Book review: The Unknowable: works by G.J. Chaitin

1. ***The Limits of Mathematics, a Course on Information Theory and Limits of Formal Reasoning*, by Gregory J. Chaitin. London: Springer-Verlag. Hardcover, November 1997.**

2. ***The Unknowable*, by Gregory J. Chaitin. London: Springer-Verlag Series in Discrete Mathematics and Theoretical Computer Science. Hardcover, August 1999.**

3. ***Exploring Randomness*, by Gregory J. Chaitin. London: Springer-Verlag. Hardcover, February 2001.**

Not only does God play dice with physics, contrary to Einstein's oft-quoted assertion, but He also plays dice with arithmetics, and even with that "hardest" part of mathematics known as number theory. So argues mathematician Gregory Chaitin, whose work has been supported for the last 30 years by the IBM research division at the Thomas J. Watson research center in New York State.

Chaitin is the main architect of a new branch of mathematics called algorithmic information theory, or "AIT. " A gifted pioneer (in 1965, while in high school, he wrote a paper on automata that is still quoted today) he obviously enjoys shaking philosophers and scientists alike by his radical statements about the incompleteness of mathematics, the need to reframe it as an experimental science rather than an exact one, and more generally the folly of ever attempting to derive complete truth from a set of axioms. As he puts it in a piece called *Letter to a daring young reader*: "I have demonstrated the existence of total randomness in the mental mindscape of pure mathematics."

Chaitin and Kolmogorov simultaneously came up with the idea that something is random if it cannot be compressed into a shorter description: "If you think of a theory as a program that calculates the observations, the smaller the program is relative to the output, which is the observations, the better the theory is," writes Chaitin.

Three overlapping books on the incompleteness of mathematics

Chaitin's three books are based on his popular lectures and must be taken together in order to assess his ideas. In *The Unknowable* he compares his work on incompleteness to that of Gödel and Turing, discussing the historical context of his research on program-size complexity; in *The Limits of Mathematics* he brings more detail on metamathematical implications; and in *Exploring Randomness* he develops algorithmic theory, further revealing its technical core.

This is important work, with implications that go far beyond the arcane arguments of one branch of mathematics. At first sight, however, the reader may be justified for feeling confused or overwhelmed. The three books are fascinating in their blend of flamboyant ideas and long chapters written in LISP, a programming language that Chaitin favors: He even developed his own dialect of it! While this provides a ready tool for his colleagues and students it makes it harder for the general reader to unravel the many threads of his ebullient arguments. Yet the sections in LISP are mandatory because the common theme of all three books is to study the size of the smallest program for calculating a given number, and "you cannot really understand an algorithm unless you can see it running on a computer."

Another weakness is the overlap of the three volumes, that would have benefited from tighter editing and structure (perhaps with the LISP developments as an appendix?) These are minor problems of presentation, however, that should not detract from the massive intellectual challenge the author is proposing. As one gets into the substance of the books it is difficult to resist Chaitin's enthusiastic style and obvious intelligence. Beyond the technicalities of the argument the reader is quickly drawn into a fundamental new landscape of ideas. What Chaitin is demanding, in effect, is nothing less than a bold reassessment of our notions about truth and logic.

The challenge to Hilbert

At the dawn of the 20^{th} century it seemed that science was about to solve, once and for all, the totality of mathematical problems. David Hilbert believed that a consistent and complete set of axioms could be drawn up, from which you could derive all of mathematics. As Chaitin summarizes it, "if all mathematicians could agree whether a proof is correct and be consistent and complete, in principle that would give a procedure for automatically solving any mathematical problem. This was Hilbert's magnificent dream, and it was to be the culmination of Euclid and Leibniz, and Boole and Peano, and Russell and Whitehead."

Hilbert's famous lecture in the year 1900 proposed a list of 23 difficult problems, a "call to arms" that inspired a generation of researchers, among them John von Neumann. In the fifties and sixties, when I studied math at the Sorbonne in the shadow of Bourbaki, this was still the dominant vision.

