DECLASSIFIED

~~CONFIDENTIAL~~ UNCLASSIFIED

TECHNICAL MEMO [illegible]

NOTS COMPUTING REQUIREMENTS

By

Bruce Oldfield

COMPUTING SECTION

MATHEMATICS DIVISION

DECLASSIFIED

Declassified by: S. FLORES Security Specialist
Declassified on: 10 Nov 2016

UNCLASSIFIED

DECLASSIFIED

U. S. NAVAL ORDNANCE TEST STATION — RESEARCH [illegible]

Inyokern, California — Februa[illegible]

February 1949

Technical Memorandum No. SHC-1

NOTS COMPUTING REQUIREMENTS

1. INTRODUCTION

This report is a continuation of the report of June 7, 1948 prepared by the Mathematics Division on the computing needs of the Station. It was undertaken in an effort to augment the first report with additional details, but primarily it was hoped that future* problems would be uncovered, problems which had not already been reported to the Mathematics Division. Each department was visited and an attempt was made to discuss and analyze all pertinent problems which would be suitable for large scale computation. Several problems were actually coded for solution on a large scale digital computer in order to accurately estimate their time of solution. Section I outlines the information gathered from each department. Section II discusses the possible ways of handling the computational load outlined in Section I, the conclusions reached and the recommendations of the Mathematics Division on a course of action for NOTS to follow.

* Meaning by future, a period approximately two years from now when a large scale digital computer could be in operation.

UNCLASSIFIED

DECLASSIFIED

SURVEY OF NOTS COMPUTING PROBLEMS, BY DEPARTMENTS

AVIATION ORDNANCE & TEST DEPARTMENT

A. Measurements Division.

The following discussion covering the Measurements Division is based on an informal paper written by Dr. John Titus.

1. Data Recording Instruments.

The major recording instruments used by this Division, either now or in the immediate future are:

(a) The Bowen camera

(b) Theodolites

(c) Hyperbolic Doppler

(d) SCR-584 radar

The reduction of the data gathered with the instruments listed above requires a large amount of computation. It is very likely that [illegible] measuring devices will be available in the future, which would [illegible] additional computations.

2. Bowen Camera.

The missile position, velocity, acceleration, and yaw a[illegible] determined by means of the Bowen camera and a rear deflection [illegible] The reduction of this data is adequately handled at the present time by the IBM equipment. The time per point required breaks down approximately as follows:

IBM time - - - - - - - - - - - - - - - - -	2 min./pt.
Desk machines - - - - - - - - - - - - - - -	3 min./pt.
Measuring, graphing, computing and yaw angle - - - - - - - - - - - - - - -	5 min./pt.
Total	10 min./pt.

DECLASSIFIED

~~CONFIDENTIAL~~

UNCLASSIFIED

If all possible computations were performed on the IBM machines, the estimated IBM machine load would be 100 hours per month. This estimate is based upon the use of the No. 602 machine. The IBM equipment would become more efficient if the number of po... handled per month were increased. However, since the number o... fired is limited by the range and photographic personnel the ... of computing required in the future should remain as a base l... approximately the level estimated. The equations involved are well suited to large machine calculation. A rough estimate of the time required on a Mark III type computer would be several seconds per point.

3. Present Calculation of Theodolite Data.

It is customary to run up to seven or eight theodolites on most guided missile tests, and to pick out the three which are b... suited for position, intersection of rays, etc. It is usually ... sary to change stations for various parts of the trajectory.

The method of data reduction currently being used by the Assessment Section minimizes the sum of the squares of the dis... connecting the three lines of sight of the three theodolite st... This is not a "least-squares" solution in the usual sense, but if ... data are good it should give a very close answer. It is doubtful if the use of three or four stations provides enough of a statistical sample to justify the added work of a least-squares solution.

This solution reduces the problem to that of finding the following coefficients and then solving a third order matrix involv... these coefficients. The final co-ordinates of the point would ... weighted average of the co-ordinates of the individual points.

[redacted]

$$A_{13} = \text{Sin } H_1 \text{ Sin } H_3 + \text{Cos } H_1 \text{ Cos } H_3 \text{ Cos } (S_1 - S_3)$$

$$B_1 = -(2X_1 - X_2 - X_3) \text{ Cos } H_1 \text{ Cos } S_1 - (2Y_1 - Y_2 - Y_3) \text{ Cos } H_1 \text{ Sin } S_1$$
$$+ (2Z_1 - Z_2 - Z_3) \text{ Sin } H_1$$

$$B_2 = -(2X_2 - X_1 - X_3) \text{ Cos } H_2 \text{ Sin } S_2 - (2Y_2 - Y_1 - Y_3) \text{ Cos } H_2 \text{ [illegible]}$$
$$+ (2Z_2 - Z_1 - Z_3) \text{ Sin } H_2$$

$$B_3 = -(2X_3 - X_1 - X_2) \text{ Cos } H_3 \text{ Sin } S_3 - (2Y_3 - Y_1 - Y_2) \text{ Cos [illegible]}$$
$$+ (2Z_3 - Z_1 - Z_2) \text{ Sin } H_3$$

$$-2\gamma_1 + A_{12}\gamma_2 + A_{13}\gamma_3 = B_1$$
$$A_{12}\gamma_1 - 2\gamma_2 + A_{23}\gamma_3 = B_2$$
$$A_{13}\gamma_1 + A_{23}\gamma_2 - 2\gamma_3 = B_3$$

H_i and S_i are the elevation and azimuth angles of the i-th theodolite station whose coordinates are x_i, y_i, z_i and γ_i is the straight line distance between the ith theodolite and the rocket. To change each straight line distance back to the X, Y, Z co-ordinates of the launcher requires several multiplications and additions. The co-ordinates of the round would be a weighted average of the X, [illegible] co-ordinates as determined by each pair of theodolite stations. This offers a convenient check when three or more theodolite stations are [illegible] Velocity and acceleration are found from first and second differ[illegible] of the computed missile coordinates.

