levels are maintained by a Joint Services Advisory Committee and by permanent representation at Lincoln of each service.

Dr. Marshall G. Holloway, an eminent nuclear scientist with a record of important contributions to the development of atomic weapons during twelve years of service at the Los Alamos Scientific Laboratory, is Director of the Laboratory.

Dr. George E. Valley, Jr., Associate Professor of Physics at M. I. T. and a specialist in nuclear physics and cosmic radiation, is Associate Director of the Laboratory.

Heads of research divisions include Dr. Carl F. J. Overhage; Dr. William H. Radford, Professor of Electrical Engineering at M. I. T.; Mr. Jerome Freedman, Dr. Stanley N. VanVoorhis; and Dr. J. W. Forrester.

A Joint Services Advisory Committee with representation from each service meets regularly with the Lincoln management. The current members of this committee are: Lt. General D. L. Putt, USAF, Chairman; Colonel J. K. Johnson, USAF, Recorder; Brig. General J. P. Daley, USA; Brig. General F. F. Uhrhane, USA; Rear Admiral R. Bennett, II, USN and Captain E. H. Eckelman, Jr., USN.

Support and close working liaison between Lincoln and the Air Force is provided by resident units of the Air Research and Development Command and the Air Defense Command. The Air Force Cambridge Research Center, one of nine major research centers scattered throughout the United States and operated by the Air Research and Development Command, provides technical liaison, contract administration, and Air Force logistical support including that required for system-wide testing. Lt. Colonel R. S. LaMontagne is in charge of the AFCRC Lincoln Project Office, and Captain H. E. Spangler is Contract Administration Officer.

The 4620th Air Defense Wing under the command of Colonel J. D. Lee maintains liaison with Lincoln for the Air Defense Command and provides Lincoln with ADC operational requirements.

Army Liaison Officers include Colonel W. L. McNamee (CONARC) and Major O. K. Gardner (Signal Corps).

Navy representatives at Lincoln include Captain E. Tatom and Commander W. C. Hilgedick.

The Program

The largest current project is research and development of the Semi-Automatic Ground Environment (SAGE) System of air defense. The SAGE System is a network of digital-computer-equipped direction centers with interconnecting communications for processing aircraft radar echoes and other information, and generating battle orders to defense weapons.

In addition to specific projects such as SAGE, one of the significant areas of work at Lincoln is radar research and development for land, sea, and airborne uses. Studies are being made to improve the transmitting, receiving and display components. Increasing useful range and improving operation under unfavorable weather conditions are other radar research objectives.

One of the first major projects at Lincoln grew out of a survey conducted in 1952 by the Summer Study Group, headed by Dr. Jerrold R. Zacharias, Professor of physics at M.I.T. This study resulted in a program for development of equipment for the Distant Early Warning (DEW) line of radars and communication circuits across the northern part of the continent. This project is now in the installation stage.

Development of the ionospheric and tropospheric scatter systems of long-range radio communications, following fundamental work in physics and engineering, is another Lincoln project. These systems provide ultra-high frequency beyond-the-horizon radio communication of extremely high reliability.

A general listing of specific kinds of research and development at Lincoln would include digital data transmission, solid-state physics including both transistors and magnetic ferrites for digital computer and radar applications, improved ground and airborne radars, long-range radio communications, anti-aircraft weapons systems, theory of sample data servo systems, psychological research on training and operator relationships to equipment and systems analysis, simulation, and evaluation.

Laboratory Facilities

Facilities at Lincoln and its affiliated M.I.T. Barta Building in Cambridge include a semiconductor physics laboratory, physical chemistry laboratory, metallurgical and ferrite laboratories, low temperature research facilities, a mechanical engineering group specializing in structures, electro-mechanical design and heat transfer, vacuum tube construction shop used for special cathode ray display tubes, microwave research facilities, three large-scale digital computers, extensive shops, drafting rooms, and a photographic laboratory.

Further facilities located in various parts of New England and the United States include more than 20 field stations for testing and development work, an experimental radar and communications network under M. I. T. operational control covering eastern Massachusetts for air defense tests, and experimental test aircraft support from the Air Force and Navy.

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AFCRC BACKGROUND

BACKGROUND ON THE AIR FORCE CAMBRIDGE RESEARCH CENTER

The Air Force Cambridge Research Center is one of ten special centers assigned to the Air Research and Development Command to carry out its mission of attaining and maintaining qualitative superiority in equipment for the United States Air Force. The Center is under the command of Major General R. C. Maude.

