THE FUNDAMENTAL OPERATIONS

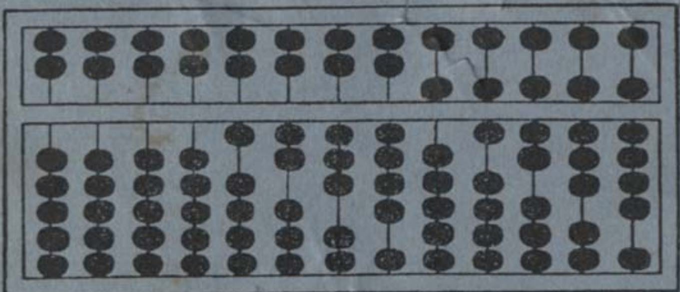
IN

BEAD ARITHMETIC

HOW TO USE THE CHINESE ABACUS

BY

KWA TAK MING



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THE

FUNDAMENTAL OPERATION IN BEAD ARITHMETIC

INTRODUCTION

Definition of Bead Arithmetic. *Bead Arithmetic* is a certain method of reckoning, in which numbers are represented by wooden beads. These beads are systematically arranged on a frame known as the *Chinese abacus*. The term Bead arithmetic is used to distinguish it from the other form of arithmetic in which written figures are used. It may be called a science of numbers but, since it is used in daily business life, it is more appropriate to speak of it as an art of reckoning.

History. The history of this subject is very meagre. For reckoning the ancient Chinese used bamboo tallies or chips; the modern Chinese use the abacus. There is no record of who invented the frame, nor is the time known at which this ingenious contrivance made its first appearance. In Dao Nan Tsang's *Cease Farming Sketch Book*, the terms *movable beads frame* and *sliding beads frame* were casually mentioned." (Chinese Cyclopedia). As this book was written during the Yuan Dynasty (which was ruling in the fourteenth century) it is clear that the abacus was in common use more than six hundred years ago.

Merits of the Abacus. In solving any problem in arithmetic, however simple it may be, two parts of our mental powers must be used, namely, the faculties of computation and of memory. We employ them when we add one and two; for in adding we must first retain the number one in our mind, then we fix our attention upon the other number, two, and finally we compute how much one and two are. With very small numbers these are very simple operations. But as the problems become more complex we soon reach the limit of our mental capacity. In dealing with difficult problems we must find some means by which the memory is relieved of the strain of computation. Fortunate are we that some wise men of past generations did find such means for us. We now merely need to learn how to use them. Among such devices are the bamboo tallies or counters used in ancient times and the Chinese Abacus. It is not an automatic machine such as the standard adding machines (in many respects it is superior to such machines) but even so it tenders no little service to us as a memorizing agent in computation. By using this instrument we can devote our entire attention to accuracy in computing and leave the memory work to the frame.

Advantages and Disadvantages of Bead Arithmetic as Compared with Written Arithmetic. Upon comparison, both the Bead and written or Pen Arithmetic, as it is called in China, have their weak as well as their strong points. In dealing with intricate problems the *Pen Arithmetic* is indeed more serviceable, but for most of the daily business transactions, Bead Arithmetic is far better suited. Its chief advantage over the pen arithmetic is the economy of time. It is quite safe to say that to solve any problem in the fundamental operations of arithmetic by using the abacus, it takes at most half the time that is needed when the written numerals are used. This is especially true in addition. For example, if two men were to solve the same problem in this process, beginning at the same time but using different methods, the result would no doubt be that one could hardly have finished copying down the numbers in the question when the other would have obtained the answer already on the frame.

Bead Arithmetic, in spite of its superiority, is not without certain disadvantages. With an unskilled operator it is liable to error because the beads are apt to be moved out of place inadvertently. However, this can be guarded against by carefulness.

Widespread Use of the Bead Arithmetic. Owing to its advantages the Bead Arithmetic is widely used in China and her neighboring countries. In almost every home throughout the length and breadth of the Celestial Land, the abacus is found. Indeed, in the past dynasties it was the only means by which the Royal Astronomers were able to tell the seasons and the days; the Treasurers of the State to clear the National Accounts, and the common *roundheads* to transact their business. So important is it considered by the tradesmen in

China that they make its use one of their necessary qualifications. In his advertisement, a Chinese merchant never fails to mention: "Only those with a fair skill in the abacus need apply." It was early introduced into Japan and Korea and has made a home there. Recently many Westerners have adopted it and have not failed to appreciate its usefulness. If a man values his business time and wants to do twice his usual work he will find in the bead arithmetic a sure means to double his output.

Field Covered by the Bead Arithmetic. Some of our foreign friends seem to consider the Chinese Abacus only as an instrument for addition, or at most for both addition and subtraction. A very few have realized that it is not only capable of solving such addition and subtraction problems but multiplication and division also. In fact, it can be used in performing any and every process in Arithmetic. Though in some places, it is rather inconvenient to use the abacus, especially in operations involving complicated fractions, yet even then it is possible. In other places the processes given in Bead Arithmetic are much simpler. For example, in extracting the square and cube roots of numbers, the process is merely a matter of subtraction.

As a science, the bead arithmetic is not so well developed as the written arithmetic. In the ordinary Chinese book on the abacus considerable attention is given to business and industrial problems, taxes and mensuration. But from the scientific point of view the literature is very meagre.

Scope of this Book. This booklet gives only the fundamental operations of bead arithmetic. The possibilities of the subject by no means end there. The aim of the writer is to familiarize the English-speaking Filipinos with this old, ingenious invention which seems to have been so long used exclusively by the Chinese. He hopes that more men may have the benefit of this labor-saving device which may be brought to a greater state of perfection by some mathematician.

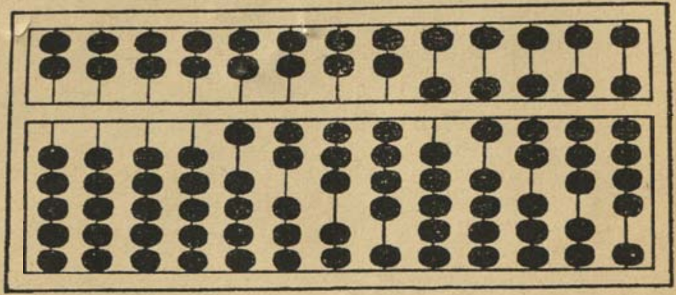
A complete treatise on the subject in the English language is still awaiting some one else to make. The author merely wishes to strike the keynote and expects his hearers to determine the melody.

CHAPTER I

GENERAL DIRECTIONS

Construction of the Abacus. The abacus as shown in the figure consists of a rectangular wooden frame which is divided lengthwise into two unequal parts by a horizontal beam or bar. It may have nine, eleven, thirteen or more ordinate columns of movable beads, usually made of wood. The number of beads in each column is seven; two above the beam and five below it. For convenience the beads above the beam are called *altobeads*, and those below it *hypobeads*. One altobead is equivalent to five hypobeads. One form of the abacus contains only six movable beads in each ordinate column, one altobead and five hypobeads.

