Model of Industrial System

Pg 1

Part I INTRODUCTION

~~Section A Need For This Type of Model~~

Work has been done on pure inventory models, on job shop manufacturing models, on flow shop manufacturing models and on conveyor models. However, we have not found in the literature an attempt to put together the basic building blocks in a multi-stage assembly model.

~~Section B Source of Information~~

Rather than try to directly simulate a real system we have instead analyzed in depth the basic elements of ~~a real~~ an experimental multi-stage assembly system. From this has evolved a control group concept[1] and a series of definitions of the ways in which one stage of a system can be linked with another.

~~Section C Control Group Concept~~

~~Refer to the Control Group Concept~~[1]

~~Section D Size of System~~

The magnitude of the system is a direct reflection of the number of stages, the variety of differentiated products (raw materials, parts, and assemblies) and the number of time periods for the operational cycle in each stage. Therefore, it was felt desirable to simulate a very small system as an initial pass, with the full expectation that this would permit a sharper, clearer understanding of the various phenomena.

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[1] February 23, 1958 Integrated Systems Project Progress Report Production Control Service, 570 Lexington Avenue, New York City

Part II GENERAL PROGRAM CONSIDERATIONS

~~Section A Purpose of Model~~

The purpose of this model is to explore strategic considerations in planning a manufacturing control system. Decisions, to be considered include: ~~relative to~~ cycle planning, placement of inventories, choice of economic or automatic ordering plans, form of scheduling rules, importance of feedback of progress information and importance of integrated versus decentralized decision planning.

~~Section B Use of IBM 702~~

The simulator was prepared for the IBM 702 ~~was used~~ because the machine was being used for ~~all~~ other Integrated Systems Project work. This was to be a small program which could be written readily in "Script" and modified by Project programmers.

~~Section C Simplification~~

The model has been intentionally simplified so that exploration could concentrate on order of magnitude changes, ~~only~~. For example, an inventory control group gives instantaneous service; all communications channels are instantaneous; cycles are either one or two periods long; all sources have infinite supplies; there is only one customer; capacity and load considerations are ignored; the number of stages in the system are limited; there are only two products each of which consists of two components; there is but one source for each input component. No provision is made for costs, set up, variable run time, or variable usage quantity.

~~Section D Use for Testing~~

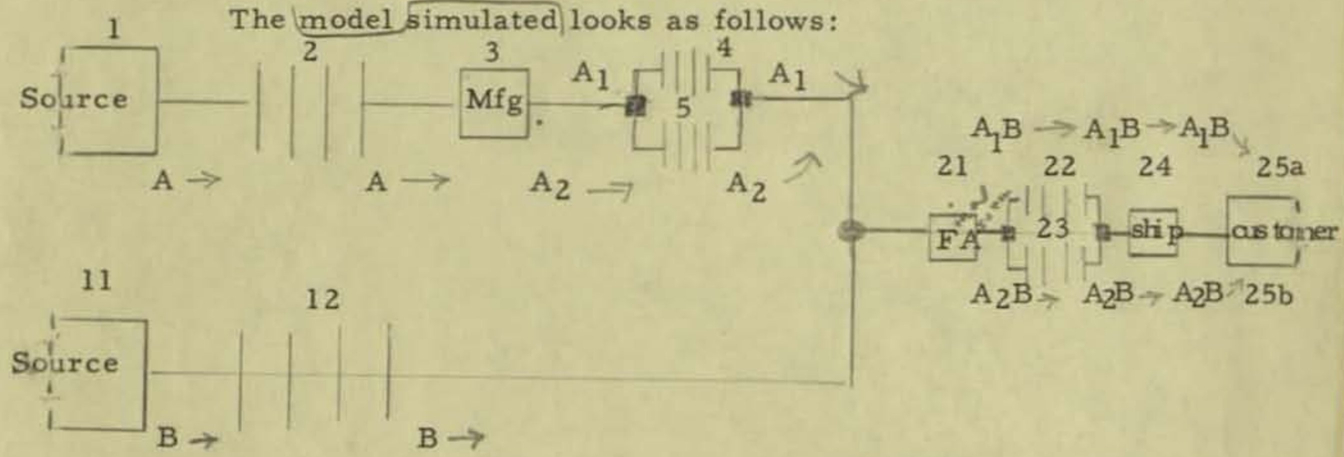
It is intended that the model be initially used for testing the form of ordering and scheduling rules. The need for allocation rules (for input components) ~~will~~ could also be tested. The factors in the scheduling and ordering rules will be examined in the light of different cycles and demand patterns.

Part III DESCRIPTION OF PHYSICAL MODEL

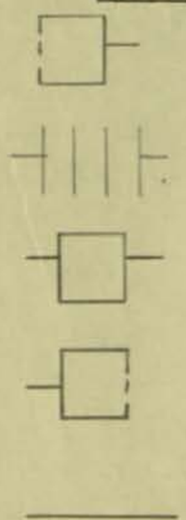
Section A Flow Chart

The model simulated looks as follows:

note - Give this more space



Section B ~~Define~~ Symbols Used in Chart



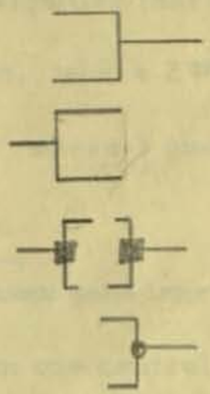
source of materials

Inventory Control Group

Manufacturing control Group

Customer; product "sink"

Physical material flow always from left to right



Summarizing junction

Distributing junction

Exclusive "or"

"and"

Section C Materials, Parts, Assemblies

Raw materials A, B

Manufactured Parts A1, A2

Finished Models A1B, A2B

Section D Precedence Charts

The physical flow precedence matrix for this product is represented as follows:

Physical Flow / From Control Group #	To Control Group # 1	2	3	4	5	11	12	21	22	23	24	25
1		1										
2			1									
3			[illegible]	1	1							
4								1				
5								1				
11							1					
12								1				
21									1	1		
22											1	
23											1	
24												1
25												

Where 1 signifies that this flow does exist and blank ~~or zero~~ indicates that it does not. where 2 ~~or~~ ones appear in the same row an "or" condition is signified. where 2 ones appear in the same column an "and" condition is indicated.

~~This paxedex par~~ The following Precedence matrix actually shows the specific product which flows from one control group to the next.

Physical Product Flow Matrix

draw lines

To Control Group #

From control group #

	1	2	3	4	5	11	12	21	22	23	24	25(a)	25(b)
1		A											
2			A										
3				A_1	A_2								
4								A_1					
5								A_2					
11							B						
12								B					
21									A_1B	A_2B			
22											A_1B		
23											A_2B		
24												A_1B	A_2B
25(a)													
25(b)													

Product Gozinto Matrix

"Finished component"

"Raw Material"

	A	B	A1	A2	A1B	A2B
A			1	1		
B					1	1
A1					1	
A2						1
A1B						
A2B						

"Finished component"

"Raw Material"

	A	B	A1	A2	A1B	A2B
A			3	3		
B					21	21
A1					21	
A2						21
A1B						
A2B						

station ~~which~~ transformation Matrix

Part IV Definition of Factors

Section A Notation

$s \equiv$ station; always used as a superscript

a separate station number is defined for each source, inventory group control group and customer.

[redacted]

$m \equiv$ "model" or part, product, material, assembly

an item changes identity after going thru any operational control group (other than shipping)

[redacted]

$t \equiv$ time period (hour, day, week, etc.)

[redacted]

Section B Description of Physical System

Cycle Tolerance

$T_m^s = 0, 1$: 0 if an order which is not scheduled immediately for minimum lead time plus cycle is a lost order

1 if an order cannot be "lost" regardless of when scheduled

$0 \equiv$ fixed cycle;

$1 \equiv$ any cycle

[redacted] T = tolerance

T = 0; tolerance for advertised cycle only

T = 1; tolerance for any cycle

need T for each Inventory Station

IV / B - C - D

XX Lead Time

L_m^s = 0 if previous station is an inventory station and the make from material is stock

L_m^s = sum of cycle times for the previous stations up to point of carrying stock; use maximum cycle if multiple source paths.

Need L for each manufacturing station

Cycle Time

[redacted]

C_m^s = manufacturing cycle *used for all operating groups*

Section C Rules Parameters

Reorder Point

[redacted] P_m^s is reorder point in units

P_m^s = 0 means non-stock (as required)

P_m^s = ∞ means stock item, reorder every period

Reorder Quantity

[redacted] R_m^{s*} = Economic Order Quantity

Leveling Factor

$0 \leq \alpha^s \leq 1$

α^s = Leveling Factor *used for all operating groups (except Source and Customer)*

α^s = 1 implies absolute leveling

α^s = 0 implies that schedule will follow demand

Section D Auxiliary *values*

~~$0 \leq U \leq 9999$~~

$U_{m,t}^s$ = unfilled orders at end of *period* t

unfilled means not yet scheduled.

~~$0 \leq I \leq 9999$~~

$I_{m,t}^s$ = inventory on hand at end of *period* t

needed for every station except for s = Source or Customer.

Section E Decision values

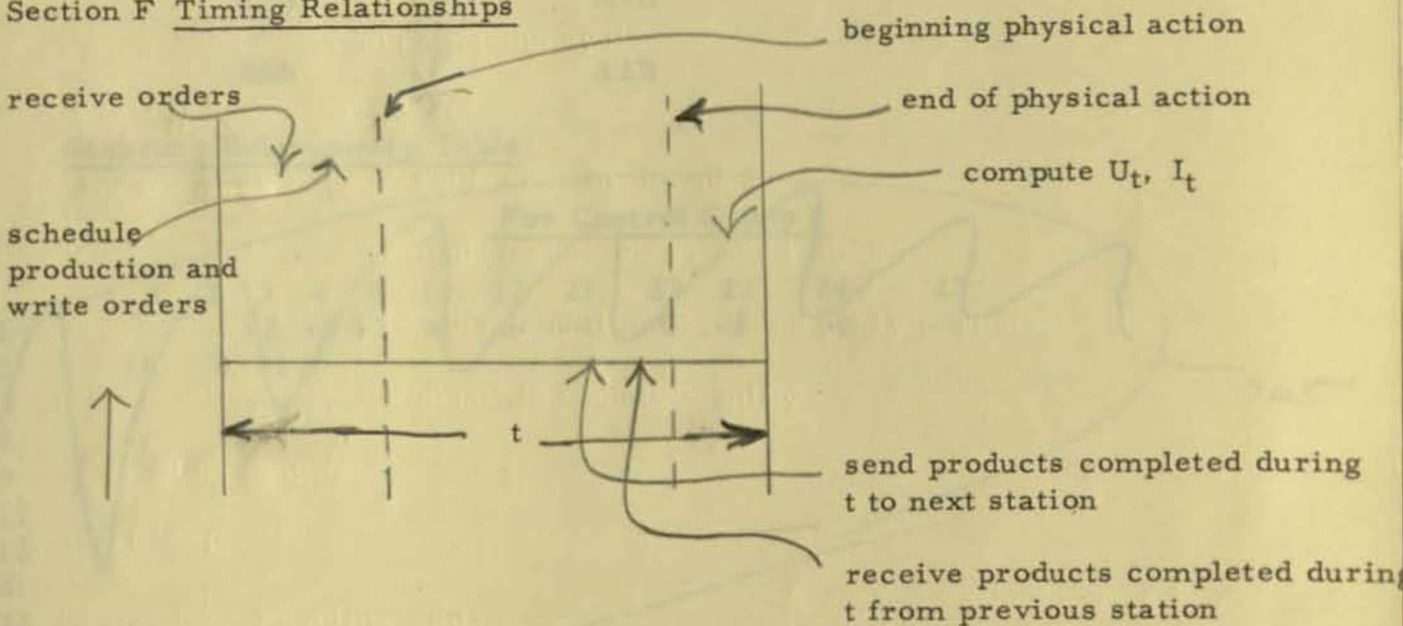
~~0 ≤ S ≤ 9999~~

$S^{s}_{m,t}$ = scheduled orders to start manufacture during period t

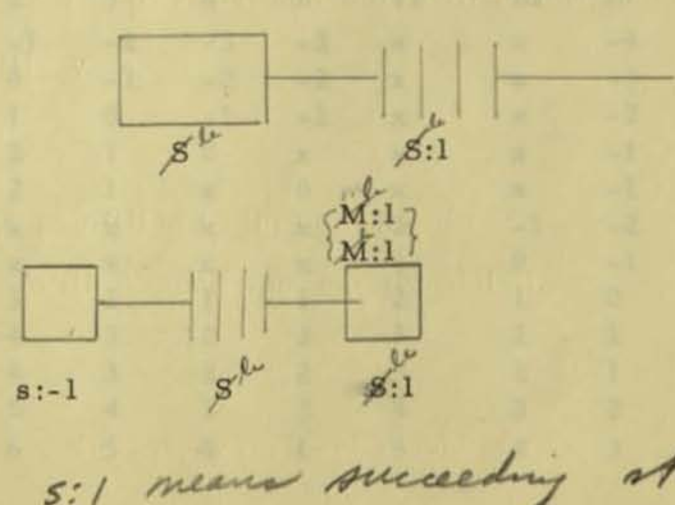
~~0 ≤ R ≤ 9999~~

$R^{s}_{m,t}$ = Quantity ordered by station s during time period t of product bm.