For the past two years, AFCRC has gradually centralized its activities at Laurence G. Hanscom Field, Bedford, Massachusetts. Although several highly specialized laboratories will be retained in other areas, the main research efforts of the Center will be conducted in modern buildings located less than half a mile from the Lincoln Laboratory.

The mission of the Center is two-fold: First, through its three major directorates, it accomplishes research, development and test in the fields of electronics, geophysics and human factors. The three components – Electronics Research Directorate, Geophysics Research Directorate, and Operational Applications Laboratory – carry out their missions within their own laboratories, or through work done on contract by universities, research foundations, and industry. Secondly, the Center provides for administration and support of the Lincoln Laboratory, operated under contract by the Massachusetts Institute of Technology. The Lincoln contract is administered through the Lincoln Project Office, manned by personnel of AFCRC, but located physically in the Lincoln Laboratory research complex. Lt. Colonel Ralph S. LaMontagne, USAF, heads the Lincoln Project Office staff.

The bulk of the Center's support to Lincoln (including materiel, flight test, and the manning of an experimental air defense network) is provided by the 6520th Test Group (Support). Using the flight facilities at expanding Hanscom Field, planes of many types assigned to the 6520th Test Group (Support) fly missions in support of the Lincoln Laboratory.

AFCRC, with approximately 4000 persons assigned to its various directorates and offices started in 1945 as the Cambridge Field Station of the Watson Laboratories. In 1949, it was transferred to the jurisdiction of the Air Materiel Command and re-named the Air Force Cambridge Research Laboratories. In April, 1951, the Air Research and Development Command assumed jurisdiction over the station, which was then primarily located in old and unsatisfactory quarters in Cambridge and Boston.

In May, 1952, having assumed its current name, the Center took command of all Air Force facilities at Laurence G. Hanscom Field in Bedford.

Growth of the Center, and especially of its facilities at Hanscom Field, has been rapid in the past few years. As recently as 1953, facilities at the field were of World War II vintage, old, unsightly, and unsatisfactory. Since a stepped-up program of construction started early in 1954, great changes have been made at the field, and today a modern new installation, costing approximately sixty million dollars, is emerging. With the Lincoln Laboratory, the Raytheon Laboratory, the buildings of the Commonwealth of Massachusetts, the recently added 49th Fighter Interceptor Squadron of the Air Defense Command, the 2234th Air Reserve Training Center, and several other smaller components, Hanscom Field is now a vital defense installation.

SAGE SCRIPT

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

WHAT IS THE SEMI-AUTOMATIC GROUND ENVIRONMENT (SAGE) SYSTEM?

(SCRIPT OF SAGE MOVIE)

Made available to members of the press for background information at SAGE Press Conference - Monday, 16 Jan 1956.

The aerial defense of America involves a variety of high performing defensive weapons like interceptors, ground-to-air guided missiles, and anti-aircraft guns. The assignment and control of these weapons is one of the basic problems of air defense. Continued study and attention to this problem has produced a new system of air defense known as the Air Force SAGE System which provides an effective "counter-balance" to the growing destructive might arrayed against us.

To understand why the Air Force has adopted this new system, let us examine the basic tasks of Air Defense. Let us look at the Aircraft Control and Warning Squadron as it evolved from World War II where men are responsible for organizing the defense of their area. How effectively they do their job depends largely on how rapidly they can process information from a variety of sources. They must find aircraft on radar scopes and follow them under all conditions. They must identify these aircraft by checking their courses against known flight plans and other identity information. They must guide airborne interceptors into attack positions. Whenever the action moves into the vision of other radars they have only a few moments to call adjacent squadrons and transfer the air battle to them. They must translate words to visual images on plotting boards. Even in a mass aerial attack each target requires manual handling of all the details associated with it.

The speed and accuracy with which men can handle these details set a natural limitation to the effectiveness of such a system. How to overcome this limitation and strengthen our air defenses was one of the most perplexing problems ever faced by those responsible for America's security. To find answers to these problems the Massachusetts Institute of Technology was asked by the three Military Services to establish a new research center known as the Lincoln Laboratory. Here, advancing electronics technology would be applied to all phases of air defense.