The first man who pointed out that Hilbert's axiomatic theory was flawed was Gödel. As early as 1931 he showed that mathematics could not be consistent and complete at the same time. More specifically, he proved that if an axiomatic system was consistent it would prove theorems that were wrong, and therefore it was incomplete. And if it was complete it would fail to prove some theorems that were true.

To put it in simplistic terms, consider the statement, "This statement is unprovable." If it turns out to be provable, then we are proving something that is false. And if it is indeed unprovable, then it is true – a true statement that escapes our system of axioms. This in turn means that they are incomplete.

Gödel's proof is difficult (refreshingly, Chaitin himself confesses that he could follow it step by step but "somehow I couldn't ever really feel that I was grasping it") but it was followed by a more clear, more devastating attack five years later, led by the father of computer theory, Alan Turing.

Gödel had shown that a formal axiomatic system for arithmetic could not be complete if it was consistent, but this still left a door open for a "decision procedure" that would tell us if a given assertion was true or not. Turing closed that door in 1936, and his proof is the springboard for Chaitin's work.

Turing posed the question in radical new terms by tackling the "halting problem," which considers a program (P) that determines whether or not a given computer program (Q) will halt or not when it is run on a particular computer. This is where computer languages with recursivity are important: In a language like LISP that is interpreted rather than compiled you can run (P) as a subprocedure of itself. If (P) stated that (Q) would never halt, then you would halt; and you would go into an infinite loop in the opposite situation, when (P) stated that (Q) would halt. Thus you would demonstrate the incompleteness of the axioms, unable to yield a fixed answer.

Chaitin refined this incompleteness result by defining a number, "Omega" as the "halting probability."

Omega is the probability that a binary program generated by tossing a coin will ever stop running. Given a specific computer, this is a well-defined real number. The computer calls for a series of binary digits and tries executing this "program." Omega is "maximally unknowable," says Chaitin, because the sequence of 0's and 1's in this number have no mathematical structure. To calculate the first N bits of Omega demands an N-bit program, in other words, N bits of axioms. This is irreducible mathematical information, a shocking idea in the Hilbertian view that assumed that all mathematical truth (hence, all computable numbers) could be derived from a small set of axioms in the same way as Pi, or the square root of 2, can be computed to arbitrary precision.

Implications beyond Mathematics

Leibniz claimed that if something was true, it was true for a reason. That reason was the "mathematical truth." But the bits in Chaitin's Omega number are not true for any reason, they are true by accident. We will never know what these bits are in the way we "know" that the first decimal in Pi is 1, the second one is 4, etc.

Summarizing the history Chaitin writes: "it turned out that not only Hilbert was wrong, as Gödel and Turing showed... With Gödel it looks surprising that you have incompleteness, that no finite set of axioms can contain all mathematical truth. With Turing incompleteness seems much more natural. But with my approach, when you look at program size, I would say that it looks inevitable. Wherever you turn, you smash up against a stone wall and incompleteness hits you in the face!"

Chaitin has shown that some mathematical truths were true by accident, that mathematics was no longer an exact science but an empirical, even an experimental science like physics. This is a nightmare for the logicians. At a time when physicists (who went through a similar revolution with the concept of randomness in the 1920s) are trying to get spacetime out of a random substratum, this work on the limits of mathematics is an inspiration.

How far can we take the implications? Chaitin himself sees no direct connection between his work and the physical concept of "random reality" but he does claim that "AIT will lead to the major breakthroughs of 21st century mathematics, which will be information-theoretic and complexity-based characterizations of what is mind, what is intelligence, what is consciousness, of why life has to appear spontaneously and then to evolve."

This last statement suggests a link with many of the topics studied by the SSE. French writer Aimé Michel had reached the conclusion that certain problems (such as the topic of "alien contact") were in the realm of the unknowable, and would remain so until humans evolved a more complex brain. But mathematical unknowability is not necessarily a consequence of human frailty.