THE CHINESE ABACUS



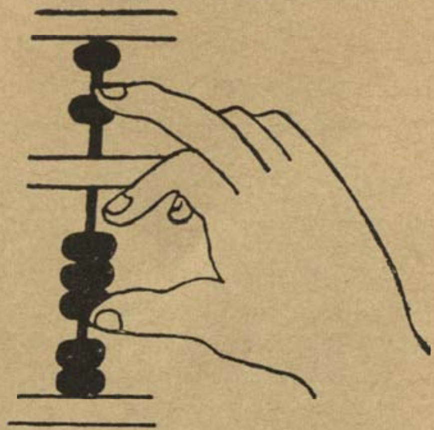
1 2 3 4 5 6 7 8 9

There is still another type which contains only five beads in each row, one above the beam and four below it. They, as the reader will find out later on, are suitable only for addition and subtraction.

Place Value. The value of a bead depends upon which column is assumed to be the unit's column. The beads in the left-hand column are greater than those in the right-hand column. One unit in the left column is always ten times as large as one unit in the adjacent column to the right. Thus, if we assume that the first column on the right-hand side of the frame to be the unit's place, one hypobead in the first column will be worth one unit; one in the second column will be worth ten units; one in the third column will equal 100 units, etc. Likewise one altobead in the first column will be five units; one in the second column will equal 50 units, and so on.

Fingering. It has been proved that it is best to move the hypobeads with the thumb and index finger and the altobeads with the middle finger. If the thumb and middle finger are employed to shift the hypobeads, the fore-finger will be idle and in the way. This incorrect method of fingering is not only awkward but also is conducive to mistakes,

CORRECT METHOD OF MOVING THE BEADS



Using the Abacus. Before using the abacus all the altobeads should be placed against the upper side of the frame and all of the hypobeads against the lower side. This having been done they are ready to be moved up or down the frame for recording any number. The middle beam is the axis along which the *active* beads that form the computation are arranged. All the *neutral* or idle beads should be against either side.

In addition and subtraction it is not necessary to move the upper one of the altobeads, nor the last one of the hypobeads. For inasmuch as one altobead is equivalent to five hypobeads, instead of moving the last hypobead to make five, an altobead may be used and the four hypobeads returned to *neutral.* Similarly, because one hypobead in the left column is equal to two adjacent right-hand altobeads, instead of using the upper altobead to make ten, one hypobead on the left column may be used and the lower altobead returned to neutral.

Check. It is always advisable, especially for a beginner to check the results of a computation. For this purpose one of two methods may be adopted. Addition and subtraction and also multiplication and division may be conducted to verify each of its opposite process. Thus, an example in addition may be checked by subtraction and *vice versa ;* and the same is true of multiplication and division. But the general method used by the abacists is to repeat the work. This method, however, is only for those who are proficient with the abacus.

EXERCISES

Represent the following numbers on the abacus :

1.	1,427
2.	7,543
3.	500,005
4.	137,005
5.	10,010
6.	16,896

Practise indicating numbers on the abacus until you can reproduce them accurately and rapidly.

CHAPTER II

ADDITION

The process of combining two or more given numbers to obtain one equal to the sum of them is called addition.

Try to add 73, 49, 21 and 58, first by using the abacus, and then by written numbers. Check the result obtained by each way and notice the time used in each ease.

You may be able to add these numbers by using the abacus, but you will find that it is rather inconvenient to do so, and that it takes a much longer time to solve by this method than by the ordinary written one. Certainly this is true, for you, as a novice to this subject, are unskilled both in hand and in mind. For a beginner there has been established a set of *guides* or hints which in Chinese books termed *secrets* show how to add the numbers to be added on the abacus. It is these that make the process in bead arithmetic easy and quick and therefore should be carefully and well committed to memory.

There are altogether seventeen guides given here. For the facility of memorizing, they are condensed into the simplest of sentences. Some words used in them need hence to be explained. To *lower five* means to move down one altobead; to *cancel* to take away from the middle beam; to *raise* to move up hypobeads; and to *forward ten* to move up one hypobead on the next column to the left. In each guide, the first number indicates the number to be added to another which is already on the frame, and the rest tells how to proceed. For illustration, take the first guide: *one; lower five, cancel four*. In this a number, four, is already on the frame in a column. To this number you want to add another number, one. Now, how will you proceed? You move down in the same column, five, that is one altobead and take away four, that is four hypobeads.

GUIDES FOR ADDITION

The following are the seventeen rules mentioned above:

One; lower five, cancel four.
Two; lower five, cancel three.
Three; lower five, cancel two.
Four; lower five, cancel one.
One; cancel nine, forward ten.
Two; cancel eight, forward ten.
Three; cancel seven, forward ten.
Four; cancel six, forward ten.
Five; cancel five, forward ten.
Six; cancel four, forward ten.
Seven; cancel three, forward ten.
Eight; cancel two, forward ten.
Nine; cancel one, forward ten.
Six; raise one, cancel five, forward ten.
Seven; raise two, cancel five, forward ten.
Eight; raise three, cancel five, forward ten.
Nine; raise four, cancel five, forward ten.

Example 1. How many dollars are three and four dollars?

In adding, we first place the number 3 on the frame by moving up three hypobeads on the first column from the right (which column for convenience sake, is taken as the units' column). To this number, 4 is to be added. But as three hypobeads on the first column have been used already, the other two beads left are not enough for four. We must, then, call out the fourth guide, *Four, lower five, cancel one.* Accordingly we move down one altobead and take away one hypobead. Then the altobead and the two remaining hypobeads on the middle bar make seven, the result desired.

Example 2. How many metres are 24 metres and 36 metres?

In a like manner we first place 24 on the frame by raising up two hypobeads on the second column and four on the first column and then proceed to add. Here there is one distinction between the bead arithmetic and the written form worthy of notice. In pen arithmetic we begin to add at the right-hand figures, but in bead arithmetic we begin at the left-hand figures. Thus we begin at the left to add: On the second column, *Three, lower five, cancel two.* On the first column, *Six, cancel four, forward ten.* They together make sixty. Hence the result is 60 metres.

Example 3. Find the sum of 783, 275, 14, 5697 and 34.

First add the first and second numbers, then take up the third, then the fourth and finally the last Take care that hundreds are added to hundreds, tens to tens, etc. The sum should be 6,803.

EXERCISES

Add, checking the work by adding in reverse order. Try to *speed up* and notice the time required to obtain the correct answer. Repeat the same operation and repeat again and as many more times as necessary until you are able to add accurately and rapidly. You may assume any column as the units column, but do not get confused.

1. 1, 2, 3, 4, 5, 6, 7, 8 and 9.
2. 45, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
3. 90, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
4. $312.75, $95.75, $21.03, and $2304.69.

5. In a city there were four schools, the first containing 396 pupils, the second 683 pupils, the third as many as the first and second together, and the fourth as many as the other three together. How many pupils attended the third school? The fourth school? All the schools?

Chapter III

SUBTRACTION

Subtraction is defined as the process of finding the difference between two numbers or the taking of a part of a number from itself and finding how many are left. The number from which another is taken is known as the minuend, and the number taken from another is known as the subtrahend. The result obtained by taking one number from another is called the remainder.

Using the abacus subtract 498 from 884.

As in additon there are 17 *guides* for subtraction. The student will soon find that they are the reverse of those given for addition. In each guide, the first number is the number to be subtracted, and the remainder of the guide tells how to proceed. For example, in the first guide. *One, cancel five, return four*, the number, one, is the number to be subtracted from a number already on the frame. This means that you are going to subtract one from a five which is already on the frame (as this guide is made for that number). Now five minus one is four, how are you going to subtract on the frame? The guide tells you to cancel the five and return four, that is to add four. Memorize these guides well as that is the secret of an efficient abacist.