The initial work of this new laboratory focused attention on the rapidly developing field of electronic computers, and in particular upon Whirlwind I, the Navy-financed all-purpose digital computer, one of the highest performing computers in existence at that time. This computer very quickly demonstrated its ability to process large quantities of all kinds of air defense data, and to do this with extreme rapidity. In the application of high speed computers to air defense problems came the prospect of hurdling what had long been a barrier to any appreciable growth in our air defense capability, the barrier of information processing. The computer opened the way for more centralized air defense data handling around which the Air Force SAGE System is built.

Before the introduction of computers, the basic building block of air defense was limited to the coverage of a single radar. The SAGE System enlarges this building block by bringing the areas of several radars under the control of a single operations center. These radars are linked by telephone lines or UHF radio directly to a high speed digital computer. The locations of aircraft anywhere within this entire area are relayed continuously and automatically from the radars. But the information from these ground-based radars is only one piece of data fed into the computer. Many other sources also supply information to the computer. These include Height Finding, Texas Towers, picket ships, AEW planes, Ground Observer Corps, flight plans and weather.

The computer digests all this information and translates it into a composite picture of the complete air situation. It generates scope displays to show this air situation as it develops and to provide the basis for the human judgments involved in tactical decisions. The computer automatically calculates for the operators the most effective application of such weapons as interceptors, anti-aircraft, Nike and other missiles. Through radio data link, all-weather interceptors and long-range missiles are guided to targets automatically by the computer. As the air battle moves out of the area served by one computer, all information pertaining to each aircraft is transferred automatically to the computer of the adjacent area.

To test this centralized data-processing system the Lincoln Laboratory and the Air Force built in eastern Massachusetts an experimental test network known as the Cape Cod System. A long range radar on Cape Cod was linked to Whirlwind I. Smaller radars for detecting low-flying aircraft were located at strategic positions and also linked to the computer. At Hanscom Air Base near the Lincoln Laboratory, a special test support wing was set up by the Air Force to operate aircraft used in evaluating the system. Additional facilities were provided by the Naval Air Development Unit at South Weymouth, Massachusetts, which supplies the Naval aircraft that participates in Lincoln's test program. A Direction Center was established next to Whirlwind I in M.I.T.'s Barta Building in Cambridge. Here the system was tested as an operational unit. The functions of detecting aircraft, identifying them, plotting and predicting their courses are all done electrically and automatically. Once weapons are committed the system directs them to their targets with a minimum of human intervention. The Cape Cod test network bridged the gap between conception and practice.

Not only was it necessary to evaluate the system as an operational unit, but the special equipment which the SAGE System required was also developed. Whirlwind I, for example, pointed the way to a new computer with greatly increased speed and capacity, and specifically designed for air defense application.

Meanwhile, at the various Cape Cod radar sites, additional important equipment was evolving. Radar designs were modified and special items of equipment developed for the automatic transmission of radar data. These sites provided a proving ground for new developments. Airmen technicians worked with Lincoln engineers in conducting tests on new developments, keeping records of system performance, and installing new experimental equipment sent to the sites for field use and evaluation.

With the successful demonstration of the Cape Cod network, the Air Force was ready to begin a far-reaching revision of our air defenses. The Western Electric Company, under Air Force contract, was brought in to engineer and supervise the installation of the SAGE System. This is a task which covers the construction program, and all planning, scheduling and procurement activities involved in bringing the new continental Air Defense network into being.

The International Business Machines Corporation, also under Air Force contract, put into production the first computer specifically designed for Air Defense. These computers, which the Air Force calls the FSQ-7 data processing equipment, are among the largest and most reliable electronic computers yet built. The many thousands of individual elements going into these computers are manufactured and assembled using the latest techniques of the electronics industry.

To supply data to the computer, air defense radars already existing in the field are used. But these are supplemented by unattended low-altitude radars, by offshore radars on platforms known as Texas Towers, by picket ships, farther out to sea, and by round-the-clock patrol of early warning aircraft.

The heart of each operational unit is the direction center - a windowless, reinforced concrete building - which houses a dual channel computer. Only one channel however furnishes data at any given time. The other operates on a stand-by basis. While one channel is working, the other is also receiving data and can take over the full air defense load in a matter of seconds.

To show how these direction centers function, let us observe the operations at the more important stations in a center. For purposes of illustration, we will show simplified examples of the scope information available to the operators and demonstrate the manner in which this information is used by those who direct our defenses. Let us imagine that three B-29's are enemy bombers approaching the east coast of the United States. They might be a part of a mass attack, and one involving more advanced, high speed aircraft. But we will follow only the activities in the direction center concerned with these three bombers.