Hilbert's First Problem (also known as "Cantor's Continuum Hypothesis") is an example of this. In transfinite arithmetic the Hebrew letter Aleph subscripted by zero ("Aleph null") is the number of integers. It can be shown that 2 raised to the Aleph-null power is another number, and is greater than Aleph-null. Hilbert asked whether there was a number between these two numbers.

In 1963 a Stanford mathematician named Paul Cohen showed that you couldn't know if such a number existed. As a scientist friend from Los Alamos reminds me, "it's not that you are not smart enough, or lack the mathematical tools to find it. It is just undecidable."

This finding challenges many philosophical positions. Materialist theoretician and Marx's co-author Friedrich Engels made the point that our subjective thought and the objective world follow the same laws and therefore cannot contradict each other in their results. That is where mathematics comes from, argues Engels: abstraction from the world of nature. Eighteenth-century materialism had already posed the principle that *nihil est in intellectu, quod not fuerit in sensu.* (nothing exists in thought that doesn't exist in sensory experience.) In a piece called *On Prototypes of Mathematical Infinity in the Real World* Engels further stressed that "our geometry starts from spatial relationships, our arithmetics and algebra begin with numerical quantities and thus correspond to our terrestrial conditions." In such a materialistic view it would seem to follow that the world itself must be unknowable.

Not all scientists will agree with this interpretation. After Gödel and Turing, you can indeed ask some well-posed questions that do not have an answer. But we should not look for implications beyond logic: "I see no connection to the existence of UFOs or the existence of God," says my Los Alamos correspondent. "But because I fail to see the connections this doesn't mean there is no connection. In the early eighteenth century Pierre Louis Moreau de Maupertuis set out to prove the existence of God, and ended up formulating the principle of least action, which provides the underpinnings for much of modern physics."

If Chaitin is right about the impact of AIT as a new discipline, his work on the Unknowable could indeed prove fundamental for 21st century science. I find it ironic that information science, which was regarded as a minor branch of "applied mathematics" when I went to graduate school, may turn out to play such a major role in the future. But the best advice Chaitin gives us comes at the end of *Exploring Randomness*, when he writes:

"Be prepared to have many false breakthroughs, which don't survive the glaring light of rational scrutiny the next morning. You have to dare to imagine many false beautiful theories before you hit on one that works; be daring, dare to dream, have faith in the power of new ideas and hard work. Get to work! Dream!"

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people read it—and perhaps ask Dr. Fontana to produce a popular version, not more than 200 pages in length, less detailed.

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Meta Math!: The Quest for Omega by Gregory Chaitin. Pantheon Books, 2005. 240 pp. $26.00 (hardcover). ISBN 0-375-42313-3.

This is the latest (number 9) of Gregory Chaitin's books on algorithmic information theory (AIT) and its implications for and his thoughts on the meaning of randomness. Chaitin writes in a refreshing way, with lots of first person, and does not hesitate to add autobiographical supporting context. One could therefore assert that these books are "non-technical" accounts of his work. I am not so sure. When I first (long ago) met pure mathematics, beginning with a course out of Landau's Foundations of Analysis[1] (with its precise German script), I marveled at the precision of set theory, issues of axiom of choice needed or not, the competing ways to describe completeness of the real number system, all that. Here was a way to get rid of the fuzziness of engineering parlance and the jumping to conclusions of physicists, which had been my previous experience. Only much later does one realize the practical limitations of the axiomatic method. Therefore, to really explain mathematics, it is then better to put it in first person, and even autobiographical, context. You can publish your technical papers in pure technicalese and with as much rigor as you can muster, or wish to muster. But the overall or final accounts should be in human context. Chaitin appears to be a romanticist who believes in his mission: that most of mathematics as we know it is true by accident. The axiomatic method touches only a tiny fraction (I don't mean here: a rational number) of mathematical truth. Therefore we should be more like physicists, allow more intuition into our mathematical lives. As Chaitin states on page 115, "Why should I believe in a real number if I can't calculate it, if I can't prove what its bits are, and if I can't even refer to it? And each of these things happens with probability one!"