GUIDES OF SUBTRACTION

One ; cancel five, return four.
Two ; cancel five, return three.
Three ; cancel five, return two.
Four ; cancel five, return one.
One ; cancel ten (i. e. take away one hypobead on the left column), return nine.
Two ; cancel ten, return eight.
Three ; cancel ten, return seven.
Four ; cancel ten, return six.
Five ; cancel ten, return five.
Six ; cancel ten, return four.
Seven ; cancel ten, return three.
Eight ; cancel ten, return two.
Nine ; cancel ten, return one.
Six ; cancel ten, return five, cancel one.
Seven ; cancel ten, return five, cancel two.
Eight ; cancel ten, return five, cancel three.
Nine ; cancel ten, return five, cancel four.

Example From 884 subtract 498.

Place the minuend 884 on the abacus. As in addition, we begin at the left to subtract, the hundreds from the hundreds, the tens from the tens, and the units from the units. Thus, to subtract 4 from eight on the hundreds' column we use, *Four cancel five, return one.* Four are left on the column. To subtract 9 from 8 on the tens' column we use, *Nine cancel ten, return one* (i. e. move down one hypobead on the hundreds' column and add one on the tens' column). Nine are left on the tens' column. To subtract 8 from 4 on the units' column, we use, *Eight, cancel ten, return five, cancel*

3, (i. e., move down one hypobead on the ten's column, add one altobead on the units' column and cancel three hypobeads on the same column. Six are left in the units' column. The whole number left on the frame is 386. Therefore 386 is the remainder.

Proof.

386 the remainder, plus 498, the subtrahend equals 884 the minuend. Hence the work is correct.

EXERCISES

Solve the following examples, proving your work by addition. Time yourself.

1. From 45 subtract 1, 2, 3, 4, 5, 6, 7, 8 and 9 successively.

2. From 90 subtract the same numbers consecutively.

3. From 135 subtract the same.

4. Find the difference between 3,402,731 and 1,924,005.

5. If you buy at a Chinese grocery store tea for 78 cents, rice for 26 cents, and sugar for as much as the difference between the amounts paid for tea and rice, giving in payment a two dollar bill, how much change should you receive ?

CHAPTER IV

MULTIPLICATION

The terms used in multiplication have been defined as follows: Multiplication is the process of taking one number as many times as there are units in another, or, is a short process of finding the sum of several equal numbers. The multiplicand is the number taken or multiplied; the multiplier is the number that shows how many times the multiplicand is taken; and the product is the result of multiplying.

Try to multiply 68 by 32 on the abacus.

The fundamental requisite in this process is to have a masterly knowledge of one-figure multiplication, i. e., the multiplication in which both the multiplicand and multiplier are each expressed by one figure. You know already how to multiply such numbers as 3 x 4, 6 x 7, 7 x 9. You ought to read the following table and remember how much the products of these numbers are. In the table, the numbers in the left-hand column may be regarded as the multipliers and those across the top as the multiplicands. The numbers in the horizontal rows opposite the multipliers and under the multiplicands will be the products. They may be read thus; 2 x 2 equals 4; 2 x 3 equals 6; 2 x 4 equals 8; 3 x 2 equals 6; 3 x 3 equals 9, etc.

MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

DRILL EXERCISES

State instantly the products of the following:

2 x 3   5 x 7   7 x 3   8 x 9   8 x 5   5 x 8
8 x 4   6 x 5   7 x 6   9 x 9   8 x 5   8 x 6
4 x 3   7 x 2   3 x 8   6 x 4   5 x 5   4 x 9
4 x 6   6 x 5   9 x 2   7 x 7   3 x 6   6 x 9

PLACE OF THE MULTIPLIER, MULTIPLICAND, AND PRODUCT ON THE ABACUS

For convenience, the multiplier is put on the left-hand side and the multiplicand on the right-hand side of the frame. But the right-hand column (that is the first column on the right) cannot be taken as the units' place of the multiplicand. Instead, this column is used as the units' column of the product. This is for the convenience of a beginner.

When the multiplier is expressed by one figure, the column on the right-hand side next to the units' place of the multiplicand is ths units' place of the product (provided that both the multiplier and the multiplicand are integral). When the multiplier is expressed by two figures, the second column from the units' place of the multiplicand (i. e. second column of the right) is the units' of the product; when expressed by three figures the third column, and so forth.

Thus when the multiplier is of one digit, the multiplicand may be put on the right-hand side of the abacus, leaving one space vacant on the right for the units' place of the product; when the multiplier is of two digits leave two columns vacant; when of three figures, leave three columns vacant, etc. This method is for the use of the beginner. The numbers may be put in any place so long as there is a place for the product. The multiplier is not placed on the frame when it is composed of only one digit.

WHEN THE MULTIPLIER IS EXPRESSED BY ONE FIGURE

Example 1. How many are 4 times 316 ?

Place the multiplicand, 316, on the right-hand side of the frame, leaving one column at the right vacant. Begin at the right to multiply in the following manner:

4 x 6 equals 24. But 24 is composed of two tens and four units. Therefore the figure 4 is placed in the units' column of the product (next column on the right from the units' place of the multiplicand) and we cancel the units' figure of the multiplicand which is 6 and in its place put the figure 2 (tens of the partial product).

4 x 1 equals 4. Here the number 4 means 4 tens, the product of 4 times one ten. Therefore the figure 4 is put in the tens' place of the product; i. e. add 4 to the tens of the preceding partial product (2 plus 4 equals 6). The tens' place of the product should now read 6. Immediately cancel the 1 in the tens' place of the multiplicand.

4 x 3 equals 12. 2 is put in the hundreds' column of the product and the figure multiplied, 3, is cancelled and in its place the figure 1 is put. Hence the product is 1264.

Note I. From the above example it can be seen that the figure in the tens' place is each a partial product and is put on the column of the figure just multiplied and that the figure in the units' place is put on the right-hand column next to the units' place of the multiplicand. At each operation that figure of the multiplicand which has just been multiplied must be cancelled at once, the multiplicand thus being converted into the product.

Note II. The method used in solving the

preceding example is called, in Chinese books, the *Break-end Method*, because we begin to multiply at the right-hand end of the number. The other method in which we begin to multiply at the left is called the *Break-head Method*. It has been proven that the former method is by far much better than the latter. Therefore the former method only is discussed in this book.

Example 2. Multiply 617, 283, 945 by 2.

Place the multiplicand, 617, 283, 945 on the frame and begin to multiply as follows:

5 : 2 x 5 equals 10 : Change 5 into 1.

4 : 2 x 4 equals 8 : Cancel the figure 4 just multiplied and add 8 to the column on the right.

9 : 2 x 9 equals 18 : Change 9 into 1 and add 8 on the column to the right.

3 : 2 x 3 equals 6 : Cancel the 3 and add 6 to the column on the right.

8 : 2 x 8 equals 16 : Change 8 into 1 and add 6 to the column on the right.

2 : 2 x 2 equals 4 : Cancel the 2 and add 4 to the column on the right.

7 : 2 x 7 equals 14 : Change 7 into 1 and add 4 to the column on the right.

1 : 2 x 1 equals 2 : Cancel the 1 and add 2 to the column on the right.

6 : 2 x 6 equals 12 : Change 6 into 1 and add 2 to the column on the right.