Section F Timing Relationships



Section G Precede and Succeed Concept



s:1 means succeeding station
s:-1 means preceding station

IV-A

where M:1 represents all finished products from station S:1 that use m as an input:

m (input)	m:1 (output)
A	A1, A2
B	A1B, A2B
A1	A1B
A2	A2B
A1B	A1B
A2B	A2B

Sequence Relationship Table

For Control Group #

control group # has this relationship

	1	2	3	4	5	11	12	21	22	23	24	25
1	-	-1	-2	-3	-3	x	x	-4	-5	-5	-6	-7
2	1	0	-1									
3												
4												
5												
11												
12												
21												
22												
23												
24												
25												

out

For Control Group #

control group # has this relationship

	1	2	3	4	5	11	12	21	22	23	24	25
1	0	-1	-2	-3	-3	x	x	-4	-5	-5	-6	-7
2	-	0	-1	-2	-2	x	x	-3	-4	-4	-5	-6
3	2	1	0	-1	-1	x	x	-2	-3	-3	-4	-5
4	3	2	1	0	x	x	x	-1	-2	-2	-3	-4
5	3	2	1	x	0	x	x	-1	-2	-2	-3	-4
11	x	x	x	x	x	0	-1	-2	-3	-3	-4	-5
12	x	x	x	x	x	1	0	-1	-2	-2	-3	-4
21	4	3	2	1	1	2	1	0	-1	-1	-2	-3
22	5	4	3	2	2	3	2	1	0	x	-1	-2
23	5	4	3	2	2	3	2	1	x	0	-1	-2
24	6	5	4	3	3	4	3	2	1	1	0	-1
25	7	6	5	4	4	5	4	3	2	2	1	0

To explain the use of the sequence relationship table we would say that control group #1 precedes control group 4 by 3; hence 1 is s: -3 from 4.

0 means that it is a one to one correspondence

x means that there is no defined relationship since the stations are in parallel or independent flow lines.

It would also be said that station 24 succeeds station 4 by 3. Hence control group 24 is s: 3 from 4.

Part V List of Factors

Section A Systems Parameters

T^{25}_{A1B}

T^{25}_{A2B}

L^{24}_{A1B}

L^{24}_{A2B}

C^{24}

α^{24}

T^{23}_{A2B}

P^{23}_{A2B}

R^{23*}_{A2B}

T^{22}_{A1B}

P^{22}_{A1B}

R^{22*}_{A1B}

L^{21}_{A1B}

L^{21}_{A2B}

C^{21}

α^{21}

T^{12}_{B}

P^{12}_{B}

R^{12*}_{B}

C^{11}

T^{5}_{A2}

P^{5}_{A2}

R^{5*}_{A2}

T^{4}_{A1}

P^{4}_{A1}

R^{4*}_{A1}

L^{3}_{A1}

L^{3}_{A2}

C^{3}

α^{3}

T^{2}_{A}

P^{2}_{A}

R^{2*}_{A}

C^{1}

Section B ~~Systems Status~~

Systems Status Variables

00	t						
01	R^{25}_{A1B}	19	R^{23}_{A2B}	37	R^{4}_{A1}	55	I^{12}_{B}
02	R^{25}_{A2B}	20	R^{22}_{A2B}	38	$S^{3}_{A1, t+2}$	56	I^{5}_{A2}
03	$S^{24}_{A1B, t+6}$	21	$S^{21}_{A1B, t+4}$	39	$S^{3}_{A1, t+1}$	57	I^{4}_{A1}
04	$S^{24}_{A1B, t+5}$	22	$S^{21}_{A1B, t+3}$	40	$S^{3}_{A1, t}$	58	U^{3}_{A1}
05	$S^{24}_{A1B, t+4}$	23	$S^{21}_{A1B, t+2}$	41	$S^{3}_{A1, t-1}$	59	U^{3}_{A2}
06	$S^{24}_{A1B, t+3}$	24	$S^{21}_{A1B, t+1}$	42	$S^{3}_{A2, t+2}$	60	I^{2}_{A}
07	$S^{24}_{A1B, t+2}$	25	$S^{21}_{A1B, t}$	43	$S^{3}_{A2, t+1}$		
08	$S^{24}_{A1B, t+1}$	26	$S^{21}_{A1B, t-1}$	44	$S^{3}_{A2, t}$		
09	$S^{24}_{A1B, t}$	27	$S^{21}_{A2B, t+4}$	45	$S^{3}_{A2, t-1}$		
10	$S^{24}_{A1B, t-1}$	28	$S^{21}_{A2B, t+3}$	46	R^{2}_{A}		
11	$S^{24}_{A2B, t+6}$	29	$S^{21}_{A2B, t+2}$	47	$S^{1}_{A, t}$		
12	$S^{24}_{A2B, t+5}$	30	$S^{21}_{A2B, t+1}$	48	$S^{1}_{A, t-1}$		

Part VI **Description of Model Operation**

~~Section A~~ **Set Initial Parameters**

Pat -
still [illegible]
TIS 303

Set all C independently

Set all P independently

$L^{3}_{A2} = C^{1}$

$L^{3}_{A1} = C^{1}$

$L^{21}_{A1B} = C$

$L^{21}_{A2B} = 0$ because $P^{4} > 0$ and $P^{12} > 0$

$L^{21}_{A2B} = \max \left[0 \text{ (because } P^{12} > 0\text{)},\ C^{3} + L^{3}_{A2} \right]$

$L^{24}_{A1B} = 0$ because $P^{22} > 0$

$L^{24}_{A2B} = 0$ because $P^{23} > 0$

$R^{*} = 0$ if $P = 0$ or $P = 99$

Set all other R^{*} independently

Set T

Set α

~~Section B~~ Set Systems Status

$S^{s}_{m,t}$ is bounded by $t = L^{s}_{m}$ and $t = 1 - C^{s}_{m}$

$U^{s}_{m} = 0$ if $T^{s:1}_{m} = 0$

$I^{s}_{m} = 0$ if $P^{s}_{m} = 0$

~~Section C~~ Provide Demands

The systems analyst must provide the demands to be tested. These can be any set of positive values for each of the two products. The number of time periods tested is a function of the demand cards inserted for testing.

~~Section D~~ Sequence of Decisions

Read demands for period t

Shift S_r to S_{r-1}

Set all $S_{t+L} = 0$

Compute all R, compute all S_{t+L}

Update U, I

Print status for period t

advance t

Part VII Formulation of Rules

Operational Rules

what → Updating System

$$U^{s}_{m,t} = T^{s:1}_{m} \cdot (U^{s}_{m,t-1} + R^{s:1}_{m,t} - S_{m,t+L^{s}_{m}})$$

$$I^{s}_{m,t} = I^{s}_{m,t-1} + S^{s:1}_{m,t+1-C^{s:1}} - \sum_{m} S^{s:1}_{m:1,t}$$

VII

VIII -A

Spread Out

<u>Decision Rules</u>

$$S^{s}_{m,\,t+L^{s}_{m}} = \min \left\{ \left[\alpha^{s}\, S^{s}_{m,\,t+L^{s}_{m}-6} + (1-\alpha^{s})\left(R^{s:1}_{m,\,t} + U^{s}_{m,\,t-1}\right) \right],\ \left[I^{s:-1}_{m:-1,\,t-1} \right] \right\}$$

$$R^{s}_{m,\,t} = \beta R^{s*}_{m} + (1-\beta) \sum_{m} \left[R^{s:2}_{m:1,\,t} - (1-T^{s:2}_{m:1}) (R^{s:2}_{m:1,\,t} - S^{s:1}_{m:1,\,t}) \right]$$

~~$\beta = 1$ if $R^{s*}_{m} \neq 0$~~

and if $\Big\{ I^{s}_{m,\,t-1} + \sum S^{s:-1}_{m} - \sum_{m} U^{s:1}_{m:1,\,t-1}$

$$- \sum_{m} \left[R^{s:2}_{m:1,\,t} - (1-T^{s:2}_{m:1}) (R^{s:2}_{m:1,\,t} - S^{s:1}_{m:1,\,t}) \right] \Big\}$$

$$\leq P^{s}_{m}$$

$\beta = 0$ if $\{\ \} > P^{s}_{m}$

or if $R^{s*}_{m} = 0$

Part IX MODEL IMPROVEMENT

Section A Error Analysis ~~Errors in Trail Run and Ways to Correct Them.~~

The following material simply analyzes certain errors which

~~Analysis for Errors~~

t	S^3_{A2}	S^3_{A1}	I^4_{A1}	S^{21}_{A1B}	R^4_{A1}	U^3_{A1}	I^2_A	S^1_A
-1	10	10		10				0
0	10	16	50	10	0	0	0	20
1	10	10	40	20	60	37	50	70
2	11	23	10	40	60	56	87	70
3	11	42	23	10	10	18	54	20
4	11	48	44	20	10	15	15	20
5	11	43	72	20	10	13	19	20
6	11	38	100	15	10	12	10	20
7	11	35	123	15	10	10	16	20
8	11	32	148	10	10	9	7	20
9	11	29	170	10	10	8	13	20
10	11	27	189	10	10	7	5	20
11	11	25	206	10	10	6	11	20
13	11	21	234	10	10	5	9	20
14	11	20	245	10	10	4	2	20
15	11	19	255	10	10	4	8	20
16	11	18	264	10	10	4	1	20
17	11	18	272	10	10	4	8	20

showed up during the initial trial run. What is needed is that these runs and others be analyzed in depth to detect and correct the systems equations and the control rules.

$$C^3 = 2,\; L^3_{A1} = 1,\; R^{4*}_{A1} = 60,\; P^4_{A1} = 60;\; T^4_{A1} = 1$$

$$L^3 = .75;\; C^1_A = 1$$

$$I^4_{A1,t} = I^4_{A1,t-1} + S^3_{A1,t+1-2} - S^{21}_{A1B,t}$$

$$I^4_{A1,1} = 50 + 10 - 20 = 40$$ ok

$$R^4_{A1} = 0,\; 60 \text{ never } 10$$ ok

Ordering Rule Error

The way R rule is set

$\beta = 0$ after second week since $\{ \quad \} > P^s_m$

and therefore second term is always ordered. This should be set up so that either R^{s*}_m is ordered or else nothing for a stock item on economic order rule.

The ordering rule ~~should~~ might take the following form:

$$R^s_{m,t} = \beta\gamma R^{s*}_m + (1-\beta) V^{s:2}_{m,t}$$

$$\beta = \begin{cases} 0 \text{ if } R^{s*}_m = 0 \\ 1 \text{ if } R^{s*}_m > 0 \end{cases}$$

$$\gamma = \begin{cases} 0 \text{ if } Q^s_{m,t} > P^s_m \\ 1 \text{ if } Q^s_{m,t} < P^s_m \end{cases}$$

Where

$$V^{s:2}_{m,t} \equiv \sum_m \left[R^{s:2}_{m:1,t} - (1 - T^{s:2}_{m:1})\,(R^{s:2}_{m:1,t} - S^{s:1}_{m:1,t}) \right]$$

$$Q^{s}_{m,t} = \left\{ I^{s}_{m,t-1} + \sum_t S^{s:-1}_{m} - \sum_m U^{s:1}_{m:1,t-1} - V^{s:2}_{m,t} \right\}$$

Schedule Rule Error

$$S^{3}_{A1,t+1} = \min. \left\{ \left[.75\, S^{3}_{A1,t} + (.25)\,(R^{4}_{A1,t} + U^{3}_{A1,t-1}) \right], \left[I^{2}_{A,t-1} \right] \right\}$$

$$U^{3}_{A1,t} = T^{4}_{A1} \left(U^{3}_{A1,t-1} + R^{4}_{A1,t} - S^{3}_{A1,t+1} \right)$$

check U rule

$$U^{3}_{A1,1} = 1\,(0 + 60 - 23) = 37$$

$$U^{3}_{A1,2} = 1\,(37 + 60 - 41) = 56 \qquad \text{ok}$$

$$S^{3}_{A1,2} = \min. \left\{ \left[.75\,(10) + .25\,(60 + 0) \right], \left[0 \right] \right\}$$

$$= \min. \left\{ (7.5 + 15),\ 0 \right\} = \min. \left\{ 23, 0 \right\}$$

should be 0

answer actually was 23;

check ~~fpx~~ for programming error on min. test.

Is there a logic error in using I_{t-1} ; wouldn't it be more logical to examine I_{t+L-1}; If this is logical, can it be predicted. Probably yes since all S data is available.

$$I^{2}_{A,16} = I^{2}_{A,15} + S^{1}_{A,16} - S^{3}_{A1,t6} - S^{3}_{A2,t6}$$

$$I^{2}_{A,16} = 8 + 20 - 18 - 11 = -1$$

$$I^{2}_{A,17} = 1 + 20 - 18 - 11 = -8$$

It isn't logical for an inventory to go negative. This should be automatically prevented by the operation of the Scheduling Rule. There is also an error in that minus numbers are not recognized and they are recorded and treated as though they were positive numbers with the same absolute value.

$$S^{3}_{A1,17} = \min.\left\{ \left[.75\ (18) + .25(10 + 4) \right], \left[8 \right] \right\}$$

$$\min.\left\{ (13.5 + 3.5),\ 8 \right\}$$

$$= \min.\ (17,\ 8)$$

answer was actually 18;

indicating that each factor was individually rounded upward and that the inventory limit was ignored.

The basic rule seems reasonable except that the unfilled should be cleared up.

$$S^{21}_{A1B,1} + S^{21}_{A2B,1} = 20 + 10 = 30$$

$$I^{4}_{B,0} = 20$$

There is no summary test for total usage

Rule should probably be corrected as follows:

$$S^{s}_{m,t+L^{s}_{m}} = \min.\left\{ \left[\alpha^{s}\ S^{s}_{m,t+L^{s}_{m}-1} + (1-\alpha^{s})\ (R^{s:1}_{m,t} + U^{s}_{m,t-1}) \right], \left[I^{s:1}_{M:-1,t-1} - S^{s}_{m:0,t+L^{s}_{m}} \right]_{\text{all } s:-1} \right\}$$

~~where M~~
where m:0 means other model using same resources.

For example:

$$S^{21}_{A1B,t} = \min.\left\{\left[\quad\right], \left[I^{4}_{A1,t-1} \right], \left[I^{12}_{B,t-1} - S^{21}_{A2B,t} \right]\right\}.$$

$$\text{for } t = 1;\ S^{21}_{A1B,t} = \min.\left\{25,\ 50,\ (20 - 10)\right\} = 10$$

$$S^{21}_{A2B,t+3} = \min.\left\{\left[.5\ (10) + .5\ (10 + 0)\right], [20], [0]\right\}$$

$$= 0\ ;$$

answer actually was 10

Therefore in programming must not have tested $I^{5}_{A2,\ t-1}$

However, this is fortunate since if test was made of a non-stock inventory there would never be any available, hence S = 0 always and no R^{5}_{A2} would ever be placed. We should not examine inventory if P = 0; so Scheduling Rule should read:

$$S \quad = \min.\left\{\left[\quad\right], \left[\quad\right], \left[\quad\right] P^{s:-1}_{m:-1} \neq 0\right\}$$

should prevent negative ~~x~~ values on unfilled calculation. Must avoid rounding errors such that order quantity for a non-stock inventory are greater than demand.