When I first received a precopy of this book to review, I started writing down some notes, also on related matters, and soon the scope grew into an examination of the whole philosophy of science, with subchapters on the various notions of complexity, randomness from the AIT as compared to meaningful randomness in quantum mechanics, a chapter on exactly where Chaitin's views place him within the various religions of mathematical logicians, how his

philosophy supported or contrasted with that of my friend Ilya Prigogine[2] and other physicists, and much more, until I realized that such an examination of this book in full wide context would require that I write a book myself! So I put the precopy down on a table and went off to more mundane mathematical and scientific matters. Now I return to a published final copy of Chaitin's book to finish this review, and to do so I shall retreat into as much brevity as I can get by with. One way to accomplish that is to cite and use the writings of others. Those that I will use are the following: (1) the excellent review in this journal of three of Chaitin's earlier books, by Jacques Vallee[3]; (2) the excellent best-selling book by John Horgan[4]; (3) the excellent recent summary of probability and randomness theories in this journal by Hans Primas[5]; (4) a relatively recent book by the logician Hintikka[6]; (5) a Turing Award Lecture by John McCarthy[7]; (6) books by Dantzig[8] and Davis[9]; (7) and some of my own writings[10–16]. In other words, I have just set in place a set of axioms (via citations) to limit the extent of this review. Not only will this review be incomplete, it will also no doubt not be free of inconsistencies. More: In my view, all human thought is inconsistent when placed in larger context, and there is always larger context, so all human thought is also incomplete. I will return to this thought (noted: it is an inconsistent and incomplete thought) below.

Turning to the task at hand, reviewing the book *Meta Math!*, and before going to the contexts (1) to (7) delineated above, we might wonder about, or even try to pin down, define, exactly what the title means, why the author Chaitin chose it. *Meta* (Gr.) in its various usages in the English language, especially when used as a prefix, can connote either an emergent entity or a foundational entity. If you consult your dictionary, you will find that its many connotations make it a very (seductive) modifier. Technically, it means *after*, powerful. Chaitin uses it in the senses: *about*, *over* (looking down on), *limitations* of, e.g., see pages 26, 27, 163–164. His historical view is that Hilbert, by formulating the axiomatic method, which of course went back to the Greeks especially for geometry but which for Hilbert was to be an attempt to formalize all of mathematics (and even physics), created the field of *metamathematics*.

If it was our desire to somewhat deromanticize the prose and content of this book, for example to make it seem more pedestrian to the reader of this review, well, we could say, this is a book that falls within the confines of the mathematical field commonly known as *logic and foundations*, a field which has been of decreasing importance ever since Godel put the kibosh on it in 1931. That is not to say Hilbert's quest for formal axiomatic systems has not been useful. As Chaitin says on page 145, "Nevertheless formalism has been a brilliant success this past century, but not in math, not in philosophy, but as computer technology, as software, as programming languages. It works for machines, but not for us!" I think that is a little too strong: axiomatic thinking, insistence by mathematicians on stating one's assumptions, has given mathematicians a niche, a distinct edge over physicists and engineers, to say nothing of more precise thinking than that of other scientists. We can examine problems on a deeper (excuse me for using this word) level. Of course, we need more time to do so. Someone has said that mathematicians are to science as accountants are to business. But that does imply that too often we come after, not before.

So I think that one could describe Chaitin's mission epitomized by his choice of title *Meta Math!* to be more than just a study of the limits of axiomatic mathematics. It is also in my view an attempt to re-energize mathematics, away from "Bourbakism" and back toward a more creative environment. I especially liked his section "On Creativity" (pp. 148–151). Being a mountain climber myself and (forgive me for saying this) having had some beautiful women in my life, the relationships of those activities to mathematics and to creativity that Chaitin describes ring true to me. So if I could give the author some advice, I would suggest: No more books for a while. You have gotten the message out there.

