Burton Grad
9/8/53

The Derivation and Use of an Economical Order Quantity Formula

The Economical Order Quantity may be defined as that quantity to be ordered which will result in a minimum annual expense to the Company. It takes into account not only the out of pocket expense related to set-up savings, the use of automatic equipment and vendor discounts, but also reflects the cost of processing the order and the cost of carrying inventory. This analysis is, of course, only applicable to "stock" items -- those purchased or made on a non-customer order basis.

The Net Yearly Cost for an item equals the sum of the following factors:

1. The Yearly Usage times the Cost per unit.

2. The number of Orders per year times the Cost of processing an order.

3. The average number of inventory units times the Cost per unit times the Cost of Carrying inventory ratio.

or

a) $Y = UT + NP + AT'I$

where

b) Y = Net Yearly Cost

c) U = Yearly Usage in units

d) T = Total out of pocket Cost per unit

e) N = Number of Orders per year

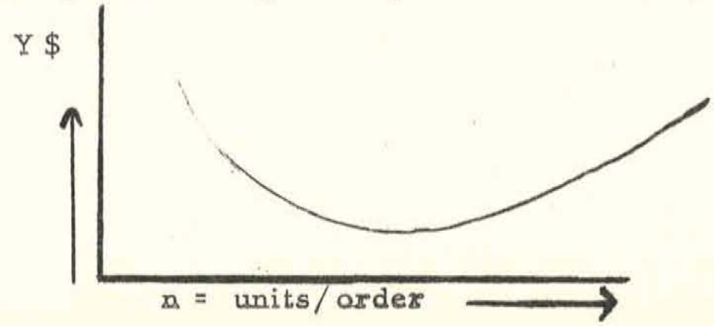
f) P = Cost of processing an order

g) A = Average Inventory in units

h) T' = Total Cost per unit (including overhead)

i) I = Cost of Carrying Inventory one year expressed as a ratio

Since the Net Yearly Cost is dependent upon the number of units per order, the graph of this equation will generally take the following shape:



Where n = units/order

The key factors can now be redefined for further calculation.

Yearly Usage (U) can be obtained by analyzing past usage or by having a measure of future requirements. The calculation of this factor should take into account any trend effects of changing business or markets. It should be expressed in the unit of measure which you will use for the actual order. For example, on screws you might use "gross", raw materials might be in pounds, or bearings might be in pieces.

Total out of pocket cost per unit (T) equals the sum of the set-up cost averaged over the number of units on the order plus the per unit cost. For items procured from a vendor an equivalent set-up must be calculated. (See Exhibit "A"). Therefore this relationship can be expressed:

(j) $T = S/n + C$

where

S = set-up (or equivalent)

n = number of units/order

C = per unit cost (or equivalent)

It should be noted that S and C do not include any overhead or burden charges.

The number of orders per year (N) equals the yearly usage divided by the number of units per order or =

(k) $N = \frac{U}{n}$

The cost of processing an order (P) is determined by examining all of the factors relating to the originating, procuring, storing, and accounting for an order. A technique for arriving at this figure is shown in Exhibit "B".

The cost of processing an order may vary with the source of the material (internal or outside vendor) and the value of the material.

The average inventory for a stock item equals the protective stock plus one half the order quantity (see Exhibit "C") or:

(l) $A = R + \frac{n}{2}$

where R = protective stock in units

n = number of units/order

The total cost per unit (T') equals the total out of pocket cost per unit plus applicable overhead. Since most businesses apply overhead as a percentage against direct labor, an outside vendor item would remain unchanged, but an internally manufactured item would have its net cost increased. Hence we can express this total cost as the sum of two costs.