It looks like an error to permit a non-stock station to order in relation to demand on user station. This permits a non-stock station to accumulate inventory in anticipation of an increase in schedule rate.

It looks like the R rule should be corrected for non-stock to depend soley on $S^{s:1}_{m:1,\ t} + L^{s:1}_{m:1}$ and nothing else. The allowed lead time already compensates for availability delays.

The ordering rule should look like this:

$$R^{s}_{m,t} = \beta\gamma R^{s*}_{m} + (1-\beta)\,\delta V^{s:2}_{m,t} + (1-\beta)\,\delta\, S^{s:1}_{m:1,\ t+L^{s:1}_{m:1}}$$

where β, γ and V are defined as before

$$\text{and } \delta = \begin{cases} 0 \text{ if } P^{s}_{m} > 0 \\ 1 \text{ if } P^{s}_{m} = 0 \end{cases}$$

Inventories should not be permitted to go negative -

Inventory Rule should be ~~xeqx~~ rewritten

$$I^{s}_{m,t} = \text{max.}\left\{\left[I^{s}_{m,t-1} - \sum_{m} S^{s:1}_{m:1,t}\right],\left[0\right]\right\} + S^{s:-1}_{m,\ t+1-\epsilon^{s:-1}}$$

Conceptually how can there be an inventory at a non-stock station.

This should be prevented

$$I = (1-\delta)\left\{\qquad\qquad\right\} \text{ etc.}$$

Section B ~~Set Up Complete Set of Test Demands~~

Demand Analysis

Take model A1B independently, model A2B independently, models jointly in various combinations.

Usage mean : low

medium

high

Usage slope: trend up

trend down

no trend

Permanent discontinuity : step up

step down

Temporary change: up early

down early

up middle

down middle

up end

down end

Magnitude of discontinuity or change: small

moderate

large

Period of change: short

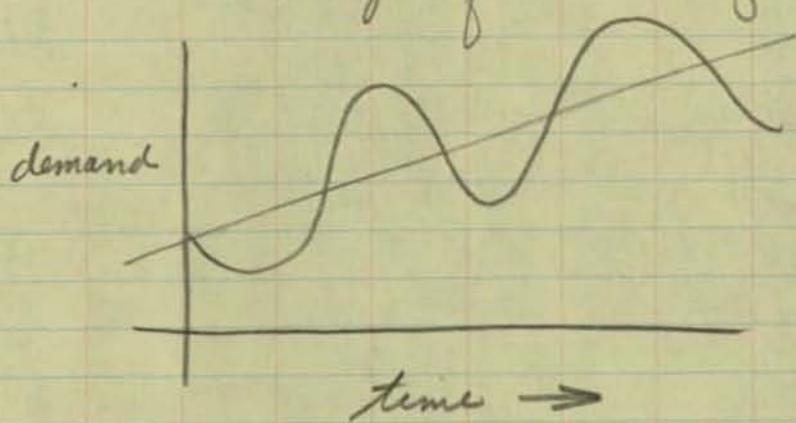
Medium

long

Random pattern

Seasonal ~~Sinusoidal~~ variation

If we view a demand curve as being a "sinusoidal" function with a trend and intercept factor included then these changes can probably be handled through equation modification.



There are four parameters needed to define a particular demand pattern (if it's consistent throughout test period):

Y = Y intercept (initial mean); high, med, low
S = Slope of mean line ; + large, + med, 0, - med, - large
F = Frequency of change ; short, med, long
A = amplitude of change ; small, med, large

To test all combinations would take 135 runs. However, using standard statistical experimental design procedures and omitting those combinations which are logically meaningful the number of runs could be ~~not~~ reduced by a ten to one ratio.

Section C Other

Multi-stage Assembly Model

1. Write - up model foundations
2. prepare effective flow chart of computer program
3. obtain computer program (written) - (punched cards)
4. obtain input format detail - sample cards used -
5. reprogram rules changes
6. Prepare test runs for demand data
7. Layout test runs for rules analysis

Section C General Rules Discussion

To actually simulate such a system requires the use of decision and operation rules for transforming the system values at time t-1 into those at time t. To do this means the establishment of decision rules for determining appropriate R's and S's and operational rules for transforming U's and I's.

The two operational rules are:

$$U^{s}_{m,t} = T^{s:1}_{m} \left(U^{s}_{m,t-1} + R^{s:1}_{m,t} - S^{s}_{m,t} + L^{s}_{m}\right)$$

Only a source or ~~non-inventory~~ manufacturing control group can have an unfilled balance.

s s:1

$$I^{s}_{m,t} = I^{s}_{m,t-1} + S^{s:-1}_{m,t-C^{s:-1}_{m}+1} - \sum_{m} S^{s:1}_{m:1,t}$$

Only an inventory control group can have an inventory balance.

To determine I^{12}_{B} both S^{21}_{A1B} and S^{21}_{A2B} must be considered.

Obviously any attempt to write completely generalized ordering rules initially would be doomed to failure. However, it did appear reasonable to establish relatively general rule forms based upon the results of previous work and try various parameters in these rules for better system understanding. The simulation model should, of course, be programmed in such a way as to permit straightforward changing of rule format.

For the purpose of scheduling and ordering rules each control group is related to every pertinent control group by a "stages before" or "stages after" representation.

The general scheduling rule without inventory limitation is:

$$S^{s}_{m,t+L^{s}_{m}} = \alpha^{s} S^{s}_{m,(t+L^{s}_{m})-1} + \left[(1-\alpha^{s}) \left(R^{s:1}_{m,t} + U^{s}_{m,t-1}\right)\right]$$

$0 \leq \alpha^{s} \leq 1$ α^{s} = Leveling factor

α^{s} = 1 implies absolute leveling

The general form of the scheduling rule has to take cognizance of leveling considerations. This format was suggested by the work done by Mills and Singleton (Operation Research and Synthesis Services). The concept is that each control group will schedule its own work rather than expecting its customers to provide a balanced set of orders. Since primary emphasis is on interstage relationships, capacity restrictions and unit loading data have been ignored. The real complexity enters in terms of availability of stock inventories or replenishment orders. If the "make-from" materials are carried on a stock basis at all directly preceding inventory positions, the maximum schedule quantity is the minimum inventory of the preceding inventory positions. Where any or all of the make-from materials are not carried on a stock basis the problem is more difficult to state. The limitation here is two-fold; first the ability of the inventory preceding the supplying station to support the quantity desired and the ability of the supplying station to make the quantity desired. For a series of non-stock products this continues to build-up until the ultimate source is reached.

There are two other complications both to do with competitive demand for a product. First is the case where one of the inventories is used in two or more finished products where an allocation rule is needed if $I^{s}_{m,t-1} < \sum_{m} S^{s:1}_{m:1,t}$

The other case is where non-stock make-from materials are involved and the make-from quantity isn't adequate to support all competing demands

Section #B **Program Logic**

1. Read R^{25}
2. Shift S
3. Set $S_{t+L} = 0$
4. Compute all R & S
5. Update U, I
6. Print
7. Advance t

Section #C **Detailed Rules Actually Programmed**

Read R_s

Shift S's (All)

Set $S_{m,\, t+L} = 0$

$$S^{24}_{A1B,\ t+L^{24}_{A1B}} = \min.\left\{\left[\alpha^{24}\ S^{24}_{A1B,\ t+L^{24}_{A1B}-1} + (1-\alpha^{24})\ (R^{25}_{A1B,t} + U^{24}_{A1B,t-1})\right],\right.$$

$$\left.\left[I^{22}_{A1B,t-1}\right]\right\}$$

S^{24}_{A2B} (similar to above)

$$R^{23}_{A2B,t} = \beta\ R^{23*}_{A2B} + (1-\beta)\left[R^{25}_{A2B,t} + (1-T^{25}_{A2B})\ (R^{25}_{A2B,t} - S^{24}_{A2B,t}\right]$$

$$\text{if}\left\{I^{23}_{A2B,t-1} + \sum_t S^{21}_{A2B,t} - U^{24}_{A2B,t-1} - \left[R^{25}_{A2B,t} - (1-T^{25}_{A2B})\ (R^{25}_{A2B,t} - S^{24}_{A2B,t})\right]\right\}$$

$$\leq P^{23}_{A2B}$$

R^{22} = (Similar to above)

$$S^{21}_{A1B,\,t+L^{21}_{A1B}} = \min.\left\{ \left[\alpha^{21}\ S^{21}_{A1B,\,t+L^{21}_{A1B}-1}\right], \left[(1-\alpha^{21})\,(R^{22}_{A1B,\,t} + U^{21}_{A1B,\,t-11})\right], \right.$$

$$\left. [illegible].\ \left[I^{4}_{A1,\,t-1}\right], \left[I^{12}_{B,\,t-1}\right] \right\}$$

S^{21}_{A2B} (similar to above)

$$R^{12}_{B,\,t} = \beta\ R^{12*}_{B}$$

$$+ (1-\beta) \left\{ \left[R^{22}_{A1B,\,t} - (1-T^{22}_{A1B})\,(R^{22}_{A1B,\,t} - S^{21}_{A1B,\,t}) \right] \right.$$

$$\left. + \left[R^{23}_{A2B,\,t} - (1-T^{23}_{A2B})\,(R^{23}_{A2B,\,t} - S^{21}_{A2B,\,t}) \right] \right\}$$

$$U^{24}_{A1B,\,t} = T^{25}_{A1B}\ (U^{24}_{A1B,\,t-1} + R^{25}_{A1B,\,t} - S^{24}_{A1B,\,t+L^{24}_{A1B}})$$

$$I^{22}_{A1B,\,t} = I^{22}_{A1B,\,t-1} + S^{21}_{A1B,\,t+C^{21}-2}$$

$$- S^{24}_{A1B,\,t}$$

$$I^{12}_{B,\,t} = I^{12}_{B,\,t-1} + S^{11}_{B,\,t+C^{11}-2}$$

$$- (S^{21}_{A1B,\,t} + S^{21}_{A2B,\,t})$$

Section D — Additional Description

Customer station

R(24) each t

T(24)

No U, S, S rule, R rule, C, 1, I

Source station

Treat as inventory station with infinite inventory.

C each source

S each source, up to C.

$S_t^s = R_t^{s:1}$

No T, [redacted], R, …, α, U ; $I = \infty$

Inventory

Supplied: I_0^s, R_0^s, S_0^s, T^s, …, U_0^s, τ^s

T ≠ 0 or ∞ means either; it is represented as T=0 or T=1

$C^s = 0$ all s

no α

$$S_t^s = \min\left\{ \left(R_t^{s:1} + U_{t-1}^s\right), I_{t-1}^s \right\}$$

$$I_t^s = I_{t-1}^s + S_{t-C^{s:2}-1}^{s:2} - S_t^s$$

$$U_t^s = T^{s:1}(U_{t-1}^s + R_t^{s:1} - S_t^s)$$

$$R_t^s = \beta\, R_t^{s:1} + \gamma(1-\beta)\, R^*$$

$\beta = 1$ if $p^s = 0$ or ∞

$\beta = 0$ if $0 \leq p^s < \infty$

$\gamma = 1$ if $(I_t^s + S_{t-\epsilon^{s:2}+1}^{s:2} + \ldots + S_{t-1}^{s:2}) \leq p^s$

= o otherwise

Manufacturing station ~~Maker~~

Supplied R_0^s (for each input model), S^s (for each output model), for all time periods up to t-ϵ

$T^s = 0$ or ∞ (represented by 1) for each model

U_0^s for " "

α^s for " "

C^s is the same for all models

No ~~β~~, β, γ, I

Rules

(for each model)

$$S_t^s = \text{Min.}\left\{ (R_t^{s:1} + U_{t-1}^s), \left[\alpha^s S_{t-1}^s + (1-\alpha^s)(R_t^{s:1} + U_{t-1}^s\right], I_{t-1}^{s:2} \right\}$$

for each model

$$U_t^s = T^{s:1}\left[U_{t-1}^s + R_t^{s:1} - S_t^s \right]$$

$$R_t^s = S_t^s \quad \text{each model}$$

DRAFT - 9/14/59

TECHNICAL REPORT

Subject: Flow Characteristics of Manufacturing Shops

A manufacturing shop can be defined by the situations within it and the transportation operations between these situations. If all of the situations in a particular shop are sequentially numbered 1, 2, ..., N and two special situations defined: a zero station which is a material source point and station N + 1 which is a destination then the following simulation is possible: A matrix can be drawn on which each row represents a source station and each column a destination station. An entry of one will be made in each block which is varied for a particular block. In other words, the complete routing for a job would be indicated by a set of ones in various blocks with the following proviso: There must be at least one entry in row zero and at least one entry in column N + 1. This set of ones defines the procedures and routing of a particular job.

It is also postulated that there is an ordering for a "flow shop" of the of the stations such that all ones will appear on or above the main diagonal. This definition of a flow shop implies that if every job which goes through an area is mapped onto a routing matrix and if the above postulate can be met, then by the introduction of the concept zero time at a station every job for which an entry occurs on or above the main diagonal can be represented by an equivalent set of ones on the main diagonal itself. It is evident that if all entries occur just at the main diagonal this is the definition of a flow shop: a straight line sequence with no reversals.

This is a very useful simplification since it can be shown that many job shops would fit this category and that whenever this is true a flow shop simulation can be used to represent its performance. Typically, these are

This holds exactly only if jobs have a uniform controlled sequence from first to last. Otherwise special provision must be made in the model to test for station skipping

faster and ~~[illegible]~~ cheaper models than those ~~provided for by~~ required for mixed routing. An illustrative example is indicated below carrying through these operations for ~~the~~ five jobs in a shop of five stations.