This cine-theodolite problem using three theodolites has been completely coded for solution on the Mark I digital computer located at Harvard University. This involved considerable work, but provides an accurate guide as to the length of time required for solving the same problem on the faster computors, namely the Mark II located at Dahlgren and the Mark III now being finished at Harvard University. A total of 981 cycles was required to solve this problem on the Mark I computer. The computer operates on a fixed cycle of .3 of a second duration. This means that 294 seconds or 4.9 minutes woul[illegible] be required to find each point. Dr. Howard Aiken of Harvard Un[illegible] and Dr. Bramble of Dahlgren have indicated that Mark II is [illegible] times as fast as Mark I and Mark III is 50 to 100 times as fa[illegible] Mark I. Summarized this means:

4.9 minutes per point on Mark I calculator

30 seconds per point on Mark II calculator

.3 to .6 seconds per point on Mark III calculator

The speed of solution on the IBM equipment is estimated to be 90 seconds per point. This is based on the new 604-416 combination that will arrive in the latter part of 1949. At the present tim[illegible] this data reduction is being done primarily on the 602 multipli[illegible] currently in the IBM room. On this equipment six minutes per poi[illegible] is the best that has ever been done and without exception there h[illegible] always been machine errors and breakdowns that increased the ov[illegible] time by a large factor. From this experience it seems safe to [illegible] that the present estimate of 90 seconds per point on the 604-416 combination should be increased by a considerable factor to take into consideration the large proportion of machine errors and machine breakdowns.

4. The Calculation of Theodolite Data as Contemplated for the Future.

The present procedure is dictated by the amount of film to b[illegible] read as well as by certain computational restrictions. The m[illegible] method bears no particular relation to the methods which will be used

 these limitations are eased. Two improvements which are expected to reduce the measurements bottleneck are:

(1) An improved film reading and recording apparatus. This is something of an interim measure until (2) is ready.

(2) A new theodolite which will record its data directly in the field, so that readings will go directly from the instrument in the field to the computer without human intervention. A theodolite of this type is being designed under the cognizance of the Army Signal Corps, and should be available sometime within the next five years.

The development of a new theodolite will result in more stations being used (rather than the present three) with a corresponding increase in accuracy. Eventually perhaps the data from all the stations used on any test will be included in the data reduction.

In general it can be said that the method of reduction of such a system is tailored to fit the computing equipment available. The present modified "least squares" solution is almost at the limit of what is practical (for mass production) on the IBM machines. For example, forty-two separate boards must now be wired to solve the problem on the IBM equipment. If a fully automatic, high speed computer were available, a theoretically rigorous solution could be made, using all the stations on a test. This would give a resulting reduction in the location error of the missile. Even more important than the increase in accuracy would be the possibility of speeding up the reduction. The test conductors will never be satisfied until the trajectory for a test is finished within 24 or 48 hours after the firing. Due to the necessity of using the information from the previous flight in planning the next test, the value of the data rapidly diminishes. Some contractors have estimated that information is 100% more valuable right after the test than it is two weeks or a month later.

The difficulty of the instrumentation program should not be underestimated. The field work will be more strict inasmuch as the corrections and calibrations will have to be made to the instrument itself rather than applied to the film readings, as is done now. Also provision will have to be made for adding and dropping stations at will as the missile moves from one part of its flight to another. A very flexible computer will be required which can be made to take care of all the different circumstances.

The test conductors have always wanted as many points per round as would be consistent with turning out the data in a reasonable time. If it were possible to turn out a trajectory with 10 points per second in a fairly short time, the user group would be extremely happy. This would mean that between 1000 and 2000 points would have to be computed for each flight. Future schedules cannot be predicted, but it may be expected that between two and ten flights per week may be anticipated.

Certain facts stand out in the foregoing discussion.

(1) A more rigorous solution using additional data would be used if computing facilities and automatic recording facilities were available.

(2) The importance of the results of a test decrease almost exponentially with time.

(3) The IBM computing facilities would be <u>inadequate if</u>: Automatic recording of data is developed; and a reasonable number of

rounds are fired per month. At the present time it seems highly probable that the IBM machines will be satisfactory until automatic recording of the theodolite data is realized. It must be remembered, however, that a large scale computing machine ordered in the near future would probably arrive at about the same time as the new theodolites. Also it is well to remember that when it is said that the IBM equipment can satisfactorily handle the theodolite reduction, the conclusion is based on this problem alone, and not on this problem considered as only one of the many problems of the base.

5. Hyperbolic Doppler.

Hyperbolic Doppler is not in operation at present, but it is expected that it will be a valuable instrument in the future. In this system the differences in the distances from the bird to the various stations are measured. A minimum of three stations is required. It is planned (to begin with, at least) to run four stations, to be combined in three pairs. If more than four stations are used, sets of four will be chosen.