(m) $T' = (1 + K)T_I + T_E$

where K = overhead expense as a ratio to Direct Labor

T_I = Total out of pocket labor costs (internal costs)

T_E = Total out of pocket material or vendor costs (external costs)

and since $T = S/n + C$ (equation j)

$$T_I = S_I/n + C_I$$

$$T_E = S_E/n + C_E$$

$\therefore$ (n) $T' = (1 + K)\ (S_I/n + C_I) + S_E/n + C_E$

or by defining:

(o) $S' = (1 + K)\ S_I + S_E$

(p) $C' = (1 + K)\ C_I + C_E$

expanding (n) $T' = (1 + K)\ S_I/n + (1 + K)\ C_I + S_E/n + C_E$

$$= \frac{(1 + K)\ S_I + S_E}{n} + (1 + K)\ C_I + C_E$$

or (q) $T' = S'/n + C'$

The Cost of Carrying Inventory Ratio is determined by examining the savings to be realized by a reduction in Inventory. (See Exhibit "D"). This ratio is expressed in terms of the net inventory value of an item including material, labor and overhead.

From this redefinition equation (a) now takes the following shape:

(r) $Y = U\left[S/n + C\right] + \frac{U}{n} P + \left[R + \frac{n}{2}\right] \left[S'/n + C'\right] I$

expanding:

$$Y = \frac{US}{n} + UC + \frac{UP}{n} + \frac{RS'I}{n} + RC'I + \frac{S'I}{2} + \frac{C'nI}{2}$$

grouping

$$\text{(s)} \quad Y = \left[UC + RC'I + \frac{S'I}{2}\right] + \left[\frac{US}{n} + \frac{UP}{n} + \frac{RS'I}{n}\right] + \left[\frac{C'nI}{2}\right]$$

now, we want the point where Y is a minimum; to do this we set $\frac{dy}{dn} = 0$ and solve for n. This "n" when substituted in equation (s) will result in a minimum value.

$$\text{(t)} \quad \frac{dy}{dn} = -\frac{US + UP + RS'I}{n^2} + \frac{C'I}{2} = 0$$

$$\text{or} \quad \frac{US + UP + RS'I}{n^2} = \frac{C'I}{2}$$

$$\text{and } n^2 = \frac{2\left[US + UP + RS'I\right]}{C'I}$$

$$\therefore \text{(u)} \quad n = \sqrt{\frac{2\left[US + UP + RS'I\right]}{C'I}}$$

where U = Yearly usage in units

S = Set Up Cost (or equivalent)

P = Cost of processing an order

R = Protective Stock in Units

S' = Internal Set Up plus overhead plus equivalent external set up.

I = Cost of Carrying Inventory Ratio

C' = Internal per unit cost plus overhead plus equivalent external per unit cost.

In order to make this formula more useful and applicable, the following definition is made:

$$\text{(v)} \quad m = \frac{R}{U}$$

where "m" thus equals the portion of a year's usage represented by the protective stock.

or

$$\text{(w)} \quad R = mU$$

and restating:

$$(u) \quad n = \sqrt{\frac{2\left[US + UP + mUS'I\right]}{C'I}}$$

$$\text{or } n = \sqrt{\frac{2U\left[S + P + mS'I\right]}{C'I}}$$

$$\text{or } n = \sqrt{U} \times \sqrt{\frac{2\left[S + P + mS'I\right]}{C'I}}$$

and if we define:

$$(x) \quad Q = \sqrt{\frac{2\left[S + P + mS'I\right]}{C'I}}$$

$$(y) \quad \therefore \; n = \sqrt{U} \times Q$$

Q is a constant for any one item unless the basic planning, price, or inventory structure changes. While Q can be easily calculated on a slide rule or a desk calculator, it may be advantageous to do this automatically on an electronic computer. (See Exhibit "E").

To apply this formula in a reasonable fashion does not mean ordering the exact amount indicated by solving for "n". This merely gives an indicated value for lowest net cost and can be adjusted as dictated by common sense and business background. Some examples are shown in Exhibit "F".

By calculating and using an economical order quantity a double benefit will be derived. First, by calculating the Cost of Processing an Order and the Cost of Carrying Inventory Ratio, clues may be given toward cost improvement of your operating expense. Secondly, by procuring items in economical order quantities your stock inventory will be reduced to the lowest value consistent with low operating expense.

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Exhibit "A"

Equivalent Set-up for Purchased Parts

Pr = Price per unit.