Hence the product is 1,234,567,890.

EXERCISES

Find the product of each of the following :

1. 3 x 41,152,263.
2. 4 x 3,086,419,725.
3. 5 x 246,913,574.
4. 6 x 205,761,315.
5. 7 x 17,636,684.
6. 8 x 15,432,098,625.
7. 9 x 13,716.421.

8. A forest nursery produced last year 675; 984 seedings of various species. At that rate how many would it produce in 7 years ?

9. A Chinese merchant sold 34,579 yards of Hangchow satin at a price of $2.00 per foot. How much did he receive for the sale ?

10. If one piece of Kiangsi linen contains 50 metres and the cost of one metre is $4.00; how much must be paid for 7,296 pieces ?

The multiplication of a number by 10,100,1000. or 1 with any number of ciphers annexed is solved by simply annexing to the multiplicand as many ciphers as there are in the multiplier.

WHEN THE MULTIPLIER IS EXPRESSED BY MORE THAN ONE FIGURE

When the multiplier is of more than one figure, the left-hand figure in it is designated as the first figure, the next one the second, still the next one the third, etc.

The right band figure of any partial product of any multiplicand by the first figure of the multiplier is placed on the first column on the right from the figure multiplied; that by the second figure of the multiplier is placed in the second column to the right; that by the third figure of the multiplier is placed in the third column, etc.

Example 1. Multiply 68 by 32.

For convenience, the multiplier is put near the left-hand side of the abacus so that it can be remembered and the multiplicand is placed near the right-hand side, leaving two columns for the product (as there are two figures in the multiplier). See above.

Since we cannot multiply 68 by 32 at one operation, we multiply part of 68, viz.; 8 units and 6 tens or 60 and at the same time add the products.

Thus, 2 x 8 equals 16. We place one (1) in the first column to the right of the multiplicand and six (6) in the second column to the right from the figure multiplied, which is 8. 3 x 8 equals 24. We at once cancel the figure multiplied, 8, and in its place put 2 and add 4 in the next column to the right.

2 x 6 equals 12. We add 1 on the first column to the right and 2 on the second column. 3 x 6 equals 18. We at once cancel the 6 and put a 1 in its place and add 8 in the next column to the right. (Rule Eight cancel two, forward ten). Hence the product is 2176.

Note. It may be more convenient in the first step to use 8 of the multiplicand, 68 to multiply first 2 and then 3 of the multiplier, 32, and in the second step to use 6 of 68 to multiply the multiplier in the same order. Their products are then recorded in their proper places as described before. In this case, the multiplicand becomes the multiplier and the multiplier the multiplicand.

Example 2. Find the product of 4,567 by 4,325.

For the first step, multiply 7 of the multiplicand 4567 by the multiplier, 4325 by taking up, as shown before 5, 2, 3, 4, figure after figure; for the second step, multiply 6 by the same figures; for the third step multiply 5; and lastly for the fourth step, multiply 4. Be sure that the product of any multiplicand figure by the fourth figure of the multiplier be placed on the fourth column on the right from the figure multiplied; that by the third figure, on the third column. etc. For example, in the first step; 5, (fourth figure of the multiplier) x 7 equals 35. The figure 5 of the product is placed on the fourth column to the right of the figure multiplied (7). The product of the above is 19,752,275.

Note I. In the multiplication of a number by a multiplier of more than two figures, the method described in old Chinese books is to multiply the number first by the second figure of the multiplier, then by the third, then by the fourth, the fifth, etc., if any, and lastly by the first. This method is simply designed from the guides given for multiplication. As the old guides are not adopted to the process described in this book, this method is not taken up.

Note II. When the multiplier contains a cipher the number is multiplied by the significant figures only, but care must be taken in placing the right-hand figure of each partial product in its proper place.

EXERCISES.

Multiply:

1. 345 x 27 3. 5,241 x 345 5. 38,613 x 206.

2. 369 x 63 4. 5,234 x 739 6. 58,325 x 5,802.

7. There are 1,440 minutes in a day. How many in a year?

8. A mile contains 5,280 feet. How many feet in 73 miles?

9. A man bought 327 bags of rice. How much did he pay for them if each bag held 3 bushels and the price was $2.59 per bushel?

10. A steamship sails at an average speed of 63 miles per hour. If another steamship sailed from the same port at the same time in an opposite direction at the rate of 45 miles per hour; how far apart were they in 6.5 days? How far apart were they if they both sailed in the same direction?

Chapter V

DIVISION

Put the number 10 on the abacus and subtract 2 from it as many times as you can. How many times do you subtract before it is finished? From the same number subtract 3 in the same manner. How many times? What is left? How much is 2 x \$5? How many times is 2 contained in \$10, 5 in \$10? How much is ½ of 10?

Division is a short form of subtraction. It is the reverse process of multiplication and has been defined as the operation by which, given the product of two numbers and one of them, the other can be found. The given number or the product which is to be divided is called the dividend, and the other number showing into how many equal parts the given number is to be divided is called the divisor. The part of the dividend left when division is not exact is called the remainder.

Short Division. When examples in division are solved without using multiplication, the process is called short division. Short division is used only when the divisor is less than 10. The process is very easy if the *guides* are well committed to memory and remembered.

Guides for Short Division. Showing how to manipulate the beads.

Divided by *one*. One by one, forward one. Two by one forward two. Three by one, forward three.........Nine by one, forward nine.

Divided by *two*. One by two is five. Two by two, forward one. Four by two, forward two. Six by two, forward three. Eight by two, forward four.

Divided by *three*. One by three is three, plus one. Two by three is six, plus two. Three by three, forward one. Nine by three forward three.

Divided by *four*. One by four, is two plus two. Two by four is five. Three by four is seven, plus two. Four by four, forward one. Eight by four, forward two.

Divided by *five*. One by five is two. Two by five is four. Three by five is six. Four by five is eight. Five by five, forward one.

Divided by *six*. One by six is one plus four. Two by six is three plus two. Three by six is five. Four by six is four, plus four. Five by six is eight, plus two. Six by six, forward one.

Divided by *seven*. One by seven is one plus three. Two by seven is two plus six. Three by seven is four plus two. Four by seven is five, plus five. Five by seven is seven plus one. Six by seven is eight, plus four. Seven by seven, forward one.

Divided by *eight*. One by eight is one plus two. Two by eight is two, plus four. Three by eight is three, plus two. Two by eight is two plus four. Three by eight is three, plus six. Four by eight is five. Five by eight is six, plus two. Six by eight is seven, plus four. Seven by eight is eight, plus six. Eight by eight, forward one.

Divided by *nine.* One by nine is one, plus one. Two by nine is two, plus two. Three by nine is three, plus three. Eight by nine is eight, plus eight. Nine by nine forward one.

EXPLANATION.

Divided by *one.* One by one equals one. Two by one equals two. As every one knows, any number divided by one is equal exactly to that number, so there seems no need to give this a guide. Yet there is a demand for it, for the word *one* may be the left-hand figure of a divisor of more than one figure. In this case the quotient cannot be equal to the dividend.