Let me now follow my context constraints (1) to (7) stated above. I do find the previous review[3] of three of Chaitin's previous books on this subject quite good. Moreover, the present book has considerable overlap with the previous books. So I refer the reader to that review to supplement this one. Here are just a few words to summarize or, if you will, to augment the discussion in that review. Independently, in the early 1960's, Ray Solomonoff, A. N. Kolmogorov, and Gregory Chaitin arrived at the notion of randomness as maximal incompressibility. A series of numbers is deemed to be random if the smallest algorithm capable of specifying it to a computer has about the same number of bits of information as the series itself. In a quest of about twenty years, Chaitin turned to Turing's halting problem. Consider all possible programs that a Turing computer could run. Consider the probability that a program chosen at random from among all such programs will halt. Chaitin showed that this "halting probability" Ω is a real number between 0 and 1. There are no computable instructions for determining the digits of Omega. Thus in its binary representation, Omega is an unending string of random 0's and 1's. There is no pattern. In the author's words (pp. 132–133) "the bits of Ω are logically irreducible, they cannot be obtained from axioms simpler than they are. Finally! We've found a way to simulate independent tosses of a fair coin, we've found 'atomic' mathematical facts, an infinite series of math facts that have no connection with each other and that are, so to speak, 'true for no reason' (no reason simpler than they are)".

To provide historical context, Chaitin traces the digital philosophy back to Liebnitz, and digital physics back to Zeno. These are not new observations. I refer the reader to the old book (originally, 1930) by Dantzig[8] and the newer book by Davis.[9] Originally I was going to develop more about these books as they relate to the one under review but I have decided not to. Dantzig[8] gives you considerable information about Liebnitz and Zeno as their views relate to numbers, mathematics, and science. Davis[9] says a lot about the historical interconnections of the development of computers and that of symbolic logic. Liebnitz described a computing machine that could do logic, long before Boole. Then Frege gave us a language of mathematical symbols, many of which we use

in proofs today. I mention that I have done some mathematical-physics work[10] on what is called Zeno's quantum paradox, an issue that has become central to the possible design of workable quantum computers.

Primas[5] describes (p. 598) the concept of algorithmic complexity (Chaitin's AIT) as "rephrase the old idea that 'randomness consists in a lack of regularity' in a mathematically acceptable way". Primas also indicates some limitations of AIT formulations, and attempts to overcome those by Martin-Lof and others. Primas' article is a good exposition of problems about our theories of probability and randomness. Some of my own views, somewhat related to those of Primas, about stochasticity and determinism in mathematics and science, are put forth in a recent article.[11] For probability from chaos, see another recent paper.[12]

As to logic and science, even though I am a computing pioneer,[13] I cannot agree with putting all my eggs into Chaitin's AIT basket. In some sense, by insisting that digital information theory can describe all of nature's complexity, he has himself fallen into what I may call "Hilbert's trap" of asserting an overall philosophy or system. There is no final theory. The famous book of Horgan[4] discusses this point. Why should Wheeler's "it from bit", a physical version of Chaitin's AIT, describe everything? The more I study quantum mechanics, the less I believe in any ultimate Zero-One Laws of randomness. However, this is just my opinion, formulated, if you will, from experience and thought over one lifetime. By the way, the book of Horgan also has a delightful Chapter 9, "The End of Limitology", which discusses Chaitin's views within a confrontational setting of a 1994 meeting at the Santa Fe Institute of Complexity. Chaitin's attacks there on axiomatic mathematics are met with a lot of hostility. In much the same reaction, I have found here in my department's small group of "logicians" a lot of hostility toward Chaitin's views the only time I mentioned it to them! But although I do not like an "information based universe" claimed by Wheeler and Chaitin and others as some final answer, even less do I believe that one must absolutely declare oneself to be absolutely Platonist, or absolutely Intuitionist, or absolutely Formalist, or even absolutely Skeptic. However, as I commented recently,[14] more and more I see intuition as richer than formal reasoning. Of course this intuition cannot be nave intuition. It is an intuition which has emerged after much formal reasoning and much experience and experimental thinking. So I am thinking more like a physicist here. With that, Chaitin would agree.