The recording instruments give

$$d_1 - d_3 = a_{13}$$

$$d_2 - d_4 = a_{24}$$

$$d_3 - d_4 = a_{34}$$

where the d's are the distances from the rocket to the stations. If x, y, z, is the position of the missile, and X_i, Y_i, Z_i the position of the ith station, then the above equations become:

$$\sqrt{(x - X_1)^2 + (y - Y_1)^2 + (z - Z_1)^2} - \sqrt{(x - X_3)^2 + (y - Y_3)^2 + (z - \ldots}$$

$$\sqrt{(x - X_2)^2 + (y - Y_2)^2 + (z - Z_2)^2} - \sqrt{(x - X_4)^2 + (y - Y_4)^2 + (z - \ldots}$$

$$\sqrt{(x - X_3)^2 + (y - Y_3)^2 + (z - Z_3)^2} - \sqrt{(x - X_4)^2 + (y - Y_4)^2 + (z \ldots}$$

where the a's are known quantities.

An iterative solution to the above set of non-linear equa has been worked out. One iteration requires the time equivalent of approximately 140 multiplications. The solution should converge rapidly due to the close initial approximation that is known. Th means approximately 90 seconds for one iteration on the new 604 combination of the IBM equipment and approximately 2.8 seconds for an iteration on the Mark III computer.

6. Future Plans for Calculation.

The present Doppler system, like the theodolites, is designed to fit the present computing equipment. Since only a radio receiver and a transmitter or wire link is required at each station, it should be possible to set up a number of stations, perhaps as many as ei or ten for a 20-mile guided missile range. As in the theodolite an increase in accuracy is to be expected if all stations are used. Since there is no hope at present of doing anything like this, the equations have not been worked out. However, they are sure to b more complicated.

In BRL Report 662, B. Garfinkel outlines another system using six stations and finding the x, y, z co-ordinates of the missile from six simultaneous radio-doppler frequencies. This system has the great advantage of eliminating the summation of Doppler cycles, and is, the fore, unaffected by radio fade-outs. Experience at White Sands has shown that in many cases the flight record is broken in a number of places, greatly reducing the value of this record. However, it is thought that a better flight recording can be obtained at NOTS.

disadvantage of this system is that the accuracy would be lower. The amount of computation required to find the x, y, z co-ordinates is roughly equivalent to that necessary in the four station system.

A system similar to Garfinkel's but using seven stations has been worked out by Dr. Haseltine of the Ballistics Division. This method has several advantages over Garfinkel's plan, but requires a large amount of computation to reduce the data. Four simultaneous, non-linear algebraic equations must be solved numerically, requiring several iterations for each point. Each iteration would require about five minutes on the new IBM equipment.

Although the various systems are not at the operating stages, there can be little doubt that in the future the computational load from Doppler data will become increasingly heavy. This will very much increase the computing requirements of the Assessment Section.

7. SCR - 584 Radar.

The equations for the SCR-584 are fairly simple, and are adequately handled by present equipment. However, the possibility in the future of improved recording equipment, as well as the combination of the SCR-584's either with each other or with the theodolites, would make the computing requirements more complicated.

B. Aviation Ordnance Division.

1. On occasion a set of simultaneous linear differential equations with variable coefficients must be solved. Fire Control is developing a simple analogue device to handle such sets of equations. The equipment will be accurate to something better than five percent. In the

future a greater degree of accuracy will be desired. This will mean going to a digital computer, or a more accurate analogue computer, for better solutions. It was felt that if a large scale digital comput... were available more nearly exact equations would be considered.

2. One part of the work being done by Fire Control consists of preparing and publishing sighting tables for fleet aircraft use. This involves making a comparison of the lead angle which the aircraft establishes at successive points along its path during a pass at the target with the lead angle which would score a hit on the target should rocket release occur at any point. The procedure at the present time consists of smoothing the values of the range with a least squares curve, taking the value of the product of the range times the target angle at the known points and differentiating this product graphically to obtain the flight path angle.* The acceleration normal to the flight path is also obtained by making a least squares fit and a graphical differentiation. These steps are all preliminary to solving for the correct lead angle. Going from the angle of attack to the sighting table involves solving a simple equation, but with a group of parameters that take on a very large number of values.

The methods being used are tedious and less accurate than a method which substitutes numerical methods for graphical methods. It seems highly desirable for the Analysis and Reports Section to send representatives to the Computing Section of the Mathematics Division

---

* For more detailed equations see the notes prepared from a lecture by M. J. Sargeant on Aug. 10, 1948.

with a typical problem. The problem will then be set up for machine computation on the IBM equipment. This should save a great deal of time, give greater accuracy, and break the bottleneck of hand computation.

One project usually consists of ten to thirty airplane passes. There are six or seven projects going on at any one time. To reduce the data and find the angle of attack for one project takes one man (P-3 level) about two months. To go from the angle of attack to the sighting table takes one man (P-3 level) one month of continuous computation. Thus for each project there is at least three months or 480 hours of high level computation. This load will continue indefinitely since a firing table must be made for each new plane that is designed.

At the present time firing tables have been made for finner rockets only. It is expected that at a future date airplane firing tables will be made for spinner rockets. This will be a much more complicated problem and hence the amount of numerical computation will be proportionately larger.