By *forward one* we mean to move up one hypobead on the left column next to the figure divided. Division on the abacus differs from the multiplication in that we begin at the left and the units' place in the quotient is the left column next to the units' place in the dividend (when the divisor is of one figure). Therefore the number in the *guide* preceded by *forward* is in the units' place in the quotient and is thereby integral. According to this guide, then, if we want to divide one by one, we, after putting the dividend on the frame, are to record one on the left row next to the figure divided. The figure having been divided must, as in any other case in Bead Arithmetic, be removed or cancelled. The number recorded there is the quotient. The rest of the sentences may be explained in the same manner.

Divided by *two*. By *one by two is five* is meant that we, to divide one by two, after placing the dividend on the frame, are to change the number divided, one, into five, in the same column. As the units' place in the quotient is the left column next to the dividend figure, the figure *five* recorded there as the quotient is in the tenths' place in the quotient. In other words, if one is divided by two, one half or five-tenths will be the quotient and so on.

The *guides* for the divisor three and other numbers may be explained in the same way, only that the number after plus, which is the remainder, is to be added to the dividend figure, if any, on the right column next to the quotient figure. For example in the guide, *One by three is three, plus one*, besides changing the dividend figure, one, into three as the quotient, we add or place the remainder, one (which means one-tenth) on the right-hand column next to the three.

Example 1. Divide 1,497 by 2.

Place the dividend 1,497 on the frame near the left side, leaving one column vacant for the quotient figure. Keep the divisor in mind, or if you cannot remember it, put it near the right-hand side of the abacus. Begin at the left to divide as follows:

"One (i. e. 1,000) by two is five"—immediately cancel the first figure and change it to five.

"Four by two, forward two"—immediately cancel four and move up two on the left column.

Now on the next column we find nine. There is no guide for it. But nine is equal to 8 plus 1; hence we divide 8 first and then take up the remainder, one. "Eight by two, forward four"—cancel eight from the nine and forward four on the left column. "One by two is five"—cancel the one and place five on the same column.

We now have left undivided only the last figure of the dividend. This figure is in the units' place in the dividend and therefore any number on the left-hand column next to it is in the units' place in the quotient and that on this column is in the tenth's place.

In the same manner as we divided 9 above we divide 7. "Six by two forward three"—cancel six and forward three on the column to the left, leaving one on the units' column of the dividend. Now "One by two is five"—cancel the one and place five on the same column. Hence the quotient is $748.5 with a remainder of one.

Proof. $748.5 the quotient multiplied by 2, the divisor, is equal to 1,497, the dividend. Therefore the work is correct.

Division of this sort, where there is a remainder is said to be *not exact*, or in other words, when there is no remainder the division is *exact*.

Example 2. Divide 123,456,789 by 3.

After placing the dividend on the abacus we begin at the left to divide as follows:

1 : 1 by 3, plus 1 : Change 1 into 3 and add 1 on the next column to the right.

2 : 3 by 3, forward 1 : Cancel the three, forward 1 in the column to the left.

3 : 3 by 3, forward 1 : Cancel the three, forward 1 in the column to the left.

4 : 3 by 3, forward 1 : Cancel 3 from 4 and forward 1 in the column to left. 1 by 3 is 3 plus 1 : Change 1 into 3 and add 1 on the next column to right.

5 : 6 by 3, forward 2 : Cancel the 6 and forward 2 in column to left.

6 : 6 by 3, forward 2 : Cancel the 6 and forward 2 in column to left.

7 : 6 by 3, forward 2 : Cancel 6 from 7 and forward 2 in column to left. 1 by 3 is 3 plus 1 : Change 1 into 3 and add one on the next column to right.

8 : 9 by 3, forward 3 : Cancel 9 and forward 3 in column to the left.

9 : 9 by 3, forward 3 : Cancel the 9 and forward 3 in column to the left.

Hence the quotient is 41,152,263.

EXERCISES

1. Divide 123,456,789 by 4, 5, 6, 7, 8, and 9 respectively.

2. Divide 3,936 by 4.

3. Divide 4,986 by 9.

4. The total cost of 4 carts of rice is $154.42. What is the cost of one bag if each cart holds 6 bags?

5. If one hectare (10,000 sq. m.) of land is to be planted with trees and the distance between every two trees is 2 metres; how many seedlings are needed?

When the Divisor is more than 10.

When the divisor is expressed by more than one figure the process of division on the abacus is entirely changed. The process of solving such a problem is a combination of short division, multiplication and subtraction. In this process, we, beginning at the left, first divide the dividend figure by the left-hand figure of the divisor, then multiply the partial quotient thus obtained by the rest of the divisor and lastly subtract the product from the dividend. This is repeated until the whole number is divided. The left-hand figure of the divisor is therefore known, in old Chinese books, as the *divisor*, and the rest of it as the *multiplier*, the whole process being termed *subtracting division.*

Example. Divide 1,536 by 12.

We first place 1,536 near the left-hand side on the frame, leaving one column vacant and the divisor near the right-hand side of the frame. Then using the guides we first divide the first figure of the dividend, 1, by the first figure of the divisor;

multiply the quotient thus obtained by the second figure of divisor, 2, and subtract the product thus obtained from the dividend on the second column from the quotient figure. We repeat the same process until the entire dividend is finished.

Here may be stated the general rule showing on which column the product of a quotient figure by any figure of the divisor is to be subtracted. The product of any quotient figure by the second figure of the divisor must be subtracted from the dividend on the second column from the quotient figure; that of the quotient figure by the third figure of the divisor subtracted on the third column from the quotient figure; that by the fourth figure on the fourth column, etc.

"One by one, forward one." 2 (the second figure of the divisor) x 1 (the quotient figure) equals 2. 5 (dividend figure on the second column from the quotient figure) minus 2 equals 3. 336 is left undivided.

Now we might divide the new left-hand figure of the dividend by the first figure of the divisor and proceed as before. However, the product, 6 obtained in this manner would be too large to be subtracted from the dividend figure, 3, on the required column; hence we must take up a part of that figure, leaving the rest of the *minuend*, a term which for convenience may be adopted for the dividend figure or figures from which the product is to be subtracted. Thus, "Two by one, forward two"—immediately we cancel two from the three and forward 2 on the left column. 2 x 2 (quotient figure) equals 4. 13 - 4 equals 9. 96 is left undivided.

Likewise: "Eight by one, forward eight"—we cancel 8 from 9 and forward 8 on the column to the left. 2 x 8 equals 16. 16—16 equals 0. Hence the quotient is 128.

Long Division. Such division in which multiplication is employed may be called *long division.* Long division is the process used to solve such problem in division where the divisor is more than 10.

In addition to the guides already given for short division there are eighteen more for long division. These are used when the left-hand figure of the dividend being equal to that of the divisor, if divided ordinarily, there is no *minuend* from which the product of the quotient figure by the rest of the divisor must be subtracted; or when the *minuend* is too small for the product.

Guides for Long Division.

Divided by *one.* One by one, if no minuend, is 9 plus 1, i. e. to change the dividend figure, 1, into 9 and add one on the next column to the right. If the minuend is insufficient (for convenience, in the rest of the guides, we say only *insufficient*), subtract 1 from the quotient figure and add one to the dividend figure on the next column. (For convenience we merely say, "*subtract 1, return 1*").

Divided by *two*. 2 by 2, if no minuend, is 9 plus 2. Insufficient subtract 1, return 2.

Divided by *three*. 3 by 3, if no minuend, is 9 plus 3. Insufficient; subtract 1, return 3.

Divided by *four*. 4 by 4, if no minuend, is 9 plus 4. Insufficient; subtract 1, return 4.