This brings me to the last discussion constraints (4) and (5) which I set in place above. If we accept Chaitin's thesis that the axiomatic foundations of mathematics are doomed, and if we at least take note of Horgan's limitologies, then where should my friends who are already inalterably committed to a lifetime career in mathematical logic and foundations go? Some interesting directions are put forth by Hintikka.[6] Among his many writings, I have only cited here one of his books, Hintikka accepts that syntactical aspects of mathematical logic are unavoidable, and moreover embraces first order languages which are more natural than the customary Tarski "truth set" higher order logics. Among the models that Hintikka discusses are game theoretic semantics (GTS). I must note here that GTS suffers the same defect as Chaitin's AIT: time, i.e., the time-length of a game or algorithm, is not in the theory. The Hintikka book is written very nicely in human language and gives a lot of pros and cons of the various logic models. It is also at times written in first person. Apparently I could say that both Chaitin and Hintikka's philosophies are nice to put in juxtaposition and both permit the addition of further intuitions or axioms to particular models.

Finally, I return to Chaitin's comment quoted above, that formalism "works for machines, but not for us!" What *does* work for us? I have a little experience with that question which I would like to share, as we close this review. Some years ago I was involved in a project to try to use neural network computer architectures to model human reasoning.[15,16] This research went on over several years and we actually ran a lot of "human tests", in contradistinction by the way to much of the artificial intelligence (AI) literature. Among our findings from these human experiments was the fact that when presented with simple classification problems, our human testers would go to great lengths to avoid accepting contradictory findings that needed to be accepted simultaneously. To quote: "humans overwhelmingly seek, create, or imagine context in order to provide meaning when presented with abstract or apparently incomplete or contradictory or otherwise untenable situations".[15]

Therefore I would assert that even if formalism does not work for us, regarding Chaitin's statement above, nonetheless there seems to be a human craving for completeness and consistency. Completeness? Witness all the religions of the world, which usually promise life eternal. But then consistency? If one religion (yours) is absolutely right, how can the differing specifics of another religion (mine) be also right? Wars have been fought to get just one (right) answer. And in a much more trivial context, wars are fought, with great rhetoric, even in academic departments, about the relative merits of one kind of mathematics versus another, and even though physical blood may not be spilled, academic careers can be killed. Why are mathematicians such absolutists? Such narrow-mindedness!

Our human versus AI research led us to the work of John McCarthy.[7] Although now a little dated, let me quote: "In my opinion, getting a language for expressing commonsense knowledge for inclusion in a general database is the key problem of generality in AI".[7] Strangely enough, I did not find *generality* explicitly discussed in either Chaitin's book or Hintikka's book, even though Hintikka's approach would not be incompatible with McCarthy's wish stated above. We had investigated[16] the problem of generalization both by human and machine, and found some new ways to do it, better it seems than some recent papers I have seen in the AI literature. However, our overriding conclusion[15] was that one cannot speak of generalization until one better understands *context*, and more to the point, how humans assign context. Of course, how they succeed in setting a context gives them power if they control that context.

So it seems to me that the future may be bright for logics which permit a not necessarily excluded middle, and for some kind of evolution of human culture which permits some ambiguity, as in quantum mechanics, for example. It has been said that humans would rather be wrong than uncertain. That has to change. In the same way, somehow we need qubits instead of bits in Chaitin's Ω.

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The Kensington Runestone: Approaching a Research Question Holistically by Alice Beck Kehoe. Long Grove, IL: Waveland Press, 2005. viii + 102 pp. $17.95 (paper). ISBN 1-57766-371-3.

The Kensington Rune Stone: Compelling New Evidence by Richard Nielsen and Scott F. Wolter. Minneapolis, MN: Lake Superior Agate Publishing, 2006. xvi + 574 pp. $29.95 (paper), $60.00 (cloth). ISBN 1-58175-562-7.

In the year 1362, in western Minnesota, members of an expedition from Norway and the island of Gotland returned from a trip and found ten of their