CONTROLLED MISSILES DEPARTMENT

A. Underwater Ordnance Division.

1. The Model Laboratory.

The model laboratory has a large amount of data reduction which is not difficult mathematically, but which occurs in large quantities. At the present time this computation is being done on hand machines. It is most desirable to have the computed data available as soon after the round has been fired as possible. [illegible] if a satisfactory method of transmitting the data back and forth could be worked out it was felt that a large scale digital computer would be of assistance.

2. Doppler Data Reduction.

The water launchings of the Underwater Ordnance Division now average six per day. About half of these rounds require the reduction of Doppler data. The equations for reducing Doppler data have been mentioned previously and a time estimate was given. As seen from these equations a Doppler computation requires the solution of simultaneous, non-linear, algebraic equations and could be conveniently handled on a large scale digital computer. The reduction of three firings per day represents considerable work.

3. Structural Analysis.

Studies in structural analysis have been going on for some time and will be continued in the future. The preliminary work is being done at the present time and from all indications will soon involve the solution of sets of partial differential equations of elasticity and plasticity with three independent variables. Needless to say a solution to this type of problem will require large scale computing facilities.

4. Hydrodynamic Coefficients.

A digital computer could be used to help determine the hydrodynamic coefficients to greater accuracies from special techniques that have been devised. These coefficients would then be used in solving other Underwater Ordnance problems. This work is not being carried on at the present time because of the large amount of computation that would be necessary.

5. Theoretical Investigations.

The following problems are of great interest to the Underwater Ordnance Division. A few are being investigated at the present time, the remainder will be investigated in the future.

(a) Free streamline flow problems which involve solving Laplace's equation with boundary conditions which are unknown and must be determined as part of the problem. This means that the boundary conditions will in all probability never be simple enough to allow an analytic solution. This will lead to problems which involve a numerical solution of partial differential equations.

(b) Gravity and viscosity flow problems which will also lead to the numerical solution of partial differential equations.

(c) Water entry problems. The first stage will be oil to water entry; the second stage will be the more difficult problem of air to water entry. This water entry problem will lead to partial differential equations in two dimensions plus time.

These problems are very difficult and can be solved satisfactorily only by a large-scale digital computer. The exact amount of computation required cannot be estimated, but it can be said with confidence that there will be a large quantity.

B. Guided Missile Division.

The Guided Missile Division is interested in the following types of problems: (1) Dynamic stability problems in three dimensions, involving three control systems with six degrees of freedom. (2) Theoretical trajectories of missiles, involving the solution of differential equations containing nine dependent variables and one independent variable. (3) Dynamic trajectory problems involving problems in pursuit, collision, and fire control. (4) Servo problems.

The examples mentioned above represent a tremendous number of unsolved problems that can be attacked successfully only with the aid of large scale computing facilities. The Guided Missile Division estimated that the number of problems that could be solved is so large that this Division alone could keep a large-scale digital computer busy <u>twenty percent</u> of the time for an indefinitely long period. This of course is assuming adequate personnel in either the Mathematics Division or the Guided Missile Division and a sufficiently large budget to support the program.

EXPLOSIVES DEPARTMENT

The needs of the Explosives Department remain essentially the same as reported before. The following problems have been mentioned by this department:

(1) Fitting of empirical equations to data by the method of least squares is a frequently recurring problem.

(2) There is a need for the efficient assessment of photo-[illegible] records, for example side-thrust in rockets.

(3) A future problem will be investigations involving heat transfer in motors, requiring the solution of partial differential equations with complicated boundary conditions. This represents an enormous computational problem.

A. Mathematics Division.

1. Applied Mathematics Section.

This section is expected to provide a steadily increasing amount of work for the Computing Section, especially if the present IBM equipment is supplemented by other devices, such as an analogue computer or a large-scale digital computer. The Reeves electronic analogue computer is particularly well adapted to the solution of systems of ordinary differential and integral equations. Such systems arise in many of the more powerful analytical and approximate methods for solving linear and non-linear partial differential equations. Among the applied mathematical problems (linear and non-linear) requiring such numerical solution are:

(a) Problems on internal ballistics of rockets; gas flow oscillations and turbulence; stresses on grains, walls, and [illegible] heat transfer and temperature distributions; erosion of grains nozzle; reaction kinetics, etc.

(b) Problems on instrumentation: signal distortions due to non-linear characteristics of pick-ups, vibrations and feed-back; optimum criteria for design, etc.

(c) Problems on the response of complex systems: servo-mechanisms in aerial combat; sources of error in computing devices; non-uniform statistical mechanics; properties of idealized molecular models, etc.

(d) Problems in aerodynamics, hydrodynamics, detonation, elasticity, plasticity, water entry, acoustics, electromagnetic [illegible] circuit analysis, radiation, combustion, aerothermodynamics, diff[illegible] etc.

These problems have not been seriously considered up to the present by the various departments and divisions of NOTS because of computational complexities. The analogue computer which the Mathematics Division has recommended purchasing will be a tremendous aid in many of these problems. The analogue computer is readily availabl.. and will be of great importance in supplying first approximations for any future digital computer. Although a certain amount of ideali.. is necessary for the theoretical treatment of relevant problems, more realistic models can be investigated with the aid of suitable computing facilities. The Applied Mathematics Section, in particular, can perform effectively in the many fields of interest to the Station only if flexible high-speed computing facilities are available.