Divided by *five*. 5 by 5, if no minuend, is 9 plus 5. Insufficient; subtract 1, return 5.

Divided by *six*. 6 by 6, if, no minueud, is 9 plus 6. Insufficient; subtract 1, return 6.

Divided by *seven*. 7 by 7, if no minuend, is 9 plus 7. Insufficient; subtract 1, return 7.

Divided by *eight*. 8 by 8, if no minuend, is 9 plus 8. Insufficient; subtract 1, return 8.

Divided by *nine*. 9 by 9, if no minuend, is 9 plus 9. Insufficient; subtract 1, return 9.

Example 1. If 136 sacks of flour cost $1028. 16; how much will one sack cost?

As before, after placing the number on the frame, we begin at the left to divide by the first figure of the divisor, multiply the quotient figure thus obtained by the second and third figures of the divisor in separate operation and subtract their respective products from the dividend figures in the second and third column from the quotient figure.

First step. If we were to divide ordinarily, using "One by one, forward one," there would be no minuend for the product. Therefore we use, instead, "One by one, if no minuend, is 9 plus 1." Accordingly we change the dividend figure, 1, into 9 and move up one on the next column to the right. As the minuend, 12, is still insufficient for the product of 9 x 3, we, basing on another guide, subtract from the quotient figure and add 1 to the dividend figure on the next column, thus making the minuend 22. But again, we find this number is too small for the product of 3 x 8; therefore we again subtract 1 from the quotient and add another 1 to the dividend making the quotient figure 7, and the minuend 32, which is now large enough for use. 3 x 7 equals 21. 32 - 21 equals 11. 6, the third figure of the divisor, times 7 equals 42. 118, the dividend figures on the third column from the quotient figure, minus 42 equals 76. We now have left undivided 76.16.

Second step. "5 (part of 7) by 1, forward 5." 3 x 5 (quotient figure) equals 15. 26 - 15 equals 11. 6 x 5 equals 30. 111 - 30 equals 81. 8.16 is left undivided.

Last step. "6 by 1, forward 6." 3 x 6 equals 18. 21 - 18 equals 3. 6 x 6 equals 36. 36 - 36 equals 0. Hence the cost of one sack of flour is $7.56.

The Units' Column of the Quotient. When the division is exact and the numbers are integral it is an easy matter to tell in which place each quotient figure is. But, when the conditions are otherwise as in the preceding exercise, it is a rather puzzling matter for a beginner to ascertain which column is the final units' place in the quotient. There is, however, a general rule for this. Invariably when the divisor is of one figure, integral of course, the first column on the left from the units' column in the dividend is the units' column of the quotient; when the divisor is of two figures, the second column from it; when of three figures the third column, and so on.

Example 2. Divide $18,144 equally among 56 men.

We proceed to divide by 5 and use 6 as a multiplier as follows: "One by five is two." 6 x 2 equals 12. 81 - 12 equals 69. But the remainder, 69, is greater than the divisor 56, so we have to take it up again.

"Five (part of 6) by five, forward one," add to the quotient figure 2. 6 x 1 (quotient just obtained) equals 6. 19 - 6 equals 13.

"One by five is two." 6 x 2 equals 12. 34-12 equals 22.

"Two by five is four." 6 x 4 equals 24. 24-24 equals 0. Hence the quotient is $324.

Example 3. The cost of 894 boxes of tea is $8,000. Find the cost of one box?

Dividend is 8,000 divisor 894.

"8 by 8, if no minuend, is 6 plus 8. Insufficient; subtract 1, return 8." (Here in order to record 16 on one column, we make one altobead represent 10 and hypobead 2. If, as sometimes occurs, this still does not answer the demand, we borrow beads from the column to the right.) 9 x 8 equals 72 160 - 72 equals 88. 4 x 8 equals 32. 880 - 32 equals 848.

"8 by 8, if no minuend, is 8." 9 x 9 equals 81. 128 - 81 equals 47. 4 x 9 equals 36. 470 - 36 equals 434. Hence the quotient is 89 with a remainder of 434. Therefore each box costs \$89 $\frac{434}{894}$

EXERCISES

Find the quotient of :

1. 1,386 divided by 21.
2. 6,473 divided by 37.
3. 749,653 divided by 721.
4. 563,217 divided by 354.
5. 150,475 divided by 5.122.
6. 4,820,805 divided by 62,413.
7. \$344.40 divided by 28.
8. \$8383.20 divided by 65.

9. At a church collection, each person on the average contributed 72 cents. If the entire sum collected was 59,112 cents. How many persons contributed ?

10. The quotient is 432, and the dividend 15,984. What is the divisor ?

CHAPTER VI

SHORT CUTS AND REVIEW EXERCISES

There have been found a great many short ways of solving certain problems, in the operations of addition, multiplication and division. These are generally the fruits of the experience of expert abacists. Here are given several examples to show a beginner how they may be used.

In Addition.

The sum of a series of consecutive numbers, or numbers, the difference between every successive pair of terms being equal, is equivalent to half the product of the sum of the first and last numbers of the series, multiplied by the number of terms in the series. Thus, 15 plus 16, plus 17, plus 18, plus 30 equals 360 equals $\frac{1}{2}$ (15 plus 30) x 16.

In Multiplication.

Multiplication by 5. Annex a cipher to multiplicand and divide by 2. Thus, 5 x 12 equals 120 divided by 2 equals 60.

By 0.5. Divide the multiplicand by 2.

By 25. Annex two ciphers to the multiplicand and divide it by 4.

By 0.25. Divide the multiplicand by 4.

By 125. Annex three ciphers to the multiplicand and divide it by 8.

By 1.25. Annex one cipher to the multiplicand and divide it by 8.

By 9. Annex one cipher to multiplicand and subtract from it the multiplicand itself. Thus, 35 x 9 equals 350 - 35.

By 0.9. Subtract from the multiplicand one tenth of the same, thus, 35 x 0.9 equals 35 - 3.5.

By 11. Annex a cipher to the multiplicand and add to it the multiplicand itself. Thus 11 x 35 equals 350 plus 35.

By 99. Annex two ciphers to the multiplicand and subtract from it the multiplicand itself.

By 9.9. Annex two ciphers to the multiplicand and subtract from it one-tenth of the multiplicand.

By 0.99. Annex two ciphers to the multiplicand and subtract from it one-hundredth of the multiplicand.

By 98. Annex two ciphers to the multiplicand and subtract from it double the multiplicand itself. Thus, 98 x 172 equals 17200-344 or 17200-172-172.

By 95. Annex two ciphers to the multiplicand and subtract from it five times the multiplicand itself.

If the first figure of the multiplier is one, multiply only by the rest of the figures, but the partial products are to be added to the multiplicand figures on the proper columns. For example; in multiplying 55 by 13 multiply only by 3, but take care that the multiplicand figure 5 should not be removed to it but 1 of the partial products 15, must be added.

In Division.

Divided by 5. Mark out one decimal place in the dividend and then multiply it by 2. Thus 125 divided by 5 equals 12.5 x 2.

By 0.5. Multiply the dividend by 2.

By 25. Mark out two decimal places in the dividend and multiply it by 4.

By 2.5. Mark out one decimal place in the dividend and multiply it by 4.

By 125. Mark out three decimal places in the dividend, multiply and divide it by 4 and 3 respectively.

There are many other short cuts that will suggest themselves to the student as he gains proficiency in the use of the abacus.