Routing Matrix

Destination Station

Source Station

	1	2	3	4	5	6
0						
1						
2						
3						
4						
5						

Job	Operations
A	2 4 3
B	5 1 2
C	1 2 4
D	4 3
E	1 2 4 3

To

From

	1	2	3	4	5	6
0	CE	A		D	B	
1						
2		BCE		ACE		B
3						ADE
4			ADE			C
5	B					

Station Old Number	Station New Number
0	0
1	2
2	3
3	5
4	4
5	1
6	6

Job	new no. operations
A	345
B	123
C	234
D	45
E	2345

New ~~Nos.~~ Routing matrix

From \ To	1	2	3	4	5	6
0	B	CE	A	D		
1		B				
2			BCE			
3				ACE		B
4					ADE	C
5						ADE

Possible Routings

	1	2	3	4	5	6
0	1	1|	1	1		
1		1				
2			1||			
3				1||		1
4					1||	1
5						1||

Jobs
A
B
C
D
E

note: ⊛ means 0 time
\# means real time

Operations (new numbers)

JOBS/STNS	1	2	3	4	5
A	⊛	⊛	#	#	#
B	#	#	#	⊛	⊛
C	⊛	#	#	#	⊛
D	⊛	⊛	⊛	#	#
E	⊛	#	#	#	#

Hence, this can be simplified to the following matrix and represented by a flow shop model.

	1	2	3	4	5	6
0	1					
1		1				
2			1			
3				1		
4					1	
5						1

An exciting by-product of this flow matrix concept is that it provides the ability to measure the "degree" of the job or flow shop with which we are dealing.

For example, if we divide each number in the matrix by the total of entries in the matrix we get the probability of making a certain move. By definition a pure job shop would have each block filled by the number $\frac{1}{(N+1)^2}$ where N is the total number of stations. This is somewhat similar to the concept of a "bit" in information theory -- that minimum amount of information needed to decide between two equally likely alternatives. Similarly if there are 64 boxes (a 8 × 8 matrix) then we would need 6 bits to identify the next move ($2^6 = 64$).

¶ However at the opposite extreme -- the pure flow shop -- there is but one filled block for each row and each column. In this case

The ~~stated~~ entries on the main diagonal ~~each entry~~ would equal $\frac{1}{n+1}$ where n is the number of stations while all other entries are 0. In this second case, given an initial state the next move is absolutely determined.

Obviously for most of our shops neither of these extreme conditions hold and it might be useful to have a measure which would express the ~~~~ nature of the shop in these terms. Factors to be considered include matrix density, probability variance and possibly location (above or below the main diagonal).

Let's work with a simple 3 station shop which yields a ~~matrix~~ 4×4 matrix.

Test Matrix 1

from \ to	1	2	3	4
0	.0625	.0625	.0625	.0625
1	.0625	.0625	.0625	.0625
2	.0625	.0625	.0625	.0625
3	.0625	.0625	.0625	.0625

This would be a pure job shop.

$$\sum_{\text{all } i,\ \text{all } j} p_{ij} = 1.0$$ must be ~~is~~ true for all matrices

where p_{ij} = probability of moving from station i to station j; i may equal j.

$p_{ij} = \frac{1}{n^2}$ (for $n \times n$ matrix)

$p_{i,j} = p_{i',j'}$ for all i, j, i', j' for job shop

(with previous page)

standard deviation ~~variance~~ of $P_{i,j} = 0$

Test Matrix 2

		to 1	2	3	4
from	0	.25			
	1		.25		
	2			.25	
	3				.25

This is a pure flow ~~job~~ shop

$P_{i,j} = P^2_{i',j'} = \frac{1}{n}$

where $i' = i+1$; $i = 0,1,2$
and $j' = j+1$; $j = 0,1,2,3$

or $P_{ij} = \frac{1}{n}$ for $j = i+1$; $i = 0,1,2,3$

~~variance of P_i~~ $P_{ij} = 0$ for $j \neq i+1$

standard deviation ~~variance~~ of P_{ij} can be calculated as follows

$$\sigma = \sqrt{\frac{\sum_{i,j} (P_{ij} - \bar{P})^2}{n^2}}$$

$$\bar{P} = \frac{\sum_{i,j} P_{ij}}{n^2} = \frac{1.0}{16} = .0625$$

4 values with $.25 - .0625 = (.1875)^2 = .0351$

12 values with $0.0 - .0625 = (-.0625)^2 = .0039$

$$\sigma = \sqrt{\frac{4(.0351) + 12(.0039)}{16}} = \sqrt{.0117} = .1082$$

~~This will probably be just about the worst possible standard deviation~~

In the following situation

Test Matrix 3

from \ to

	1	2	3	4
0	.49	~~.01~~		
1		.01 ~~.01~~		.48
2			.01	
3				.01

This illustrates that $\sum_{\substack{i=k \\ \text{all } j}} p_{ij} = \sum_{\substack{j=k \\ \text{all } i}} p_{ij}$; $k \neq 0, 4$

— or what goes in must come out.

This is also an extremely unbalanced case

$\bar{p}$ for $n^2 = 16$ = .0625

σ = .1597

This is probably approaching a limiting case for the standard deviation for a 4x4 matrix

also $\sum_{\substack{i=0 \\ \text{all } j}} p_{ij} = \sum_{\substack{j=4 \\ \text{all } i}} p_{ij}$

~~It~~ Since the mean is obviously not a good measure when taken across all values, it may be useful to consider the mean of only those blocks which have a $p_{ij} > 0$. Let's call this $\bar{p}*$

for Test Matrix 1:

$\bar{p}* = .0625$; $\bar{p} = .0625$

for test matrix 2 ~~and Test~~

$\bar{p}* = .25$; $\bar{p} = .0625$

It is evident that this simply measures $\frac{b*}{b}$ where $b*$ = number of blocks with non-zero entries and b = total number of blocks $= n^2$

This gives a measure of matrix density but still doesn't really differentiate matrix 3 ($\bar{p}* = .2$) from Matrix 2 ($\bar{p}* = .25$). We need some way of indicating the distribution of values among those blocks which have non-zero entries

We might calculate the standard deviation of just the non-zero entries.

For Test matrix 2

$\sigma_{p*} = 0$

For Test matrix 3

$\sigma_{p*} \cong .23$

This seems to give a somewhat better measure of internal variance.

Initial Conclusions

(A) The Pure job shop is characterized by:

(1) $\bar{p}* = \bar{p} = \frac{1}{n^2} =$ minimum value

(2) $\sigma_p = 0$

(3) $\sigma_{p*} = 0$

~~(B)~~

The pure flow shop is characterized by:

(1) $\bar{p}* = \frac{1}{n} =$ max value

(2) $\sigma_{p_2} = .250$; $\sigma_{p_3} = .155$; $\sigma_{p_4} = .108$

(3) $\sigma_{p*} = 0$

(10A)

(A)

All flow matrices must satisfy the following conditions:

(1) $\sum_{\substack{\text{all } i \\ \text{all } j}} p_{i,j} = 1.0$

(4) ~~(2)~~ $\bar{p} = \frac{1}{n^2}$ where n = number of stations + 1

(5) ~~(3)~~ $\bar{p}^* = \frac{b^*}{n^2}$ where b^* = ~~equals~~ number of blocks with non zero entries.

~~(4)~~

(2) $\sum_{\substack{i=k \\ \text{all } j}} p_{i,j} = \sum_{\substack{j=k \\ \text{all } i}} p_{i,j}$; $k = 1, 2, \ldots, n-1$

(3) $\sum_{\substack{i=0 \\ \text{all } j}} p_{i,j} = \sum_{\substack{j=n \\ \text{all } i}} p_{i,j}$

For a given matrix compute

(1) $\frac{\bar{P}* - P_{MIN}}{P_{MAX}}$

where $P_{MIN} = \frac{1}{n^2}$

$P_{MAX} = \frac{1}{n}$

(2) σ_p

Consider other measures

AN EXPERIMENTAL MULTI-STAGE ASSEMBLY SIMULATOR

by Burton Grad
Technical Counselor
Manufacturing Services
General Electric Company

This program was written by D. D. McCracken, then of the General Electric Company, and many of the formulations are a result of his suggestions. The original concept for such a simulator was suggested by H. Mills of Market Research Corporation of America. The work described was completed prior to June 1, 1958.

Contents

Part I INTRODUCTION

Work has been done on pure inventory models, on job shop manufacturing models, on flow shop manufacturing models and on conveyor models. However, we have not found in the literature an attempt to put together the basic building blocks in a multi-stage assembly model.

Rather than try to directly simulate a real system we have instead analyzed in depth the basic elements of an experimental multi-stage assembly system. From this has evolved a control group concept[1] and a series of definitions of the ways in which one stage of a system can be linked with another.

The magnitude of the system is a direct reflection of the number of stages, the variety of differentiated products (raw materials, parts, and assemblies)and the number of time periods for the operational cycle in each stage. Therefore, it was felt desirable to simulate a very small system as an initial pass, with the full expectation that this would permit a sharper, clearer understanding of the various pheno mena.

Part II GENERAL PROGRAM CONSIDERATIONS

The purpose of this model is to explore strategic considerations in planning a manufacturing control system. Decisions to be considered include: cycle planning, placement of inventories, choice of economic or automatic ordering plans, form of scheduling rules, importance of feedback of of progress information and importance of integrated versus decentralized decision planning.

The simulator was prepared for the IBM 702 because the machine was being used for other Integrated Systems Project work. This was to be a small program which could be written readily in "Script" and modified by Project programmers.

---

[1] February 23, 1958, Integrated Systems Project Progress Report, Production Control Service, 570 Lexington Avenue, New York 22, New York.

BG

~~Do you want to draw this figure typeset up?~~

BG

Pl. put in arrows

P

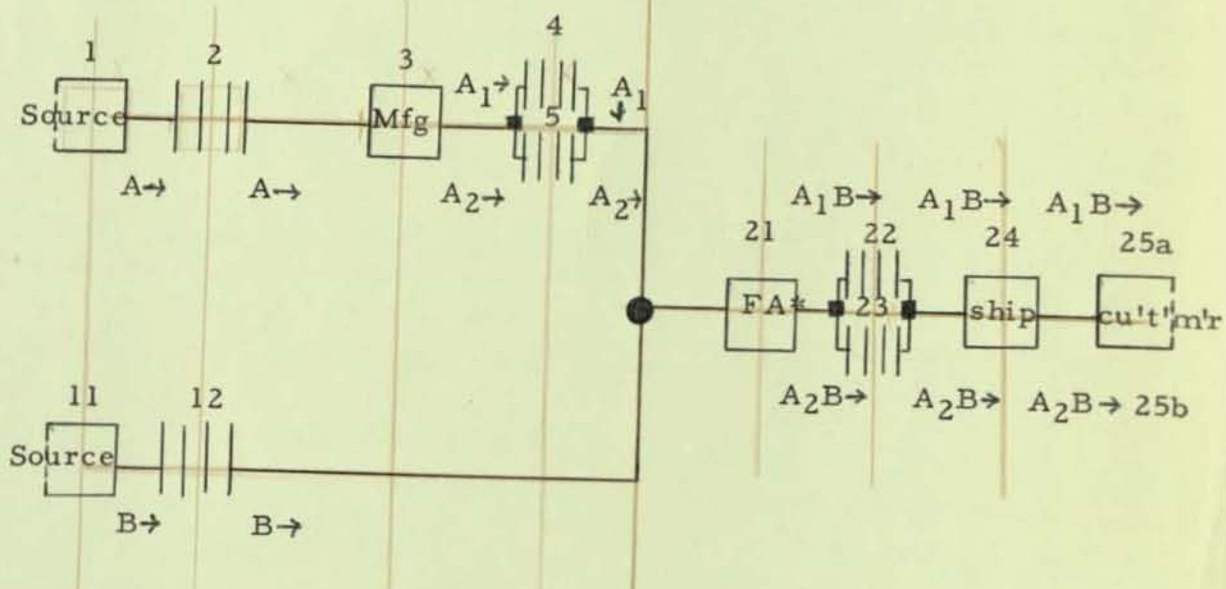
The model has been intentionally simplified so that exploration could concentrate on order of magnitude changes. For example, an inventory control group gives instantaneous service; all communications channels are instantaneous; cycles are either one or two periods long; all sources have infinite supplies; there is only one customer; capacity and load considerations are ignored; the number of states in the system are limited; there are only two products each of which consists of two components; there is but one source for each input component. No provision is made for costs, set up, variable run time or variable usage quantity.

It is intended that the model be initially used for testing the form of ordering and scheduling rules. The need for allocation rules (for input components) could also be tested. The factors in the scheduling and ordering rules will be examined in the light of different cycles and demand patterns.

Part III DESCRIPTION OF PHYSICAL MODEL

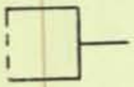
Section A Flow Chart

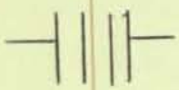
The simulated model looks as follows:

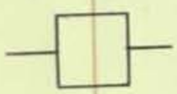


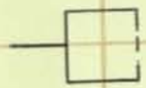
* Final Assembly

Section B Symbols Used In Chart


source of materials

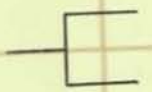

inventory control group

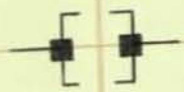

manufacturing control group


customer; product "sink"


physical material flow, always from left to right


summarizing junction


distributing junction


exclusive "or"


"and"

Section C <u>Materials, Parts, Assemblies</u>

Raw materials	A, B
Manufactured Parts	A1, A2
Finished Models	A1B, A2B

Section D <u>Precedence Charts</u>

The physical flow precedence matrix for this product is represented as follows:

To Control Group #

Physical Flow / From Control Group #	1	2	3	4	5	11	12	21	22	23	24	25
1		1										
2			1									
3				1	1							
4								1				
5								1				
11							1					
12								1				
21									1	1		
22											1	
23											1	
24												1
25												

Where 1 signifies that this flow does exist and blank indicates that it does not. Where 2 ones appear in the same row an "or" condition is signified. Where 2 ones appear in the same column an "and" condition is indicated.

The following precedence matrix actually shows the specific product which flows from one control group to the next.