The future computing time requirements of this section will be largely governed by the types of computing machines available, present IBM equipment requiring a considerably longer time than an analogue computer to solve most of the problems now under consideration by this section. As a reasonable guess it may be predicted that the computing needs of this section will be covered during the next six months by an average of one-tenth of the time of the present computin.. staff, with a subsequent rise within two years to the full time of . or two computers, depending on the type of equipment available.

2. Analysis Section.

Examples of future problems to be considered by the Analysis Section include problems of pursuit, collision, fire-control and the evaluation of weapons taking into account all of the many variables. These problems will be sufficiently complex that they will benefit by the availability of large-scale computing machinery.

3. Statistics Section.

No large-scale computation is now performed, but an occasionally recurring problem is the determination of equations of regression connecting correlated variables. The computations involved are [illegible] but have been set up for IBM computation. Research on the effect of application of various statistical techniques to non-normal distributions would be possible (though not of first priority) with IBM or a large-scale computer. This problem might be considerably expanded if adequate computing facilities were available.

B. Chemistry Division.

The Chemistry Division has been using the IBM facilities for the past fourteen months to determine the equilibrium composition and the thermodynamic properties of dissociating combustion gases. This has been a study of the combustion gases produced when Aniline is ignited with nitrogen-tetroxide. It is estimated that three more months will be required to finish this problem.

A similar problem using hydrazine ignited with nitrogen-tetroxide will be done if funds permit. Thus the Chemistry Division may have over a year of IBM computation ahead. The problem of computing the composition of combustion gases will very likely re-occur in the future for different liquid fuels. This problem would be very well suited for solution on a large scale digital computer.

Very recently Re2d has expressed an interest in having a similar study made of a number of composite propellants. The present problem which will require sixteen months for solution contains four components resulting in nine simultaneous equations of which a number are non-linear.

The investigations proposed by Reed would contain six components, resulting in a combination of sixteen simultaneous, linear and non-linear equations to be solved for each change in a parameter. There might be as many as twelve of these problems to be solved.

These problems are far too big for the IBM equipment and thus will no doubt be done on a large scale digital computer. If NOTS can purchase a computer of this type, there is reason to believe that this entire problem and other similar problems would be sent to NOTS for solution. This is an important source of future computational problems for which the present IBM equipment is entirely inadequate.

No other large scale computations are required at the present time. However, problems in thermodynamics and crystallography will be occuring in the future, requiring extensive numerical calculations, quite suitable for a large-scale digital computer. No adequate quantitative estimate of the number of such problems that could be expected to occur can be made at the present time.

C. Applied Science Division.

1. Optics Section.

The equations for trigonometric ray tracing are simple recursion relations. In the design of a lens system, however, the number of parameters is quite large. The index of refraction of each glass, the radius of curvature of each surface, the thickness of each lens, and the air spaces between lenses are all parameters. This means that lens design is very well suited for a large scale digital computer. For example: The Mark I computer at Harvard University has done an involved problem in trigonometric ray tracing. The Eastman Kodak Company is working on a computing machine to be used for their own lens design problems.

With the aid of a large scale computer, extensive investigations could be conducted which would be impossible to carry on by hand. Furthermore, the use of a large machine would in all probability completely change the approach to a lens design problem. The Optics Section was very interested in making use of a large scale computer if it should become available. However, the future work load in lens design is unknown and thus no satisfactory estimate of future problems could be given.

2. Mechanics Section.

The Mechanics Section has reported that it will be working on a study of high temperature heat flow and heat transfer in the future. It was stated that ten problems per year would be a reasonable estimate of the work load. These problems will involve partial differential equations with difficult boundary conditions. This means that a numerical approach using finite differences would probably be used. As accuracy, and hence the number of points in the mesh, is increased the amount of numerical computation involved increases at a tremendous rate. This makes a solution practicable only if a large scale computer can be used.

It was very difficult for the Mechanics Section to predict their computing needs so far in the future. For this reason the estimate given may be much too large. However, if this estimate of ten heat flow problems per year is substantially correct then considerable computing facilities will be required by this Section in the future.

3. Aerophysics Section.

The Aerophysics Section is now working on an extensive project investigating the light of the night sky. The present computing needed for this project is a very large conversion table which would convert terrestial coordinates to either Ecliptic or Galactic. The IBM equipment is now working on this table. The computation will include approximately 200,000 multiplications, 100,000 divisions, and 200,000 values to be printed. A Mark III type computer could handle this entire problem in about three hours.

It is interesting to note that if a large scale digital computer were available, this conversion table would be unnecessary. The coordinate conversion would be programmed in the computer and thus the machine would convert each point as the need arose in less than one second. This would eliminate the computation of the original table and the table lookup necessary for the conversion of each point.

The light of the night sky project will continue until February of 1950. Data is being continually compiled with four recording instruments. No attempt is being made to keep the data reduction work up with the incoming data. The Aerophysics Section estimates that at the end of the project, or February 1950, there will be <u>eighteen man years</u> of computing to be done. This reduction will lend itself to either IBM or large machine computation. Limited personnel, limited appropriations, and limited time will make a machine solution to this problem mandatory.