EXERCISES.

Solve each of the following by "short cuts."

1. \$27.50 plus \$28 plus \$28.50 plus \$29 plus....plus \$40.
2. 25,000 - 317 - 347 - 377 -........917.
3. 968 multiplied by 5, 125, and 17.
4. 163 x 5,954.
5. 167,250 divided by 12.5 and 0.75.

REVIEW EXERCISES

Find the value of the following:

Note. The parenthesis ( ), or vinculum ——, or brackets [ ] show that the number or numbers within are to be treated as a single number.

1. $18 - \dfrac{(3 \times 4) - (2 \times 3)}{3}$

2. (99 - 3) divided by 8 - (86 plus 10) divided by 12.

3. 78 - 2 (equals 78 x 78) - 146 - 147 - 148 - ....170.

4. [(82 x 76) divided by (192 - 11)] divided by 42.

5. (5 plus 2 plus 6) x 4 - (7 plus 8 - 3) divided by (6 plus 10).

6. A man paid $375 for a piano, $13 for freight and cartage, and $2 for tuning it. He then rented it for 7 quarters at $15 a quarter and afterwards sold it for $325. Did he gain or lose and how much?

7. A carpenter alone can build a house in 25 days, but with the assistance of his son he can build it in 15 days. In how many days can the son alone build it?

8. Two boys start from the same place and run in the same direction around a pool, the circumference of which is 36 yds. one at the rate of 58 yds. a minute, and the other at the rate of 43 yds. a minute. How far apart will they be after 5 minutes?

9. If the boys in the above problem run in opposite directions, how far apart will they be in five minutes?

10. Twenty men can build a schoolhouse in 56 days. In how many days can 70 men build it?

CHAPTER VII

SQUARE AND CUBE ROOT

The process of extracting the square and cube roots of numbers by the Bead Arithmetic, as has been stated before (page 4) is merely a matter of repeated subtraction. However, as the reader will find; it is rather cumbersome to use. It is given here only to show how it can be done on the abacus.

TO EXTRACT SQUARE ROOT

Method of Procedure.

1. Place the number on the right side of the abacus (for convenience call it the "square number,") and separate it into periods of two figures or columns each, beginning from the decimal point, the same as we do in pen arithmetic.

2. Mark one (1) on the left side of the frame (call it the "root number,") and subtract it from the left-hand period of the square number.

3. Add two (2) to the root number and subtract the sum again from the left-hand period of the square number. Add two again to the root number and again subtract the sum from the same period, repeating this process until the root number (which increases at every such operation) is greater than the number in that period.

4. Now, bringing down the next period, annex one cipher to the root number and add eleven (11), and from the new period subtract the sum. Repeat the process described in (3) until the root number is again too large to be subtracted from that period. Then repeat (4), thus continuing adding and subtracting until the whole number is finished.

5. If, as is sometimes the case, after bringing down the next period, the root number is still too large to be subtracted, bring down another period, but instead of annexing one cipher to the root number and adding 11; annex two ciphers and add 101, and then proceed on as in (3) and (4).

6. After the whole square number is thus used up, add one (1) to the final root number and divide this sum by two (2). The result is the square root of the number sought

Example 1. Find the square root of 625.

In accordance with the method of procedure just given, after placing the square number 625, on the right side of the frame, we separate it into two periods, the first period, containing one figure, 6, and the second period two figures, 25. These two periods indicate that there will be two figures in the square root.

We mark one on the left-hand side of the frame as the root number and subtract it from the left-hand period, 6, thus leaving 5.

We then add 2 to the root number making it 3, and then subtract it from the left-hand period, 5, thus leaving 2.

Now the root number is greater than the square number up to that period. Therefore, we annex one cipher to the root number, 3, making it 30, and add 11, thus obtaining 41. We now subtract 41 (the sum) from the next period of the square number 225, in leaving 184. Proceeding as the method indicates, 41, the root number plus 2 equals 43, and 184 minus 43 equals 141. Root number 43 plus 2 equals 45. Square number 141 minus 45 equals 96. Root number 45 plus 2 equals 47. Square number 96 minus 47 equals 49. Root number 47 plus 2 equals 49. Square number 49 minus 49 equals 0. Therefore the number is a perfect square.

The final root number plus 1 equals 50 (that is 49 plus 1), 50 divided by 2 equals 25 which is the square root of the number 625.

Example 2. Find the square root of 363,690.

Proceeding as before, after the first period is finished, we have 11 in the root number and 36 in the next period. If we were to add 11 to 110 (root number plus 0) we would find that the sum is greater than 36. Therefore we know that there is zero in the square root. Therefore we bring down another period, making the square number 3609. However, we do not subtract 121 but, instead, 1201 which is the sum of 1100 and 101 (see the 5th step of method of procedure). Now proceed as usual.

3609 minus 1201 equals 2408, etc. When the square number is finished we have 1205 for the final root number. Adding 1 to this and dividing by 2 equals 603 which is the square root of 363,609.

TO EXTRACT CUBE ROOT

Method of Procedure.

1. Place the number on the right-hand side of the abacus (for convenience call it the "cube number,") and separate it into periods of three figures each, beginning from the decimal point.

2. Mark one on the left-hand side of the frame (call it the "root number,") and subtract it from the left-hand period of the cube number. Also mark one on the middle of the frame and call it the "square number."

3. Add one to the root number and add the sum to the square number. Again add two to the root number and add the sum to the square number. Subtract the sum of the square from the left-hand period. Repeat the same process until the square number is greater than the cube number up to that period.

4. Now, bring down the next period; add one to the root number and add the sum to the square number. Then add one cipher to the root number and add 11 and again add the sum to the square number on the second column to the right from its right-hand figure. Subtract the final sum from the new period of the cube number. Then proceed as in (3) or (4), if necessary.

5. If, after bringing down the next period, the square number is still larger, bring down another period, but instead of annexing one cipher and adding 11, annex two ciphers and add 101 to the root number. The sum is then added to the square number on the fourth column to the right from the right-hand figure. Then proceed as in (3) or (4).

6. When the whole cube number has been used up, add 2 to the final root number and divide by 3. The result is the cube root.

Example 1. Find the cube root of 42,875.

Note. In solving this sort of problem an abacus of at least 15 columns is needed, or else we record the square number on a piece of paper or use two abaci.

Having placed the cube number, 42,875, on the right-hand side of the frame, we separate it into periods of three figures each. There are two periods, the first one consisting of 42, and the second of 875.

We then mark one on the left-hand side of the frame as the root number and subtract it from the left-hand period of the cube number, 42, leaving 41. Again we mark one on the middle of the frame as the square number.

We now add 1 to the root number and add the sum, 2, to the square number, obtaining 3. Again we add 2 to the root number, 2, obtaining 4, which is again added to the square number, 3, obtaining 7. We then subtract the sum, 7, from the first period of the cube number, 41, obtaining 34. Repeating the same process, the root number, 4 plus 1 equals 5. 5 plus 7, the square number, equals 12. Root number, 5, plus 2 equals 7. 7 plus 12, the square number, equals 19. The first period of the cube number, 34, minus 19 equals 15 which is now smaller than the square number.

Therefore we bring down the next period making the cube number 15,875.