Physical Product Flow Matrix

To Control Group #

From Control Group #

	1	2	3	4	5	11	12	21	22	23	24	25a	25b
1		A											
2			A										
3				A_1	A_2								
4								A_1					
5								A_2					
11							B						
12								B					
21									A_1B	A_2B			
22											A_1B		
23											A_2B		
24												A_1B	A_2B
25a													
25b													

Product Gozinto Matrix

"Finished component" (columns); "Raw Material" (rows)

	A	B	A_1	A_2	A_1B	A_2B
A			1	1		
B					1	1
A_1					1	
A_2						1
A_1B						
A_2B						

Station Transformation Matrix

"Finished component" (columns); "Raw Material" (rows)

	A	B	A_1	A_2	A_1B	A_2B
A			3			
B					21	21
A_1					21	
A_2						21
A_1B						
A_2B						

Part IV DEFINITION OF FACTORS

Section A Notation

s ≡ station; always used as a superscript
a separate station number is defined for each source, inventory group, control group and customer.

s = 1, 2, ..., 25

m ≡ "model" or part, product, material, assembly
an item changes identity after going thru any operational control group (other than shipping)

m = A, B, A_1, A_2, A_1B, A_2B

t ≡ time period

$0 \leq t \leq 9999$

Section B Description of Physical System

Cycle Tolerance

$T_m^s = 0, 1$: 0 if an order which is not scheduled immediately for minimum lead time plus cycle is a lost order.

1 if an order cannot be "lost" regardless of when scheduled.

0 ≡ fixed cycle; 1 ≡ any cycle

T = tolerance

T = 0; tolerance for advertised cycle only

T = 1; tolerance for any cycle

need T for each Inventory Station

Lead Time

$0 \leq L^{s}_{m} \leq 6$; 0 if previous station is an inventory station and the make from material is stock

L^{s}_{m} = sum of cycle times for the previous stations up to point of carrying stock; use maximum cycle if multiple source paths.

need L for each manufacturing station

Cycle Time

$1 \leq C^{s} \leq 2$

C = manufacturing cycle used for all operating groups

Section C Rules Parameters

Reorder Point

$P^{s}_{m} \leq 9999$; $P^{s}_{m} \geq 0$ is reorder point in units

$P^{s}_{m} = 0$ means non-stock (as required)

$P^{s}_{m} = \infty$ means stock item, reorder every period

Reorder Quantity

$R^{s*}_{m} \leq 9999$; R^{s*}_{m} = Economic Order Quantity

Leveling Factor

$0 \leq \alpha^{s} \leq 1$

α^{s} = Leveling factor used for all operating groups (except source and customer)

α^{s} = 1 implies absolute leveling

α^{s} = 0 implies that schedule will follow demand

BG

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I'll type it in
when you finish

Section D Auxiliary Values

$0 \leq U \leq 9999$

$U^s_{m,t} \equiv$ unfilled orders at end of period t

unfilled means not yet scheduled.

$0 \leq I \leq 9999$

$I^s_{m,t} \equiv$ inventory on hand at end of period t

Section E Decision Values

$0 \leq S \leq 9999$

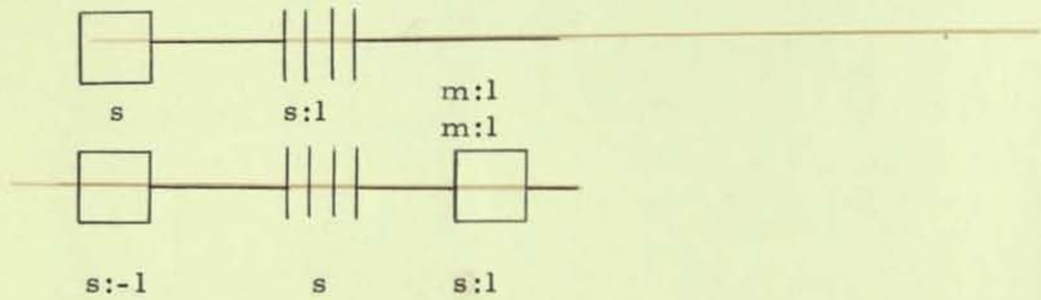
$S^s_{m,t}$ = scheduled orders to start manufacture during period t

$0 \leq R \leq 9999$

$R^s_{m,t}$ = Quantity ordered by station s during time period t of product m.

Section F Timing Relationships

Section G **Precede and Succeed Concept**



s:1 means succeeding station

s:-1 means preceding station

m:1 represents all finished products from station s:1 that use m as an input:

m (input)	m:1 (output)
A	A_1, A_2
B	A_1B, A_2B
A_1	A_1B
A_2	A_2B
A_1B	A_1B
A_2B	A_2B

Sequence Relationship Table

		For Control Group #											
		1	2	3	4	5	11	12	21	22	23	24	25
	1	0	-1	-2	-3	-3	x	x	-4	-5	-5	-6	-7
	2	-	0	-1	-2	-2	x	x	-3	-4	-4	-5	-6
	3	2	1	0	-1	-1	x	x	-2	-3	-3	-4	-5
	4	3	2	1	0	x	x	x	-1	-2	-2	-3	-4
	5	3	2	1	x	0	x	x	-1	-2	-2	-3	-4
Control	11	x	x	x	x	x	0	-1	-2	-3	-3	-4	-5
group # has	12	x	x	x	x	x	1	0	-1	-2	-2	-3	-4
this rela-	21	4	3	2	1	1	2	1	0	-1	-1	-2	-3
tionship	22	5	4	3	2	2	3	2	1	0	x	-1	-2
	23	5	4	3	2	2	3	2	1	x	0	-1	-2
	24	6	5	4	3	3	4	3	2	1	1	0	-1
	25	7	6	5	4	4	5	4	3	2	2	1	0

To explain the use of the sequence relationship table we would say that control group #1 precedes control group 4 by 3; hence, 1 is s:-3 from 4.

0 means that it is a one to one correspondence.

x means that there is no defined relationship since the stations are in parallel or independent flow lines.

It would also be said that station 24 succeeds station 4 by 3. Hence, control group 24 is s:3 from 4.

Part V LIST OF FACTORS

Section A Systems Parameters

T_{A1B}^{25}	R_{A1B}^{22*}	R_{A2}^{5*}
T_{A2B}^{25}	L_{A1B}^{21}	T_{A1}^{4}
L_{A2B}^{24}	C^{21}	R_{A1}^{4*}
C^{24}	α^{21}	L_{A1}^{3}
α^{24}	T_{B}^{12}	L_{A2}^{3}
T_{A2B}^{23}	P_{B}^{12}	C^{3}
P_{A2B}^{23}	R_{B}^{12*}	α^{3}
R_{A2B}^{23*}	C^{11}	T_{A}^{2}
T_{A1B}^{22}	T_{A2}^{5}	P_{A}^{2}
P_{A1B}^{22}	P_{A2}^{5}	R_{A}^{2*}
		C^{1}

Section B **Systems Status Variables**

00	t	14	$S^{24}_{A2b, t+3}$	28	$S^{21}_{A2B, t+3}$
01	R^{25}_{A1B}	15	$S^{24}_{A2B, t+2}$	29	$S^{21}_{A2B, t+2}$
02	R^{25}_{A2B}	16	$S^{24}_{A2B, t+1}$	30	$S^{21}_{A2B, t+1}$
03	$S^{24}_{A1B, t+6}$	17	$S^{24}_{A2B, t}$	31	$S^{21}_{A2B, t}$
04	$S^{24}_{A1B, t+5}$	18	$S^{24}_{A2B, t-1}$	32	$S^{21}_{A2B, t-1}$
05	$S^{24}_{A1B, t+4}$	19	R^{23}_{A2B}	33	R^{12}_{B}
06	$S^{24}_{A1B, t+3}$	20	R^{22}_{A2B}	34	$S^{11}_{B, t}$
07	$S^{24}_{A1B, t+2}$	21	$S^{21}_{A1B, t+4}$	35	$S^{11}_{B, t-1}$
08	$S^{24}_{A1B, t+1}$	22	$S^{21}_{A1B, t+4}$	36	R^{5}_{A2}
09	$S^{24}_{A1B, t}$	23	$S^{21}_{A1B, t+2}$	37	R^{4}_{A1}
10	$S^{24}_{A1B, t-1}$	24	$S^{21}_{A1B, t+1}$	38	$S^{3}_{A1, t+2}$
11	$S^{24}_{A2B, t+6}$	25	$S^{21}_{A1B, t}$	39	$S^{3}_{A1, t+1}$
12	$S^{24}_{A2B, t+5}$	26	$S^{21}_{A1B, t-1}$	40	$S^{3}_{A1, t}$
13	$S^{24}_{A2B, t+4}$	27	$S^{21}_{A2B, t+4}$	41	$S^{3}_{A1, t-1}$

Section B Systems Status Variables (cont.)

42	$S^{3}_{A2,t+2}$	48	$S^{1}_{A,t-1}$	54	U^{21}_{A2B}
43	$S^{3}_{A2,t+1}$	49	U^{24}_{A1B}	55	I^{12}_{B}
44	$S^{3}_{A2,t}$	50	U^{24}_{A2B}	56	I^{5}_{A2}
45	$S^{3}_{A2,t-1}$	51	I^{23}_{A2B}	57	I^{4}_{A1}
46	R^{2}_{A}	52	I^{22}_{A1B}	58	U^{3}_{A1}
47	$S^{1}_{A,t}$	53	U^{21}_{A1B}	59	U^{3}_{A2}
				60	I^{2}_{A}

Part VI DESCRIPTION OF MODEL OPERATION

Set Initial Parameters

Set all C independently

Set all P independently

$L^{3}_{A2} = C^{1}$

$L^{3}_{A1} = C^{1}$

$L^{21}_{A1B} = C$

$L^{21}_{A2B} = 0$ because $P^{4} > 0$ and $P^{12} > 0$

$L^{21}_{A2B} = \max. \left[0 \text{ (because } P^{12} > 0\text{)},\ C^{3} + L^{3}_{A2} \right]$

$L^{24}_{A1B} = 0$ because $P^{22} > 0$

$L^{24}_{A2B} = 0$ because $P^{23} > 0$

$R^{*} = 0$ if $P = 0$ or $P = 99$

Set all other R^{*} independently

Set T

Set α

Set Systems Status

$S^{s}_{m,t}$ is bounded by $t = L^{s}_{m}$ and $t = 1 - C^{s}_{m}$

$U^{s}_{m} = 0$ if $T^{s:1}_{m} = 0$

$I^{s}_{m} = 0$ if $P^{s}_{m} = 0$

Provide Demands

The systems analyst must provide the demands to be tested. These can be any set of positive values for each of the two products. The number of time periods tested is a function of the demand cards inserted for testing.

Sequence of Decisions

Read demands for period t

Shift S_r to S_{r-1}

Set all $S_{t+L} = 0$

Compute all R, compute all S_{t+L}

Update U, I

Print status for period t

advance t

Part VII FORMULATION OF RULES

Operational Rules

Updating System

$$U^{s}_{m,t} = T^{s:1}_{m} \times (U^{s}_{m,t-1} + R^{s:1}_{m,t} - S_{m,t} + L^{s}_{m})$$

$$I^{s}_{m,t} = I^{s}_{m,t-1} + S^{s:1}_{m,t+1-C^{s:1}} - \sum_{m} S^{s:1}_{m:1,t}$$

Decision Rules

$$S^{s}_{m,t+L^{s}_{m}} = \min.\left\{\left[\alpha^{s} S^{s}_{m,t+L^{s}_{m}-1} + (1-\alpha^{s})(R^{s:1}_{m,t} + U^{s}_{m,t-1})\right]\left[I^{s:-1}_{m:-1,t-1}\right]\right\}$$

$$R^{s}_{m,t} = \beta R^{s*}_{m} + (1-\beta) {}_{m} R^{s:2}_{m:1,t} - (1-T^{s:2}_{m:1})(R^{s:2}_{m:1,t} - S^{s:1}_{m:1,t})$$

$\beta = 1$ if $R^{s*}_{m} \neq 0$

and if $\left\{ I^{s}_{m,t-1} + \sum S^{s:-1}_{m} - \sum_{m} U^{s:1}_{m:1,t-1} - \sum_{m}\left[R^{s:2}_{m:1,t} - (1-T^{s:2}_{m:1})(R^{s:2}_{m:1,t} - S^{s:1}_{m:1,t})\right]\right\}$

$\leq P^{s}_{m}$

$\beta = 0$ if $\{\quad\} > P^{s}_{m}$

or if $R^{s*}_{m} = 0$

Part VIII INITIAL TEST RUNS

Section A Parameters Values

Initial Systems Parameters for Test Runs

Set Parameters

T^{25}_{A1B}	0	L^{21}_{A1B}	0	P^{4}_{A1}	30
T^{25}_{A2B}	1	L^{21}_{A2B}	3	R^{4*}_{A1}	60
L^{24}_{A1B}	0	C^{21}	2	L^{3}_{A1}	1
L^{24}_{A2B}	0	α^{21}	.5	L^{3}_{A2}	1
C^{24}	1	T^{12}_{B}	1	C^{3}	2
α^{24}	0	P^{12}_{B}	9999	α^{3}	.75
T^{23}_{A2B}	1	R^{12*}_{B}	0	T^{2}_{A}	1
P^{23}_{A2B}	9999	C^{11}	1	P^{2}_{A}	0
R^{23*}_{A2B}	0	T^{5}_{A2}	1	R^{2*}_{A}	0
T^{22}_{A1B}	1	P^{5}_{A2}	0	C^{1}	1
P^{22}_{A1B}	30	R^{5*}_{A2}	0		
R^{22*}_{A1B}	40	T^{4}_{A1}	1		