D. Physics Division.

Practically all of the problems of the Physics Division which will require large-scale computational facilities will be problems involving the solution of partial differential equations. Examples of the type of future problems which will require a large amount of computation are:

(1) Wave propagation - elastic and plastic

(2) Plastic flow of metals when impulsively loaded

(3) Airflow around hypersonic missiles and the heat flow resulting

(4) Acoustic pulses through a dissipative medium

(5) Computation of diffraction patterns

(6) Solution of integral-differential equations in the solidification of a sphere. The more rigorous solution to general problems of solidification are of great interest.

(7) Re-occuring problems in heat flow.

1. Electricity and Magnetism Section.

The following problems will be investigated in the future by this section:

(a) Study of the initiation of explosives by an electrical pulse on a fine wire.

(b) Solution of transmission line equations and Maxwell's equations.

(c) Studies of the inhomogeneities in a wire.

(d) Thermodynamic studies - a statistical mechanical approach.

(e) Optical characteristics - computation of phase fronts b[illegible] up of uniaxial crystals. This is the equivalent of ray tracing p[illegible] for light in crystalline optics where the medium is non-isotropic.

This represents a considerable number of future problems which would be solved on a large scale digital computer if such a computer should become available.

E. Ballistics Division.

1. Ordnance Aerodynamics Section.

Nothing was found out from this section that was not covered by the Mathematics Division's informal report of June 7. This report states, "at present computation is not a routine matter. Wind tunnel data is obtained from Daingerfield, Texas, at a rate sufficient to occupy a professional person with a desk machine one week per month, obtaining best-fitting straight lines. Wind tunnel gauges have not yet been standardized, so that each set of data presents a separate problem. However, in less than a year it is planned to have forty days of wind tunnel testing per year, leading to 66 man-weeks per year of desk computation. This larger mass of computation could be standardized for the use of IBM or other larger machines, and it was felt efficient to do so. A second set of computations will arise in less than a year from applying the method of characteristics to the solution of non-linear partial differential equations of fluid flow. This could entail considerable, but less predictable, use of a large machine".

2. Exterior Ballistics Section.

*The Field Laboratory. The field laboratory will eventually work up to a firing schedule of one round per day. There will be two hundred pictures taken per round. The reduction of one round by the simplest means will require 200 man hours of computation. This solution makes a number of approximations which save an immense amount of computation. However, in order to obtain results which are not merely repetition of the work being done at Aberdeen, more exact equations will be used. For example, an approximation is now used which treats the yaw residuals and the swerve residuals separately. This requires the solution of a tenth order triangular matrix. If this simplification is not introduced the matrix becomes one of eighteenth order. Another approximation introduced in the present Aberdeen solution is that an almost linear function is chosen to approximate the yaw. An involved solution using more exact equations has been worked out by Dr. Alexander Wundheiler of the Bureau of Ordnance. This method is a modification of the Picard method of successive approximations, Picardian step alternating with solutions of systems of arithmetic equations. Dr. Wundheiler thinks this more exact solution can be handled practicably only on a large scale computing machine.

In addition to the work mentioned above there will be computations involving mean square fits, numerical integration, and solution of second order, non-linear differential equations. The

* A comprehensive report on the field laboratory is being prepared at the present time by the Aerodynamics Section.

aerodynamic coefficients that are wanted are: the axial drag coefficient; the spin deceleration coefficient; the normal force coefficient; the overturning moment coefficient; the cross spin force coefficient; the damping moment coefficient; the magnus force coefficient; the coefficient of magnus cross torque due to cross velocity; the coefficient of magnus cross torque due to cross spin; coefficient of magnus force due to cross spin.

An analogue computer would present certain advantages in solving the second order, non-linear differential equations involved, but its use would be limited to this type of problem only. On the other hand a digital computer could solve the same problems more accurately, though not as easily, and in addition handle all the other types of numerical problems which may come up.

The field laboratory must measure rounds in space to extreme accuracies. They will correct for all of the sources of error such as camera plate tilt and lens aberrations. This gives a very <u>involved</u> and <u>lengthy</u> transcendental equation which must be computed at <u>each point</u> in order to get the data into a form in which regular computation can begin.

The computations of the field laboratory are ideal for solution on a digital computer. They are involved computations, but computations which can be standardized and thus coded up only once on a magnetic tape and stored in the computer's "library", to be used again and again whenever necessary.

There is every indication that the new field laboratory will be one of the largest sources of computational problems for any future large scale computer. Furthermore, the field laboratory will require the services of a large scale computer if it is to turn out large quantities of useful, original work.

General Ballistic Problems. (The following discussion covering the Exterior Ballistics Section was prepared by Mr. Edwin Winkel. His advice and assistance in obtaining information for this report are greatly appreciated.)

The general problem of exterior ballistics is the solution of the differential equations of motion of a projectile and a correlation of these results with appropriate experimental data. It should be realized that there are no unique differential equations of motion for the theory is being constantly reviewed and revised to accommodate the new knowledge uncovered by current research.