Before entering our detection network, the bomber formation splits into individual elements. Within seconds after being picked up by our radars, the courses of the enemy bombers are plotted by the computer, which also shows that these aircraft cannot be identified as friendlies. On the basis of this information an Identification Officer declares them hostile and selects a Weapons Assigner to organize the attack against them. The computer gives the Weapons Assigner complete information on the hostiles and the defensive weapons he has available. The Weapons Assigner controls several Intercept Director teams who in turn can be assigned responsibility for individual interceptions. Thus, even a mass raid can be broken down into easily manageable parts.

To understand the completeness of the information available to the Weapons Assigner for making tactical decisions, let us take a close look at his scope. At the extreme right, the three bombers appear, designated by the computer as tracks 12, 13, and 14. The computer graphically shows the course and speed of each bomber by the direction and length of the line accompanying each track number. The Weapons Assigner's scope also shows certain selected geographical references: the ADIZ line, the coverage of available anti-aircraft fire, indicated by the circle, and two airfields, Westerly and Oberlin, indicated by the two X's. The small squares enclosing the letters O and W tell the Weapons Assigner the points at which fighters scrambled from either Oberlin and Westerly will intercept the hostiles. Beneath these squares, the time to interception in minutes is shown. Vector lines from the two air bases indicate the initial heading to be assumed by any fighters scrambled from these bases.

With this information, which normally is presented less than a minute after enemy aircraft are first detected, the Weapons Assigner makes his decision to scramble interceptors. The Oberlin Intercept Director, assigned Hostiles Two and Three, scrambles four F-94's. Meanwhile the Westerly Intercept Director, assigned Hostile One, scrambles four F-86's. The interceptors

speed toward their assigned targets. The Oberlin Intercept Director notes position and heading of his four aircraft, plus the two hostiles they are to intercept. The computer has given track numbers to the two pairs of F-94 interceptors. The pair assigned to Hostile Two is Track 70, and the pair assigned to Hostile Three is Track 71. The Westerly Intercept Director has a similar scope presentation covering the action on Hostile One for which he is responsible. Here, the F-86 interceptors have been assigned track numbers 60 and 61. Actually, the job of the Intercept Director is largely one of monitoring action since the mid-course flight of the interceptors is under the control of computer-generated directions transmitted by radio to the automatic pilot in the lead aircraft.

The Direction Center Commander and his assistants analyse these attacks in relation to other threats that may be developing in his area. He can call upon the computer to display the overall air situation or whatever portion of it he wishes to examine in detail. He sees the F-94's move toward Hostiles Two and Three. The 94's will make individual attacks. Consequently, two additional interception points for Hostiles Two and Three are established by the computer, and a new track number is assigned to each interceptor which peeled off. The 94's close in on the invader. As the fighters reach the final phase of action airborne equipment takes over from the computer. The 94 picks up the Hostile. Hostile No. Three is now in the pilot's scope. In this manner, one of the F-86's also intercepts and destroys Hostile No. One. Another F-94 intercepts and destroys Hostile No. Two.

Back in the Direction Center, the Westerly Intercept Director watches while the computer, still attending to the multiple details, guides all Westerly aircraft back to base. Likewise, the Oberlin intercept team need only monitor as the computer directs the return of all Oberlin aircraft. The high-performing, large capacity

air defense computers together with the radars and other individual components of the SAGE System represent one of the farthest advances yet made in electronics technology. Electronics is helping reduce to a minimum the human effort required for rapid assimilation and processing of information, a basic demand of modern air defense. The SAGE System fulfills this demand but it also fulfills a demand of greater significance to our nation's security. In releasing men for jobs they can do best, machines are helping to overcome what has long been an imbalance between the increasing "destruction potential" directed against America and our ability to counter this potential. In achieving this lies the promise of "Counterbalance" and this is the ultimate goal of SAGE - our new Air Defense System.

QUESTIONS AND ANSWERS

QUESTIONS AND ANSWERS

Status

- Q. What is the current status of SAGE? Are any direction centers now operating? When will the first subsector be placed in operation? What is the time schedule?
- A. SAGE has been accepted by the Air Force to be a basic component of the continental air defense program. At the present time it is in operation as an experimental unit, undergoing final evaluation tests. The urgency of the situation requires that final testing, production of equipment, operator training and installation be carried on simultaneously. The specific time schedule, both for the system as a whole and any area, is classified information.