Initial Status Values For Test Runs

all other $\left.\begin{matrix} U \\ I \\ S \end{matrix}\right\} = 0$

Set Initial Conditions

$U^{24}_{A1B,0}$	0	$S^{21}_{A1B,-1}$	10	$U^{3}_{A1,0}$	0
$U^{24}_{A2B,0}$	0	$S^{21}_{A2B,3}$	10	$U^{3}_{A2,0}$	0
$S^{24}_{A1B,0}$	10	$S^{21}_{A2B,2}$	10	$S^{3}_{A1,1}$	10
$S^{24}_{A2B,0}$	10	$S^{21}_{A2B,2}$	10	$S^{3}_{A1,0}$	10
$I^{23}_{A2B,0}$	20	$S^{21}_{A2B,0}$	10	$S^{3}_{A1,-1}$	10
$I^{22}_{A1B,0}$	20	$S^{21}_{A2B,-1}$	10	$S^{3}_{A2,1}$	10
$U^{21}_{A1B,0}$	0	$I^{12}_{B,0}$	20	$S^{3}_{A2,0}$	10
$U^{21}_{A2B,0}$	0	$S^{11}_{B,0}$	20	$S^{3}_{A2,-1}$	10
$S^{21}_{A1B,0}$	10	$I^{5}_{A2,0}$	0	$I^{2}_{A,0}$	0
				$S^{1}_{A,0}$	20

Section B Demand Patterns Tried

Trial 1 65 time periods

Demand A_1B 10 all time periods

Demand A_2B 10 all time periods

Trial 2 65 time periods

Demand A_1B for $t = 1$ is 100; for $t = 2\text{-}65$, demand is 10

Demand A_2B is 10 for all time periods

Trial 3 65 time periods

Demand A_1B is 10 for all time periods

Demand A_2B for $t = 1$ is 100; for $t = 2\text{-}65$, demand is 10

Trial 4 65 time periods

Demand A_1B for $t = 1\text{-}29$ is 20; for $t = 30\text{-}65$, demand is 10

Demand A_2B for $t = 1\text{-}29$ is 20; for $t = 30\text{-}65$, demand is 10

Trial 5 125 time periods

Demand A_1B for $t = 1$ is 2; for $t = 2$, demand is 4; demand = 2t

Demand A_2B for $t = 1$ is 2; for $t = 2$, demand is 4; demand = 2t

Section C <u>Results</u>

<u>Trial Runs 1-5</u>
<u>Initial Parameters</u>

Name	Value	Name	Value	Name	Value
T_{A1B}^{24}	0	R_{A1B}^{22*}	40	T_{A1}^{4}	1
T_{A2B}^{25}	1	L_{A1B}^{21}	0	P_{A1}^{4}	30
L_{A1B}^{24}	0	C^{21}	2	R_{A1}^{4*}	60
L_{A1B}^{24}	0	α^{21}	500	L_{A1}^{3}	1
C^{24}	1	T_{B}^{12}	1	L_{A2}^{3}	1
α^{24}	0	P_{B}^{12}	9999	C^{3}	2
T_{A2B}^{23}	1	R_{B}^{12*}	0	α^{3}	750
P_{A2B}^{23}	9999	C^{11}	1	T^{2}	1
R_{A2B}^{23*}	0	T_{A2}^{5}	1	P^{2}	0
T_{A1B}^{22}	1	P_{A2}^{5}	0	R^{2*}	0
P_{A1B}^{22}	30	R_{A2}^{5*}	0	C_{A}^{1}	1

Systems Status
Trial Run 1

t	0	1	10	20	40	65
R^{25}_{A1B}	0	10	10	10	10	10
R^{25}_{A2B}	0	10	10	10	10	10
$S^{24}_{A1B, t+6}$	0	0	0	0	0	0
$S^{24}_{A1B, t+5}$	0	0	0	0	0	0
$S^{24}_{A1B, t+4}$	0	0	0	0	0	0
$S^{24}_{A1B, t+3}$	0	0	0	0	0	0
$S^{24}_{A1B, t+2}$	0	0	0	0	0	0
$S^{24}_{A1B, t+1}$	0	0	0	0	0	0
$S^{24}_{A1B, t}$	10	10	10	10	10	10
$S^{24}_{A1B, t-1}$	0	10	10	10	10	10
$S^{24}_{A2B, t+6}$	0	0	0	0	0	0
$S^{24}_{A2B, t+5}$	0	0	0	0	0	0
$S^{24}_{A2B, t+4}$	0	0	0	0	0	0

Trial Run I (cont.)

$S^{24}_{A2B, t+3}$	0	0	0	0	0	0
$S^{24}_{A2B, t+2}$	0	0	0	0	0	0
$S^{24}_{A2B, t+1}$	0	0	0	0	0	0
$S^{24}_{A2B, t}$	10	10	10	10	10	10
$S^{24}_{A2B, t-1}$	0	10	10	10	10	10
R^{23}_{A2B}	0	10	10	10	10	10
R^{22}_{A2B}	0	40	10	10	10	10
$S^{21}_{A1B, t+4}$	0	0	0	0	0	0
$S^{21}_{A1B, t+3}$	0	0	0	0	0	0
$S^{21}_{A1B, t+2}$	0	0	0	0	0	0
$S^{21}_{A1B, t+1}$	0	0	0	0	0	0
$S^{21}_{A1B, t}$	10	20	10	10	10	10
$S^{21}_{A1B, t-1}$	10	10	10	10	10	10
$S^{21}_{A2B, t+4}$	0	0	0	0	0	0
$S^{21}_{A2B, t+3}$	10	10	10	10	10	10

Trial Run I (cont.)

$S^{21}_{A2B,t+2}$	10	10	10	10	10	10
$S^{21}_{A2B,t+1}$	10	10	10	10	10	10
$S^{21}_{A2B,t}$	10	10	10	10	10	10
$S^{21}_{A2B,t-1}$	10	10	10	10	10	10
R^{12}_{B}	0	50	20	20	20	20
$S^{11}_{B,t}$	20	50	20	20	20	20
$S^{11}_{B,t-1}$	0	20	20	20	20	20
R^{5}_{A2}	0	10	10	10	10	10
R^{4}_{A1}	0	60	10	10	10	10
$S^{3}_{A1,t+2}$	0	0	0	0	0	0
$S^{3}_{A1,t+1}$	10	23	25	18	18	18
$S^{3}_{A1,t}$	10	10	27	18	18	18
$S^{3}_{A1,t-1}$	10	10	29	18	18	18
$S^{3}_{A2,t+2}$	0	0	0	0	0	0
$S^{3}_{A2,t+1}$	10	11	11	11	11	11

Trial Run 1 (cont.)

$S^3_{A2,t}$	10	10	11	11	11	11
$S^3_{A2,t-1}$	10	10	11	11	11	11
R^2_A	0	70	20	20	20	20
S^1	20	70	21	20	20	20
$S^1_{A,t-1}$	0	20	20	20	20	20
U^{24}_{A1B}	0	0	0	0	0	0
U^{24}_{A2B}	0	0	0	0	0	0
I^{23}_{A2B}	20	20	20	20	20	20
I^{22}_{A1B}	20	20	90	90	90	90
U^{21}_{A1B}	0	20	0	0	0	0
U^{21}_{A2B}	0	0	0	0	0	0
I^{12}_B	20	40	10	10	10	10
I^5_{A2}	0	0	8	18	38	63
I^4_{A1}	50	40	189	296	456	656
U^3_{A1}	0	37	7	4	4	4

Trial Run 1 (cont.)

U^{3}_{A2}	0	1	0	0	0	0
I^{2}_{A}	0	50	5	1	1	8

Systems Status
Trial Run 2 and 3

0	1	10	20	40	65	t	0	1	10	20	40	65
0	0	10	10	10	10	R^{25}_{A1B}	0	10	10	10	10	10
0	10	10	10	10	10	R^{25}_{A2B}	0	100	10	10	10	10
0	0	0	0	0	0	$S^{24}_{A1B, t+6}$	0	0	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A1B, t+5}$	0	0	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A1B, t+4}$	0	0	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A1B, t+3}$	0	0	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A1B, t+2}$	0	0	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A1B, t+1}$	0	0	0	0	0	0
10	20	10	10	10	10	$S^{24}_{A1B, t}$	10	10	10	10	10	10
0	10	10	10	10	10	$S^{24}_{A1B, t-1}$	0	10	10	10	10	10

Trial Run 2 and 3 (cont.)

0	0	0	0	0	0	$S^{24}_{A2B, t+6}$	0	0	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A2B, t+5}$	0	0	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A2B, t+4}$	0	0	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A2B, t+3}$	0	0	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A2B, t+2}$	0	0	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A2B, t+1}$	0	0	0	0	0	0
10	10	10	10	10	10	$S^{24}_{A2B, t}$	10	100	10	10	10	10
0	10	10	10	10	10	$S^{24}_{A2B, t-1}$	0	10	10	10	10	10
0	10	10	10	10	10	R^{23}_{A2B}	0	100	10	10	10	10
0	40	10	10	10	10	R^{22}_{A2B}	0	40	10	40	40	10
0	0	0	0	0	0	$S^{21}_{A1B, t+4}$	0	0	0	0	0	0
0	0	0	0	0	0	$S^{21}_{A1B, t+3}$	0	0	0	0	0	0
0	0	0	0	0	0	$S^{21}_{A1B, t+2}$	0	0	0	0	0	0
0	0	0	0	0	0	$S^{21}_{A1B, t+1}$	0	0	0	0	0	0

Trial Run 2 and 3 (cont.)

10	20	10	10	10	10	$S^{21}_{A1B,t}$	10	20	3	7	7	7
10	10	10	10	10	10	$S^{21}_{A1B,t-1}$	10	10	2	7	7	7
0	0	0	0	0	0	$S^{21}_{A2B,t+4}$	0	0	0	0	0	0
10	10	10	10	10	10	$S^{21}_{A2B,t+3}$	10	55	15	13	13	13
10	10	10	10	10	10	$S^{21}_{A2B,t+2}$	10	10	15	13	13	13
10	10	10	10	10	10	$S^{21}_{A2B,t+1}$	10	10	18	13	13	13
10	10	10	10	10	10	$S^{21}_{A2B,t}$	10	10	18	13	13	13
10	10	10	10	10	10	$S^{21}_{A2B,t-1}$	10	10	23	13	13	13
0	50	20	20	20	20	R^{12}_{B}	0	140	20	50	50	20
20	50	20	20	20	20	$S^{11}_{B,t}$	20	140	20	50	50	20
0	20	20	20	20	20	$S^{11}_{B,t-1}$	0	20	20	20	20	20
0	10	10	10	10	10	R^{5}_{A2}	0	100	10	10	10	10
0	60	10	10	10	10	R^{4}_{A1}	0	60	10	40	40	10
0	0	0	0	0	0	$S^{3}_{A1,t+2}$	0	0	0	0	0	0

Trial Run 2 and 3 (cont.)

10	23	25	18	18	18	$S^3_{A1,t+1}$	10	23	25	25	22	26
10	10	27	18	18	18	$S^3_{A1,t}$	10	10	27	18	14	29
10	10	29	18	18	18	$S^3_{A1,t-1}$	10	10	29	18	15	30
0	0	0	0	0	0	$S^3_{A2,t+2}$	0	0	0	0	0	0
10	11	11	11	11	11	$S^3_{A2,t+1}$	10	33	23	15	14	14
10	10	11	11	11	11	$S^3_{A2,t}$	10	10	26	16	14	14
10	10	11	11	11	11	$S^3_{A2,t-1}$	10	10	26	16	14	14
0	70	20	20	20	20	R^2_A	0	160	20	50	50	20
20	70	20	20	20	20	S^1	20	160	20	50	50	20
0	20	20	20	20	20	$S^1_{A,t-1}$	0	20	20	20	20	20
0	0	0	0	0	0	U^{24}_{A1B}	0	0	0	0	0	0
0	0	0	0	0	0	U^{24}_{A2B}	0	0	0	0	0	0
20	20	20	20	20	20	I^{23}_{A2B}	20	70	235	281	341	416
20	10	80	80	80	80	I^{22}_{A1B}	20	20	80	30	30	75

0	20	0	0	0	0	U^{21}_{A1B}	0	20	17	93	153	198
0	0	0	0	0	0	U^{21}_{A2B}	0	45	1	1	1	2
20	40	10	10	10	10	I^{12}_{B}	20	130	2	37	37	7
0	0	8	18	38	63	I^{5}_{A2}	0	0	26	83	107	132
50	40	189	296	456	656	I^{4}_{A1}	50	40	206	359	596	855
0	37	7	4	4	4	U^{3}_{A1}	0	37	7	19	21	12
0	1	0	0	0	1	U^{3}_{A2}	0	67	12	4	3	1
0	50	5	1	1	8	I^{2}_{A}	0	140	7	27	30	11

Systems Status
Trial Run 4 and 5

0	1	10	20	40	65	t	0	1	10	20	40	65	124
0	20	20	20	10	10	R^{25}_{A1B}	0	2	36	40	80	130	248
0	20	20	20	10	10	R^{25}_{A2B}	0	2	36	40	80	130	248
0	0	0	0	0	0	$S^{24}_{A1B,t+6}$	0	0	32	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A1B,t+5}$	0	0	18	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A1B,t+4}$	0	0	18	0	0	0	0

Trial Run 4 and 5 (cont.)

0	0	0	0	0	0	$S^{24}_{A1B,t+3}$	0	0	0	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A1B,t+2}$	0	0	20	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A1B,t+1}$	0	0	20	0	0	0	0
10	20	20	20	10	10	$S^{24}_{A1B,t}$	10	2	21	34	34	34	34
0	10	20	20	10	10	$S^{24}_{A1B,t-1}$	0	10	0	34	34	34	34
0	0	0	0	0	0	$S^{24}_{A2B,t+6}$	0	0	17	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A2B,t+5}$	0	0	14	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A2B,t+4}$	0	0	12	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A2B,t+3}$	0	0	36	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A2B,t+2}$	0	0	36	0	0	0	0
0	0	0	0	0	0	$S^{24}_{A2B,t+1}$	0	0	32	0	0	0	0
10	20	20	20	10	10	$S^{24}_{A2B,t}$	10	2	0	40	80	130	248
0	10	20	20	10	10	$S^{24}_{A2B,t-1}$	0	10	0	38	78	128	246
0	20	20	20	10	10	R^{23}_{A2B}	0	2	10	40	80	130	248

Trial Run 4 and 5 (cont.)