The results of the computations yield information about the trajectory of the projectile including velocity, position, orientation and time. In the case of shell ballistics such computations are aided by the use of the exterior ballistic tables based on numerical integration which have been prepared by the War Department (1924). These tables yield information about certain points on the trajectory. When one considers the case of shells fired from aircraft a more complete knowledge of the trajectory is required and one must turn to a more direct solution of the equations. For rocket ballistics there is an additional phenomenon to consider, namely, the period of burning in which the rocket acquires full velocity as well as orientation, which profoundly affects the subsequent free flight trajectory. A simplified ballistic theory has provided, to date, much valuable information about fin-stabilized and spin-stabilized rockets, but in efforts to explore the behavior of shells and rockets when fired into such strong cross winds as to induce large initial yaws the simplified theory breaks down and it is impossible to avoid a great deal of numerical work. Furthermore with the advent of extensive supersonic wind tunnel testing and indoor controlled ballistic range testing it will be possible to obtain such a knowledge of aerodynamic and ballistic coefficients as to demand their inclusion in future computations of rocket performance characteristics. Indeed, with an appropriate computing machine it should be possible, given an adequate ballistic theory, to arrive at some very definite conclusions about optimum missile design by a process of varying the ballistic parameters and observing the influence on performance characteristics.

At present, the Exterior Ballistics Section is working on experimental as well as theoretical assignments in about a 50-50 ratio. The experimental work on the firing ranges is dependent upon data reduction to a very high degree and suffers greatly when the results from tests are not forthcoming within a reasonably short time. Here, as before with the assessment problem, large scale computing machinery is the indicated answer so that this data may be reduced in a very short time yet not at the expense of other high priority work. Some experimental work is concerned with design of measuring devices to study such things as mechanical and jet malalignments. For this purpose which is of non-routine nature it appears that the desk computer is a satisfactory facility even though it usually requires two to four weeks of computing time. The theoretical work has recently produced a large yaw theory which is to be checked against experimental data. This will require the solution of six simultaneous differential equations and is expected to consume four months or more time for one person. On the face of it, each point of the trajectory computed will probably demand about an hour's time, for the variables appearing are numerous and thoroughly interlaced. In order to keep the number of iterations at a minimum it will be necessary to keep to fairly small intervals in the independent variable.

Even for more routine trajectory computations, i.e., ones in which only a variable axial drag and possibly a variable thrust characteristic are considered, lengthy computations are involved. One case in particular is remembered to have required about three months of calculation time for one person to investigate four different trajectories computed to observe the effect of variations in drag functions. On the order of 75 steps with from one to five iterations per step were required for each trajectory. At 15-20 minutes per step this amounts to about 4000 minutes per trajectory or 16000 minutes (2½ mos.) for the problem. The time per step includes lookup time and interpolation for the trigonometric and drag functions involved as well as time necessary to run down obvious blunders.

Part of the time computations were run by two persons in parallel for check purposes. Thus it is seen that a great deal of time is requ[illegible] for the computation of trajectories and the quantity of work of th[illegible] nature that can be accepted by a small group as we now have is limited. Ideally it would be preferred to seek and accept work of this nature on a greater scale and thus serve the purpose of exter[illegible] ballistics much more fully rather than try to avoid tasks which would be a bind on a limited force - limited in the number of personnel and limited in computation facilities.

SECTION II

CONCLUSIONS REGARDING FUTURE COMPUTING EQUIPMENT REQUIRED BY NOTS

Section I shows rather conclusively that with the inauguration of automatic data recording, field laboratory testing, and the research problems mentioned for each section, there will be a computing load of sufficient severity to demand the services of a large scale digital computer or a tremendously expanded IBM installation. The IBM equipment can be expanded in two ways. (1) The machines can be kept in operation twenty-four hours per day at just twice the rental paid for an eight hour day. (2) Additional machines (newer models in some cases) can be rented and thus the entire IBM equipment might be doubled or tripled, etc. Needless to say, either type of expansion would require an increase in personnel.

The question immediately arises, could the IBM equipment be expanded economically to handle the future computational load outlined in Section I. Certainly the IBM equipment could be expanded indefinitely and thus handle most of the future computing needs of the Station. This, however, is not an economical procedure. Assuming that a large scale digital calculator should pay itself off in six years, setting the price of the calculator at $300,000 means that the computer is costing $50,000 per year. The future IBM rental including the new equipment ordered will be about $25,000 per year. Thus for $50,000 dollars per year using the IBM equipment, the output would be increased by three and with the same $50,000 per year using a large digital computer the output would be increased by a factor of about 30 to 100 depending upon the type of problems to be solved.

From Section I it is quite clear that the new IBM equipment working twenty-four hours per day could not begin to handle the future computing load that was reported by the various sections. To expand this equipment greatly beyond the present facilities, would seem to be very poor economics.

Another very important thing to remember is that a rather large group of problems are sufficiently complex that a solution on the IBM equipment becomes undesirable or even completely impossible. For example: (1) the problems connected with the new field laboratory; (2) The Chemistry Division's problem of determining the equilibrium composition and the thermodynamic properties of dissociating combustion gases for six or more components; (3) The large number of problems mentioned requiring a numerical solution of partial differential equations in two or three variables and time, etc. This means that a considerable group of future NOTS problems will have to be solved on a large scale computer, or not be solved at all.

An arrangement whereby NOTS could send its problems to be worked out on the Mark II or the Mark III at Dahlgren might be set up, but to make this a permanent arrangement raises a number of serious objections: NOTS problems might wait for weeks or months before they could be solved on the Dahlgren computers, since the problems of Dahlgren would come first and only the spare time of the computers would be available to other establishments; The large group of assessment problems could not be handled satisfactorily by Dahlgr... due to the urgency of their solutions and it would be extremely difficult to establish an efficient method of coding the problem and finding their correct solutions at an establishment so far distant from the source of these problems.