Q = Quantity in units

$Pr = f(Q)$

For simplification, assume that the price variance is a straight line function or can be made to approximate one.

then, (1) $Pr = C + \frac{S}{Q}$

where C = per unit cost

S = set-up cost

If for Q the cost per unit is Pr_1
and for Q_2, the cost per unit is Pr_2
and $Q_2 > Q_1$

then $Pr_1 = C + \frac{S}{Q_1}$

$Pr_2 = C + \frac{S}{Q_2}$

and $Pr_1 - Pr_2 = \frac{S}{Q_1} - \frac{S}{Q_2}$

$Pr_1 - Pr_2 = \frac{Q_2 S - Q_1 S}{Q_1 Q_2}$

or $Pr_1 - Pr_2 = \frac{S(Q_2 - Q_1)}{Q_1 Q_2}$

and $\boxed{S = \frac{(Pr_1 - Pr_2)\, Q_1 Q_2}{Q_2 - Q_1}}$ Equivalent set-up

$Pr_1 = C + \frac{S}{Q_1}$

$\boxed{C = Pr_1 - \frac{S}{Q_1}}$

To prove your solution, substitute the derived values of S and C in the Equation for Pr_2 in terms of Q_2.

Exhibit "B"

Cost of Processing an Order

There are four major functions involved and the evaluation should only contain those functions which are variable as the number of orders increases or decreases.

1. Originate the order.
2. Process and/or Procure the item.
3. Store and Disburse the item.
4. Record and pay for the order.

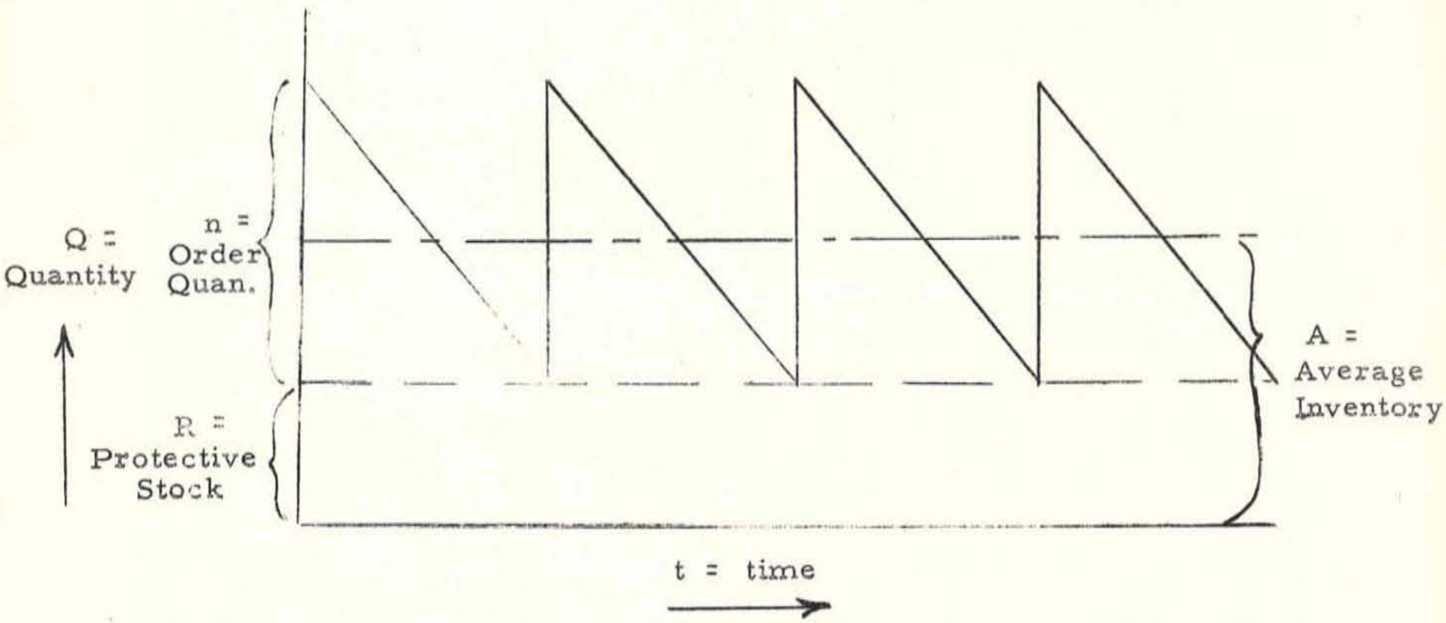
This will probably have to be subdivided into Outside Vendor orders
Other Department orders
and Internal machined orders.