0	40	20	20	10	10	R^{22}_{A2B}	0	40	49	34	34	34	34
0	0	0	0	0	0	$S^{21}_{A1B, t+4}$	0	0	0	0	0	0	0
0	0	0	0	0	0	$S^{21}_{A1B, t+3}$	0	0	1	0	0	0	0
0	0	0	0	0	0	$S^{21}_{A1B, t+2}$	0	0	29	0	0	0	0
0	0	0	0	0	0	$S^{21}_{A1B, t+1}$	0	0	15	0	0	0	0
10	20	17	17	7	7	$S^{21}_{A1B, t}$	10	20	103	34	34	34	34
10	10	21	17	6	7	$S^{21}_{A1B, t-1}$	10	10	1	34	34	34	34
0	0	0	0	0	0	$S^{21}_{A2B, t+4}$	0	0	7	0	0	0	0
10	15	23	23	13	13	$S^{21}_{A2B, t+3}$	10	6	5	40	80	130	248
10	10	23	23	13	13	$S^{21}_{A2B, t+2}$	10	10	10	38	78	128	246
10	10	23	23	13	13	$S^{21}_{A2B, t+1}$	10	10	20	36	76	126	244
10	10	23	23	13	13	$S^{21}_{A2B, t}$	10	10	20	34	74	124	242
10	10	23	23	13	13	$S^{21}_{A2B, t-1}$	10	10	0	32	72	122	240
0	60	40	40	20	20	R^{12}_{B}	0	42	0	74	114	164	282

20	60	40	40	20	20	$S^{11}_{B,t}$	20	42	0	74	114	164	282
0	20	40	40	20	20	$S^{11}_{B,t-1}$	0	20	0	72	112	162	280
0	20	20	20	10	10	R^{5}_{A2}	0	2	0	40	80	130	248
0	60	20	20	10	10	R^{4}_{A1}	0	60	0	34	34	34	34
0	0	0	0	0	0	$S^{3}_{A1,t+2}$	0	0	20	0	0	0	0
10	23	36	26	29	17	$S^{3}_{A1,t+1}$	10	23	18	37	37	37	37
10	10	37	26	29	17	$S^{3}_{A1,t}$	10	10	0	37	37	37	37
10	10	40	26	34	18	$S^{3}_{A1,t-1}$	10	10	0	36	37	37	37
0	0	0	0	0	0	$S^{3}_{A2,t+2}$	0	0	0	0	0	0	0
10	13	24	24	14	14	$S^{3}_{A2,t+1}$	10	9	0	40	80	130	248
10	10	24	24	15	14	$S^{3}_{A2,t}$	10	10	0	38	78	128	246
10	10	24	24	15	14	$S^{3}_{A2,t-1}$	10	10	0	36	76	126	244
0	80	40	40	20	20	R^{2}_{A}	0	62	20	74	114	164	282
20	80	40	40	20	20	S^{1}	20	62	18	74	114	164	282
0	20	40	40	20	20	$S^{1}_{A,t-1}$	0	20	20	72	112	162	280

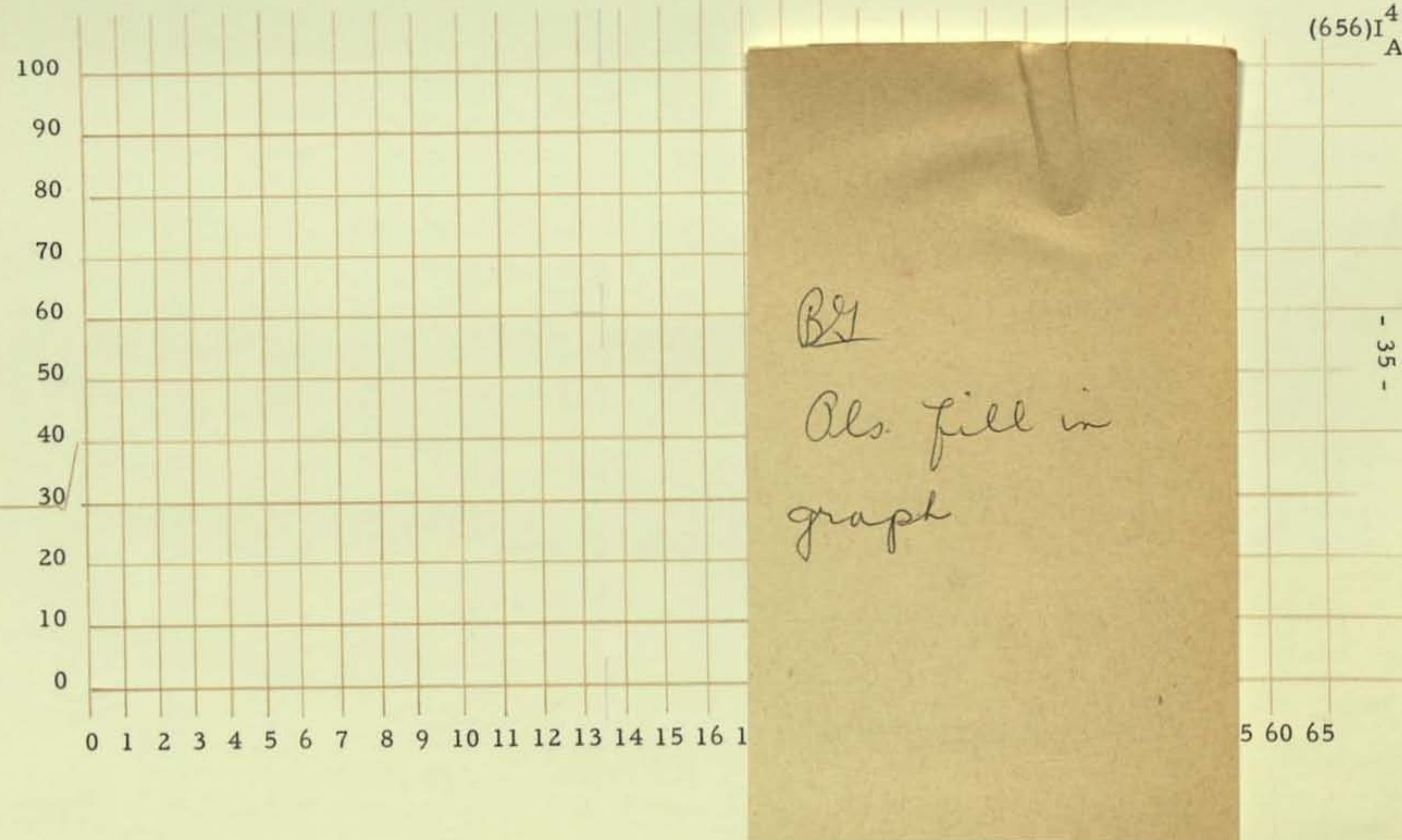
Trial Run 4 and 5 (cont.)

0	0	0	0	0	0	U^{24}_{A1B}	0	0	20	0	0	0	0
0	0	0	0	0	0	U^{24}_{A2B}	0	0	0	0	0	0	0
20	10	18	48	164	239	I^{23}_{A2B}	20	28	0	1	1	7	1
20	10	105	75	67	52	I^{22}_{A1B}	20	28	0	34	34	34	34
0	20	4	34	112	187	U^{21}_{A1B}	0	20	0	0	0	0	0
0	5	2	2	2	1	U^{21}_{A2B}	0	4	20	1	1	1	1
20	50	17	17	7	7	I^{12}_{B}	20	32	19	95	215	365	719
0	0	9	19	47	73	I^{5}_{A2}	0	0	0	36	76	126	244
50	40	135	276	501	827	I^{4}_{A1}	50	40	20	72	132	207	384
0	37	6	2	1	2	U^{3}_{A1}	0	37	18	2	2	1	2
0	7	2	2	3	1	U^{3}_{A2}	0	7	16	5	5	5	5
0	60	2	0	5	0	I^{2}_{A}	0	42	14	40	20	1	0

Section D **Graphs** **Trial 1**

————	R^{25}_{A1B}	xxxxxxx	I^{2}_{A}	ooooooooo	I^{5}_{A2}	-----------	I^{23}_{A2B}
————	R^{25}_{A2B}	-------	I^{4}_{A1}	————	I^{22}_{A1B}		

$(656)I^{4}_{A1}$



Part IX MODEL IMPROVEMENT

Section A Error Analysis

The following material simply analyzes certain errors which showed up during the initial trial run. What is needed is that these runs and others be analyzed in depth to detect and correct the systems equations and the control rules.

t	S^3_{A2}	S^3_{A1}	I^4_{A1}	S^{21}_{A1B}	R^4_{A1}	U^3_{A1}	I^2_A	S^1_A
-1	10	10		10				0
0	10	10	50	10	0	0	0	20
1	10	10	40	20	60	37	50	70
2	11	23	10	40	60	56	87	70
3	11	42	23	10	10	18	54	20
4	11	48	44	20	10	15	15	20
5	11	43	72	20	10	13	19	20
6	11	38	100	15	10	12	10	20
7	11	35	123	15	10	10	16	20
8	11	32	148	10	10	9	7	20
9	11	29	170	10	10	8	13	20
10	11	27	189	10	10	7	5	20
11	11	25	206	10	10	6	11	20
13	11	21	234	10	10	5	9	20
14	11	20	245	10	10	4	2	20
15	11	19	255	10	10	4	8	20
16	11	18	264	10	10	4	1	20
17	11	18	272	10	10	4	8	20

$C^3 = 2,\ L^3_{A1} = 1,\ R^{4*}_{A1} = 60,\ P^4_{A1} = 60;\ T^4_{A1} = 1$

$L^3 = .75;\ C^1_A = 1$

$$I^4_{A1,t} = I^4_{A1,t-1} + S^3_{A1,t+1-2} - S^{21}_{A1B,t}$$

$$I^4_{A1,1} = 50 + 10 - 20 = 40$$

$$R^4_{A1} = 0,\ 60 \text{ never } 10$$

<u>Ordering Rule Error</u>

The way R rule is set

$\beta = 0$ after second week since $\{ \quad \} > P^s_m$

and therefore second term is always ordered. This should be set up so that either R^{s*}_m is ordered or else nothing for a stock item on economic order rule.

The ordering rule might take the following form:

$$R^s_{m,t} = \beta \gamma R^{s*}_m + (1 - \beta) V^{s:2}_{m,t}$$

$$\beta = \begin{cases} 0 \text{ if } R^{s*}_m = 0 \\ 1 \text{ if } R^{s*}_m > 0 \end{cases}$$

$$\gamma = \begin{cases} 0 \text{ if } Q^s_{m,t} > P^s_m \\ 1 \text{ if } Q^s_{m,t} \leq P^s_m \end{cases}$$

Where

$$V^{s:2}_{m,t} \quad \sum_m \left[R^{s:2}_{m:1,t} - (1 - T^{s:2}_{m:1})(R^{s:2}_{m:1,t} - S^{s:1}_{m:1,t}) \right]$$

$$Q^s_{m,t} = \left\{ I^s_{m,t-1} + \sum_t S^{s:-1}_m - \sum_m U^{s:1}_{m:1,t-1} - V^{s:2}_{m,t} \right\}$$

Schedule Rule Error

$$S^3_{a1,t+1} = \min. \left\{ \left[.75\, S^3_{A1,t} + (.25)\,(R^4_{A1,t} + U^3_{A1,t-1}) \right], \left[I^2_{A,t-1} \right] \right\}$$

$$U^3_{A1,t} = T^4_{A1} \quad (U^3_{A1,t-1} + R^4_{A1,t} - S^3_{A1,t+1})$$

check U rule

$$U^3_{A1,1} = 1\ (0 + 60 - 23) = 37$$

$$U^3_{A1,2} = 1\ (37 + 60 - 41) = 56 \qquad \text{ok}$$

$$S^3_{A1,2} = \min. \left\{ \left[.75\,(10) + .25\,(60 + 0) \right], \left[0 \right] \right\}$$

$$= \min. \left\{ (7.5 + 15),\ 0 \right\} = \min. \left\{ 23, 0 \right\}$$

should be 0

answer actually was 23;

check for programming error on min. test.

Is there a logic error in using I_{t-1}; wouldn't it be more logical to examine I_{t+L-1}; if this is logical, can it be predicted? Probably yes since all S data is available.

$$I^2_{A,16} = I^2_{A,15} + S^1_{A,16} - S^3_{A1,t6} - S^3_{A2,t6}$$

$$I^2_{A,16} = 8 + 20 - 18 - 11 = -1$$

$$I^2_{A,17} = 1 + 20 - 18 - 11 = -8$$

It isn't logical for an inventory to go negative. This should be automatically prevented by the operation of the Scheduling Rule. There is also an error in that minus numbers are not recognized and they are recorded and treated as though they were positive numbers with the same absolute value.

$$S^{3}_{A1,17} = \min.\left\{\left[.75\,(18) + .25\,(10+4)\,,\ \left[8\right]\right]\right\}$$
$$\min.\left\{(13.5 + 3.5),\ 8\right\}$$
$$= \min.\ (17, 8)$$

Answer was actually 18;

indicating that each factor was individually rounded upward and that the inventory limit was ignored.

The basic rule seems reasonable except that the unfilled should be cleared up.

$$S^{21}_{A1B,1} + S^{21}_{A2B,1} = 20 + 10 = 30$$

$$I^{4}_{B,0} = 20$$

There is no summary test for total usage.
Rule should probably be corrected as follows:

$$S^{s}_{m,t} + L^{s}_{m} = \min.\left\{\left[\alpha^{s}\ S^{s}_{m,t} + L^{s}_{m} - 1 + (1-\alpha^{s})\,(R^{s:1}_{m,t} + U^{s}_{m,t-1})\right],\ \left[I^{s:-1}_{m:-1,t-1} - S^{s}_{m:0,\,t+L^{s}_{m}}\right]\ \text{all } s:-1\right\}$$

where m:0 means other model using same resources.