According to step 4 of the method of procedure, we add 1 to the root number, making 8, which is at once added to the square number, 19, thus obtaining 27. Then we annex one cipher to the root number making it 80 and add 11. The sum 91 is added to the square number in the second column to the right from it, thus obtaining 2791. We subtract this from the cube number, 15,875, leaving 13,084. Then we proceed as follows:

Root number 91 plus 1 is 92. Square number 2791 plus 92 equals 2883. Root number 92 plus 2 equals 94. Square number 2883 plus 94 is 2977. Cube number 13084 minus 2977 is 10107.

Root number 94 plus 1 is 95. Square number 2977 plus 95 is 3072. Root number 95 plus 2 is 97. Square number 3072 plus 97 is 3169. Cube number 10107 minus 3169 equals 6938.

Root number 97 plus 1 is 98. Square number 3169 plus 98 is 3267. Root number 98 plus 2 is

100. Square number 3267 plus 100 is 3367. Cube number 6938 minus 3367 is 3571.

Root number 100 plus 1 is 101. Square number 3367 plus 101 is 3468. Root number 101 plus 2 is 103. Square number 3468 plus 103 is 3571. Cube number 3571 minus square number 3571 equals 0. Therefore the number is a perfect cube.

The final root number, 103, plus 2 equals 105. 105 divided by 3 equals 35. Therefore 35 is the cube root of 42,875.

Example 2. Find the cube root of 28,934,443.

Proceeding as in the above example, when bringing down the second period we find that the square number which is 2791 is greater than the cube number up to that period which is 1934. Therefore we bring down the third period. But we annex two ciphers and add 101, instead of annexing one cipher and adding 11 to the root number which is 27 in the fourth column to the right from it, making it 270901, which is to be subtracted from the cube number. We then proceed as before. The cube root should be 207.

APPENDICES

TABLE OF BINARY & DECIMAL FRACTIONS

$\frac{1}{64}$=.015625	$\frac{23}{64}$=.359375	$\frac{53}{64}$=.828125
$\frac{1}{32}$=.03125	$\frac{3}{8}$=.375	$\frac{27}{32}$=.84375
$\frac{3}{64}$=.046875	$\frac{33}{64}$=.515625	$\frac{55}{64}$=.859375
$\frac{1}{16}$=.0625	$\frac{17}{32}$=.53125	$\frac{7}{8}$=.875
$\frac{5}{64}$=.078125	$\frac{35}{64}$=.546875	$\frac{9}{64}$=.150625
$\frac{3}{32}$=.09375	$\frac{9}{16}$=.5625	$\frac{5}{32}$=.15625
$\frac{7}{64}$=.109375	$\frac{37}{64}$=.578125	$\frac{11}{64}$=.171875
$\frac{1}{8}$=.125	$\frac{19}{32}$=.59375	$\frac{3}{16}$=.1875
$\frac{17}{64}$=.265625	$\frac{39}{64}$=.600375	$\frac{13}{64}$=.203125
$\frac{9}{32}$=.28125	$\frac{5}{8}$=.625	$\frac{7}{32}$=.21875
$\frac{19}{64}$=.296875	$\frac{49}{64}$=.765625	$\frac{15}{64}$=.234375
$\frac{5}{15}$=.3125	$\frac{25}{32}$=.78125	$\frac{1}{4}$=.25
$\frac{21}{64}$=.328125	$\frac{51}{64}$=.796875	$\frac{25}{64}$=.390625
$\frac{11}{32}$=.34375	$\frac{13}{17}$=.8125	$\frac{13}{32}$=.40625
$\frac{27}{64}$=.421875	$\frac{21}{32}$=.65625	$\frac{57}{64}$=.890625
$\frac{7}{16}$=.4375	$\frac{43}{64}$=.671875	$\frac{29}{32}$=.90625
$\frac{29}{64}$=.453125	$\frac{11}{16}$=.6875	$\frac{59}{64}$=.921875
$\frac{15}{32}$=.46875	$\frac{45}{64}$=.703125	$\frac{15}{16}$=.9375
$\frac{31}{64}$=.484375	$\frac{23}{32}$=.71875	$\frac{61}{64}$=.953125
$\frac{1}{2}$=.5	$\frac{47}{64}$=.734375	$\frac{31}{32}$=.96875
$\frac{41}{64}$=.640625	$\frac{3}{4}$=.75	$\frac{63}{64}$=.984375
		1 =.1000000

ROMAN CARDINAL NUMBERS

I	1	XXII	22
II	2	XXX	30
III	3	XL	40
IV	4	L	50
V	5	LX	60
VI	6	LXX	70
VII	7	LXXX	80
VIII	8	XC	90
IX	9	C	100
X	10	CC	200
XI	11	CCC	300
XII	12	CCCC	400
XIII	13	D	500
XIV	14	DC	600
XV	15	M	1,000
XVI	16	MM	2,000
XVII	17	$\overline{\text{V}}$	5,000
XVIII	18	$\overline{\text{X}}$	10,000
XIX	19	$\overline{\text{L}}$	50,000
XX	20	$\overline{\text{C}}$	100,000
XXI	21		

The Romans contrived to express all numbers by these seven letters—I, one ; V, 5 ; X, 10 ; L, 50 ; C, 100 ; D, 500 ; M, 1,000.

The repetition of a letter repeats its value ; thus II signifies 2; XXX 30 etc.; V and L are never repeated.

When a letter of less value is placed before another of greater value, the value of the less is taken from the greater. When placed after it, the value of the less is added to the greater.

END

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語 論   CONFUCIAN ANALECTS

子曰、道千乘之國、敬事而信、節用而愛人、使民以時。

子曰、弟子、入則孝、出則弟、謹而信、汎愛衆、而親仁、行有餘力、則以學文。

CHAPTER V. The Master said, "To rule a country of a thousand chariots, there must be reverent attention to business, and sincerity; economy in expenditure, and love for men; and the employment of the people at the proper seasons."

CHAPTER VI. The Master said, "A youth, when at home, should be filial, and, abroad, respectful to his elders. He should be earnest and truthful. He should overflow in love to all, and cultivate the friendship of the good. When he has time and opportunity, after the performance of these things, he should employ them in polite studies."

5. FUNDAMENTAL PRINCIPLES FOR THE GOVERNMENT OF A LARGE STATE. 道 is used for 導, "to rule," "to lead," and is marked in the fourth tone, to distinguish it from 道, the noun, which was anciently read with the third tone. It is different from 治, which refers to the actual business of government, while 導 is the duty and purpose thereof, apprehended by the prince. The standpoint of the principles is the prince's mind. 乘, in fourth tone, "a chariot," different from its meaning in the second tone, "to ride." A country of one thousand chariots was one of the largest fiefs of the empire, which could bring such an armament into the field. The last principle,—使民以時, means that the people should not be called from their husbandry at improper seasons, to do service on military expeditions and public works.

6. RULES FOR THE TRAINING OF THE YOUNG:—DUTY FIRST AND THEN ACCOMPLISHMENTS. 弟子, "younger brothers and sons," taken together, = *youths, a youth*. The second 弟 is for 悌, as in chap. ii. 入 出, "coming in, going out," = at home, abroad. 汎 is explained by Chû Hsî by 廣, "wide," "widely"; its proper meaning is "the rush or overflow of water." 力, "strength," here embracing the idea of *leisure*. 學文, not *literary studies* merely, but all the accomplishments of a gentleman also:—ceremonies, music, archery, horsemanship, writing, and numbers.