It may also be necessary to distinguish between Stock and Non-Stock orders.

The Cost Summary should be made on a basis of average cost over a period of time -- one year is a good criteria. Naturally, these figures will have to be adjusted as operating costs vary.

Average Stock Inventory

The Average Inventory of a stock item can be represented by the following graph:



Of course this is an idealized situation; but it serves as a good approximation.

Therefore: $A = R + \frac{n}{2}$

Exhibit "D"

Cost of Carrying Inventory Ratio

The Cost of Carrying Inventory is dependent upon four basic factors:

- I Possession Costs
- II Value Losses
- III Return on Investment
- IV General Business Influences

This ratio will vary with business conditions and the nature of your business. For a more detailed discussion, you are referred to an article entitled "The Cost of Carrying Inventory" by R. C. Hartigan and B. Grad in the Sept. 1953 issue of Mill & Factory magazine.

Program for Electronic Calculation of "Q" Factor

For each item the following data must be established:

S_I = internal set-up

S_E = equivalent external set-up

P = Cost of processing the order

m = Portion a year's usage represented by the protective stock.

K = Overhead Expense ratio to Direct Labor

I = Cost of Carrying Inventory Ratio

C_I = internal per unit cost

C_E = equivalent external per unit cost

A punched card should be initiated for each stock item containing S_I, S_E, P, C_I and C_E. The other factors can probably be gang punched, as they are usually constants for any given line of product.

The formula is:

$$Q = \sqrt{\frac{2\left\{ S_E + S_I + P + m \left[(1 + K) S_I + S_E\right] I \right\}}{\left[(1 + K)\ C_I + C_E\right] I}}$$

(1) spread load

S_E	into acc	# 1
S_I		2
P		3
m		4
K		5
I		6
C_I		7
C_E		8
2		9
1		10

Oper No.	Oper	Acc	Acc	Results into
2	add	5	10	11
3	mult	4	6	12
4	mult	2	11	13
5	add	1	2	14
6	add	1	13	13
7	add	3	14	14
8	mult	12	13	12
9	mult	7	11	11
10	add	12	14	12
11	add	8	11	11
12	mult	9	12	12
13	mult	6	11	11
CPC delay				
14	divide	12 by	11	11
15	sq. rt.	11		11

16 take data from acc #11 and punch result.

Examples:

made from a forging purchased from a vendor internally machined to finished dimensions.

Vendor price:

Q'ty 1 $3.00 ea
Qty 15 2.00 ea

$$\therefore S_E = \frac{(3.00 - 2.00)\ 1 \times 15}{15 - 1} = \frac{1500}{14} = 1.07$$

$$C_E = 3.00 - \frac{1.07}{1} = 1.93$$

proof: $C_E = 2.00 - \frac{1.07}{15} = 1.93$

Mchg Cost

SMW -	S/u	.30	1.20 ea
grind fin	S/u	.50	.80 ea
dress seat			.50 ea
	S_I =	.80;	C_I = 2.50

$U = 40$

$S = \$1.07 + .80 = \$1.87$

$P = \$20.15$

$m = .083$ (1 month)

$S' = 1.07 + (.80)(3.2) = 3.63$

$I = .45$

$C' = 1.93 + (2.50)(3.2) = 9.93$

$$\therefore n = \sqrt{40} \times Q$$

$$Q = \sqrt{\frac{2\ \boxed{1.87 + 20.15 + .083 \times 3.63 \times .45}}{9.93 \times .45}}$$

$$= \sqrt{\frac{2\ \boxed{22.16}}{4.47}} = \sqrt{\frac{44.32}{4.47}} = \sqrt{9.915}$$

$Q = 3.15$

$n = 3.15\sqrt{40} = 3.15 \times 6.32$

$n = 20$

You should order a six months supply.