For example:

$$S^{21}_{A1B,t} = \min.\left\{\left[\quad\right],\left[I^{4}_{A1,t-1}\right],\left[I^{12}_{B,t-1} - S^{21}_{A2B,t}\right]\right\}$$

$$\text{for } t = 1;\ S^{21}_{A1B,t} = \min.\left\{25,\ 50,\ (20-10)\right\} = 10$$

$$S^{21}_{A2B,\,t+3} = \min.\left\{\left[.5\ (10) + .5\ (10 + 0)\right],\ \left[20\right],\ \left[0\right]\right\} = 0;$$

Answer actually was 10

Therefore, in programming must not have tested $I^{5}_{A2,\,t-1}$. However, this is fortunate since if test was made of a non-stock inventory there would never be any available, hence, S = 0 always and no R^{5}_{A2} would ever be placed. We should not examine inventory if P = 0; so Scheduling Rule should read:

$$S = \min.\left\{\left[\quad\right],\left[\quad\right],\left[\quad\right] \quad P^{s:-1}_{m:-1} \neq 0\right\}$$

should prevent negative values on unfilled calculation. Must avoid rounding errors such that order quantity for a non-stock inventory are greater than demand.

It looks like an error to permit a non-stock station to order in relation to demand on user station. This permits a non-stock station to accumulate inventory in anticipation of an increase in schedule rate.

It looks like the R rule should be corrected for non-stock to depend soley on $S^{s:1}_{m:1;\,t} + L^{s:1}_{m:1}$ and nothing else. The allowed lead time already compensates for availability delays.

The ordering rule should look like this:

$$R^{s}_{m,\,t} = \beta\gamma R^{s*}_{m} + (1-\beta)\,\delta V^{s:2}_{m,\,t} + (1-\beta)\,\delta\ S^{s:1}_{m:1,\,t} + L^{s:1}_{m:1}$$

where β, γ and V are defined as before

$$\text{and } \delta = \begin{cases} 0 \text{ if } P^{s}_{m} > 0 \\ 1 \text{ if } P^{s}_{m} = 0 \end{cases}$$

Inventories should not be permitted to go negative - Inventory Rule should be rewritten -

$$I^{s}_{m,\,t} = \max.\left\{\left[I^{s}_{m,\,t-1} - \sum_{m} S^{s:1}_{m:1,\,t}\right],\ \left[0\right]\right\}$$

$$+ S^{s:-1}_{m,\,t+1} - C^{s:-1}$$

Conceptually, how can there be an inventory at a non-stock station? This should be prevented

$I = (1 - \delta)$ etc.

Section B **Demand Analysis**

Take model A1B independently, model A2B independently, models jointly in various combinations.

Usage mean: low
medium
high

Usage slope: trend up
trend down
no trend

Permanent discontinuity:

step up
step down

Temporary change:

up early
down early
up middle
down middle
up end
down end

Magnitude of discontinuity or change:

small
moderate
large

Period of change: short
medium
long

Random pattern

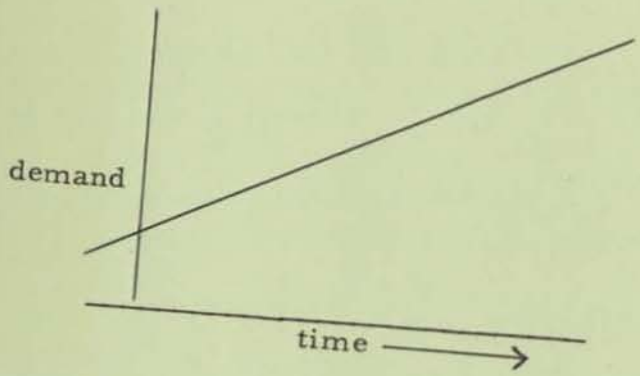
Seasonal variation

BG

Pls draw the

Curvy line

If we view a demand curve as being a "sinusoidal" function with a trend and intercept factor included then these changes can probably be handled through equation modification.



There are four parameters needed to define a particular demand pattern (if it's consistent throughout test period):

Y = Y intercept (initial mean); high, medium, low

S = Slope of mean line; + large, + medium, 0, - medium, - large

F = Frequency of change; short, medium long

A = Amplitude of change; small, medium, large

To test all combinations would take 135 runs. However, using standard statistical experimental design procedures and omitting those combinations which are logically meaningful, the number of trials could be reduced by a ten to one ratio.

Section C General Rules Discussion

To actually simulate such a system requires the use of decision and operation rules for transforming the system values at time t-1 into those at time t. To do this means the establishment of decision rules for determining appropriate R's and S's and operational rules for transforming U's and I's.

The two operational rules are:

$$U^{s}_{m,t} = T^{s:1}_{m} \; (U^{s}_{m,t-1} + R^{s:1}_{m,t} - S^{s}_{m,t} + L^{s}_{m})$$

Only a source or a manufacturing control group can have an unfilled balance.

$$I^{s}_{m,t} = I^{s}_{m,t-1} + S^{s:-1}_{m,t} - C^{s:-1}_{m+1} - \sum_{m} S^{s:1}_{m:1,t}$$

Only an inventory control group can have an inventory balance.

To determine I^{12}_{B} both S^{21}_{A1B} and S^{21}_{A2B} must be considered.

Obviously, any attempt to write completely generalized ordering rules initially would be doomed to failure. However, it did appear reasonable to establish relatively general rule forms based upon the results of previous work and try various parameters in these rules for better system understanding. The simulation model should, of course, be programmed in such a way as to permit straightforward changing of the rule format.

For the purpose of scheduling and ordering rules each control group is related to every pertinent control group by "stages before" or "stages after" representation.

The general scheduling rule without inventory limitation is:

$$S^{s}_{m,t+L^{s}_{m}} = \alpha^{s} \; S^{s}_{m,(t+L^{s}_{m})-1} + \left[(1-\alpha^{s}) \, (R^{s:1}_{m,t} + U^{s}_{m,t-1}) \right]$$

$0 \leq \alpha^{s} \leq 1$ $\qquad$ α^{s} = Leveling factor

$\alpha^{s} = 1$ implies absolute leveling

The general form of the scheduling rule has to take cognizance of leveling considerations. This format was suggested by the work done by Mills and Singleton (Operations Research and Synthesis Services). The concept is that each control group will schedule its own work rather than expecting its customers to provide a balanced set of orders. Since primary emphasis is on interstage relationships, capacity restrictions and unit loading data have been ignored. The real complexity enters in terms of availability of stock inventories or replenishment orders. If the "make-from" materials are carried on a stock basis at all directly preceding inventory positions, the maximum schedule quantity is the minimum inventory of the preceding inventory positions. Where any or all of the make-from materials are not carried on a stock basis the problem is more difficult to state. The limitation here is two-fold; first, the ability of the inventory preceding the supplying station to support the quantity desired and the ability of the supplying station to make the quantity desired. For a series of non-stock products this continues to build-up until the ultimate source is reached.

There are two other complications both to do with competitive demand for a product. First is the case where one of the inventories is used in two or more finished products where an allocation rule is needed if

$$I^{s}_{m,t-1} < \sum_{m} S^{s:1}_{m:1,t}$$

The other case is where non-stock make-from materials are involved and the make-from quantity isn't adequate to support all competing demands.

Part X PROGRAM

Section A Memory Assignments

Initial Systems Parameters

60.01.0 T^{25}_{A1B}

60.02. T^{25}_{A2B}

60.34. C^{1}

Systems Status

61.00.0 t

61.01.0 R^{25}_{A1B}

61.02.0 R^{25}_{A2B}

61.60.0 I^{2}_{A}

S-Rule Subroutine Quantities (all unsigned)

$S^{s}_{m,\,t+L^{s}_{m}}$ 70.01.0

^{s} 70.02.0

$S_{m,\,t+L^{s}_{m}-1}$ 70.03.0

$R^{s:1}_{m,\,t}$ 70.04.0

$U^{s}_{m,t-1}$	70.05.0	
$I^{s:-1}_{m-1,t-1}$	70.06.0	first
$I^{s:-1}_{m:-1,t-1}$	70.07.0	second
spare	70.08.0	
spare	70.09.0	
spare	70.10.0	

R-rule Subroutine Quantities

$R^{s}_{m,t}$	71.01.0	
R^{s*}_{m}	71.02.0	
$R^{s:2}_{m:1,t}$	71.03.0	first
$R^{s:2}_{m:1,t}$	71.04.0	second
$T^{s:2}_{m:1}$	71.05.0	first
$T^{s:2}_{m:1}$	71.06.0	second
$S^{s:1}_{m:1,t}$	71.07.0	first
$S^{s:1}_{m:1,t}$	71.08.0	second
$I^{s}_{m,t-1}$	71.09.0	

R-rule Subroutine Quantities (cont.)

$\sum_t S_m^{s:-1}$	71.10.0	
$U^{s:1}_{m:1,t-1}$	71.11.0	first
$U^{s:1}_{m:1,t-1}$	71.12.0	second
P^s_m	71.13.0	
spare	71.14.0	
spare	71.15.0	
spare	71.16.0	

Input Formats

cols. 1-20	:	any desired identification
cols. 21-24	:	time period
cols. 25-28	:	R^{25}_{A1B}
cols. 29-32	:	R^{25}_{A2B}

Parameters loaded into 7000 -- as bxxxx, for convenience in keypunching on standard card forms, and convenience in inspecting cards.

Initial conditions loaded into 8000 -- same way.

Section B Program Logic

1. Read R^{25}
2. Shift S
3. Set $S_{t+L} = 0$
4. Compute all R & S

5. Update U, I

6. Print

7. Advance t

Section C **Detailed Rules Actually Programmed**

Read R_s

Shift S's (All)

Set $S_{m, t+L} = 0$

$$S^{24}_{A1B, t+L^{24}_{A1B}} = \min . \left\{ \left[\alpha^{24} S^{24}_{A1B, t+L^{24}_{A1B}-1} + (1-\alpha^{24}) (R^{25}_{A1B, t} + U^{24}_{A1B, t-1}) \right], \left[I^{22}_{A1B, t-1} \right] \right\}$$

S^{24}_{A2B} (similar to above)

$$R^{23}_{A2B, t} = \beta R^{23*}_{A2B} + (1-\beta) \left[R^{25}_{A2B, t} + (1-T^{25}_{A2B}) (R^{25}_{A2B, t} - S^{24}_{A2B, t} \right]$$

$$\text{if } \left\{ I^{23}_{A2B, t-1} + \sum_t S^{21}_{A2B, t} - U^{24}_{A2B, t-1} - \left[R^{25}_{A2B, t} - (1-T^{25}_{A2B}) (R^{25}_{A2B, t} - S^{24}_{A2B, t}) \right] \right\} \leq P^{23}_{A2B}$$

R^{22} = (similar to above)

$$S^{21}_{A1B,\,t+L^{21}_{A1B}} = \min.\left\{ \left[\alpha^{21}\, S_{A1B,\,t+L^{21}_{A1B}} - 1\right],\right.$$

$$\left[(1-\alpha^{21})\,(R^{22}_{A1B,\,t} + U^{21}_{A1B,\,t-11})\right],$$

$$\left.\left[I^{4}_{A1,\,t-1}\right],\ \left[I^{12}_{B,\,t-1}\right]\right\}$$

S^{21}_{A2B} (similar to above

$$R^{12}_{B,\,t} = \beta\quad R^{12*}_{B}$$

$$+ (1-\beta)\left\{\left[\ R^{22}_{A1B,\,t} - (1-T^{22}_{A1B})\,(R^{22}_{A1B,\,t} - S^{21}_{A1B,\,t})\right]\right.$$

$$\left.+\left[R^{23}_{A2B,\,t} - (1-T^{23}_{A2B})\,(R^{23}_{A2B,\,t} - S^{21}_{A2B,\,t})\right]\right\}$$

$$U^{24}_{A1B,\,t} = T^{25}_{A1B}\left(U^{24}_{A1B,\,t-1} + R^{25}_{A1B,\,t} - S^{24}_{A1B,\,t+L^{24}_{A1B}}\right)$$

$$I^{22}_{A1B,\,t} = I^{22}_{A1B,\,t-1} + S^{21}_{A1B,\,t+C^{21}-2}$$

$$- S^{24}_{A1B,\,t}$$

$$I^{12}_{B,\,t} = I^{12}_{B,\,t-1} + S^{11}_{B,\,t+C^{11}-2}$$

$$- (S^{21}_{A1B,\,t} + S^{21}_{A2B,\,t})$$

Section D **Additional Description**

Customer station

R(24) each t

T(24)

No U, S, S rule, R rule, C, 1, I

Source Station

Treat as inventory station with infinite inventory

C each source

S each source, up to C.

$S_t^s = R_t^{s:1}$

No T, R, P, α, U; $I = \infty$

Inventory

Supplied: I_0^s, R_0^s, S_0^s, T^s, P^s, U_0^s, T^s

T means either 0 or ∞ ; it is represented as T = 0 or T = 1

$C^s = 0$ (all s)

no α

$S_t^s = \min.\left\{ (R_t^{s:1} + U_{t-1}^s) , I_t^s - 1 \right\}$

$I_t^s = I_t^s - 1 + S_{t-C^{s:2}-1}^{s:2} - S_t^s$

$U_t^s = T^{s:1} (U_t^s - 1 + R_t^{s:1} - S_t^s)$

$R_t^s = \beta R_t^{s:1} + \gamma (1 - \beta) R*$

$\beta = 1$ if $P^s = 0$ or ∞

$\beta = 0$ if 0 $P^s < \infty$

$\gamma = 1$ if $(I_t^s + S_{t-C^{s:2}+1}^{s:2} + \dots + S_{t-1}^{s:2}) \leq P^s$

$= 0$ otherwise

Manufacturing Station

Supplied R_0^s for each input model, S^s for each output model, for all time periods up to t-C

$T^s = 0$ or ∞ (represented by 1) for each model

U_0^s " " "

α^s " " "

C^s is the same for all models

No P, β, γ, I

Rules

$$S_t^s = \min.\left\{ (R_t^{s:1} + U_{t-1}^s), \left[\alpha^s S_{t-1}^s + (1-\alpha^s)(R_t^{s:1} + U_{t-1}^s \right], I_{t-1}^{s:2} \right\}$$

for each model

$$U_t^s = T^{s:1} \left[U_{t-1}^s + R_t^{s:1} - S_t^s \right]$$ for each model

$R_t^s = S_t^s$ for each model