Cost

- Q. What is the cost of SAGE?
- A. Lincoln's responsibility does not include determination of capital or operating costs. However, it has been estimated that the capital costs of the ground environment for the system will exceed one-billion dollars. It has been estimated that the annual recurring charges to provide the leased facilities for SAGE will total somewhere between \$160 million and \$240 million.

Operation

- Q. How large an area is served by a single installation?
- A. Each installation or direction center serves an area designated as a subsector. There will be 32 subsectors in the continental United States. The exact size and location of each subsector are influenced by many considerations and are classified information.

- Q. Does SAGE reduce the personnel requirement for air warning and defense?
- A. No. The personnel requirement may be increased by a small percentage. The primary objective of SAGE is to provide an improved system of air defense. By automatizing such routine tasks as observing, recording, calculating, and transmitting information, personnel are relieved of tiring and boring routines and are freed to handle more effectively the tasks requiring intelligence and judgment.
- Q. What is the maximum distance at which planes can be detected? How much warning time is available in the event of enemy attack?
- A. The answer to this question involves two classified areas - the distance potential of the radars and the deployment of the defensive network.
- Q. In the movie, each interceptor succeeded in its mission. What procedure is followed when an interceptor fails?
- A. The Intercept Director, immediately aware of the failure, continues to direct the air battle, using whatever defense weapons may be indicated at that time.
- Q. How does SAGE perform its identification functions and how great is the possibility of error?
- A. When an unidentified plane enters the detection network, its course is displayed by the computer together with data concerning "friendlies" known to be in the area. On the basis of this information, and with the aid of other classified means and procedures, identification is made.
- Q. What methods are used to communicate instructions to the computer?
- A. Depending upon circumstances and conditions, the most common methods are by punched cards, magnetic tape, push-buttons, and a "light gun."

General

Q. Will SAGE be integrated with or available to Canada?

A. Montreal Star 18 January 1956 – According to a spokesman from the Royal Canadian Air Force – "The RCAF has been kept in the picture by the US Air Force with regard to the development of SAGE and continental discussions with the USAF have been carried out in this connection."

"The question of whether SAGE or some other type of air defense equipment will be introduced into the air defense system in Canada and if so, when, is a matter for future decision, and one to which the RCAF cannot give an answer at this time. Any such equipment would be compatible with the USAF equipment to insure the continental coordination of the Canadian and American parts of the North American air defense systems."

Q. Do potential enemy countries have an air defense system similar to SAGE?

A. The answer to this question is not known to Lincoln scientists. It is presumed that an enemy country choosing to devote like efforts in this area could do as well as in any other area in which it tries to equal or exceed our accomplishments.

Q. Can SAGE be used to control commercial air traffic?

A. Yes – from a technical point of view. Other considerations might or might not make such control advisable.

Q. Have other organizations participated in the development of SAGE?

A. Yes – Many industrial and research organizations have cooperated with Lincoln Laboratory in this effort, either as prime contractors or as subcontractors. Important contributions have been made by the

International Business Machines Corporation, the Burroughs Adding Machine Company, the Western Electric Company, Bell Telephone Laboratories, the RAND Corporation, Bendix Radio, and Hazeltine Corporation, to mention a few. The close and effective cooperation and working relationships between Lincoln and the armed services and, in particular, the Air Force which has the major service responsibility, has been a basic factor in the successful prosecution of this project.

Vulnerability

- Q. What provisions are made against system failure?
- A. With duplex computer installation in each direction center, a second computer is always available to carry the defense load. A further safeguard is back-up by a manual system. In the event a direction center is destroyed or cannot function for any reason, the defense load will be carried by adjacent centers. Evaluation tests have indicated ample reliability of all electronic and mechanical parts.
- Q. Is SAGE jam-proof?
- A. Any question concerning vulnerability including possibility of jamming cannot be answered for security reasons.
- Q. Can aircraft avoid detection by flying at low altitudes?
- A. The SAGE radar network includes small radars, strategically located to detect low-flying aircraft.

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Digital Equipment Corporation, Maynard, Massachusetts, is offering the first commercial version of LINC, the digital computer designed at the Massachusetts Institute of Technology for biomedical research. The LINC (for Laboratory Instrument Computer) was developed under sponsorship of the National Institutes of Health.

The computer, as offered by Digital, incorporates all of the features of the original LINC and can use all existing LINC programs. Digital reports that LINC's key features—compactness, flexibility, versatility—make it an effective tool in the biomedical research laboratory and the operating room.