The cost of a large computer might be balanced against the costs of a few of the many programs which it would contribute to. For example, the theodolite system for obtaining data costs over $1,000,000; one SCR-584 radar unit costs $100,000. The field laboratory will cost upwards of $500,000. These data-obtaining facilities are used on an average of five minutes per day. In this connection it might be mentioned that the costs of the larger guided missile tests are reaching $50,000 to $75,000 per test and the tests are becoming increasingly complex. A very large amount of money is being spent to achieve automatic data recording. Thus it seems that the expenditure for a large scale calculating machine should be considered as a ... part of the overall picture of obtaining and reducing data efficiently, to say nothing of the tremendous contribution it would make to the many problems mentioned in part one that are not concerned with data reduction.

In case of a national emergency the value of a large scale computing machine could not be overestimated. The labor shortage would be acute and the problems needed would have the highest priority. The two large digital computing machines available during the last war were in ... use solving problems for the armed services.

To keep a large computing machine busy would require a sta[illegible] staff of high level mathematicians. The staff would be large, b[illegible] the output of the computer would be tremendous. There is eve[illegible] reason to believe that the problems to be put on the computer woul[illegible] "snowball" as more and more people become acquainted with the power o[illegible] numerical methods. For example, the IBM equipment at NOTS has bee[illegible] increasingly busy as more and more people became acquainted wi[illegible] capabilities. More and more people would begin to search fo[illegible] mathematical approach to parallel the experimental approach [illegible] research problems. It might be said that there are computation[illegible] problems in every drawer that are not recognized at the pres[illegible] and these problems will only be brought to light when more [illegible] spend more time thinking of new ways of applying mathematical m[illegible] to further scientific research. Thus if the computing machine wer[illegible] occupied only half of every day at the beginning, it would not b[illegible] unreasonable to expect a slow increase, picking up speed over sev[illegible] years, until the computing machine would be in operation twenty-fo[illegible] hours per day solving important problems.

A large amount of research is now being done in the compu[illegible] field. This makes one hesitant to order something now which [illegible] considerably improved upon five years from now. The fact r[illegible] however, that the computer purchased by NOTS would be the be[illegible] available at the time of delivery and although this compu[illegible] not be the so-called "ultimate computer" it would be a very [illegible] computer which would give many years of excellent service. The E[illegible] could be cited as an example; it was the first electronic digital computer ever built, and, although at the present time it is al[illegible] completely outdated, it is still turning out a large number of u[illegible] answers.

It has been previously stated that the number of personnel requ- to keep a large-scale digital computer in operation is quite large. order of magnitude would probably be ten or twenty competent mathem in addition to several electronic engineers. This crew might be abl keep the computer busy if many of the problems to be solved could be handled by tapes taken from the computer's "tape library". This would include assessment problems which would require the same computational routine over and over again. A majority of the mathematicians would spe their time considering problems which required a great amount of prep tion both in the numerical analysis and the machine preparation. It be useless to purchase a large-scale digital computer and then att operate this computer with a limited number of low level personnel. Considerable personnel would be required, but the output from the comp would be extremely large.

SUMMARY

The amount of future computation required by the various sections has been outlined in Section I. The future needs of these sections amount to an impressive computational load. The future computational load will be great and the problems to be solved will be so complex that it will completely impossible for an economically sound, expanded IBM set-up to handle them satisfactorily. This report was undertaken in an attempt to come to a definite conclusion as to whether the future computations of NOTS would justify the purchase of a large-scale digital computer. Section II has shown rather conclusively that the only way this large amount of computation can be satisfactorily handled is by a large scale digital computer. Thus the Mathematics Division considers that the need for such a machine has now been established and consequently recommends that NOTS purchase a large-scale, automatic sequencing, digital computer.

It is very desirable to take early action to obtain <u>funds</u> and <u>authorization</u> to purchase a large computer since it can be expected from the time of purchase one and one half to two years will be required for its delivery. Unfortunately, no concrete recommendations can be made at the present time as to the purchase of a <u>particular</u> digital computer. A number of high speed, digital computers are being developed at the present time, but as yet it would be premature to select a computer and recommend its purchase. It is hoped that in the near future more will be known of actual performances which will enable NOTS to make an intelligent selection of a particular digital computer. At the present time NOTS should keep in close contact with the various computer development programs and also with other government agencies who are interested in the purchase of a digital computer.

Original Distribution of NMC-1 at U.S. Naval Ordnance Test Station

Dr. L. T. E. Thompson (4)
Dr. C. T. Elvey
Dr. J. H. Shenk
Dr. B. H. Sage
Dr. A. H. Warner
W. B. Saylor
Dr. N. A. Renzetti
Dr. Roger Estey
Dr. Arthur L. Bennett
Dr. W. H. Cady
Dr. G. B. L. Smith
Dr. A. Vazsonyi
Dr. W. B. McLean
Dr. J. H. Wayland
Dr. L. E. Ward
Dr. E. P. Cooper
Dr. F. A. Shaffer
H. A. Van Dyke
R. C. Davis
Dr. John Titus
E. A. Nyswee
W. D. Herrick
E. F. Finkel
Mathematics Division (14)