APPENDIX C

APPLICATION OF ANALYZER TO A SET OF EQUATIONS FOR EXTERNAL BALLISTICS

The purpose of this appendix is to indicate one of many possible ways of setting up a program for the step-by-step solution of a pair of equations commonly used in external ballistics. It is assumed that the steps, defined by the increment in the independent variable, time, are taken to be so small that linear extrapolations may be used, and that when the steps are so small, the order in which the various calculations entering into a single step are performed is relatively unimportant. When this is true, then the order actually chosen should be one for which the total time required for a single step is minimized. In genera, the minimum time is attained by causing as many operations such as multiplication and division to proceed simultaneously. This is illustrated in Appendix B, where all the multiplications are done at the same time. If it is desired to complete the entire integration within some limited time, and to obtain the best possible accuracy in so doing, then one must choose between a large number of small, linear, steps, each planned to occupy a minimum time, or a smaller number of more complicated steps, each planned to yield a greater accuracy per step. Without entering into a detailed investigation of the theoretical optimum, it may nevertheless be noted that, (a) the speed of operation is usually great enough so that accuracy may be assured by using a large number of simple steps in preference to some more complex procedure, (b) the more complicated procedure will probably require a greater amount of equipment, and (c) more care and thought is required properly to set up the more complex methods, so that in some cases the time spent in attempting a more complex method would be greater than the operating time saved by the used of the complex method.

The procedure or program for each step that is suggested here is therefore not proposed as a unique or optimum method of attaining the desired solution, but rather as a possible method on which to base an estimate of the time required for solution. It is quite likely that, for any type of equation that is to be run a great many times, it would be well worth while to study carefully the relative merits of various ways of setting up the problem. Those who have had experience in setting up equations for the mechanical differential analyzer are well aware that some set-ups are greatly to be preferred to others.

In the following explanation concerning the external ballistic equations, it will be assumed that the material in the main body of this report has already been covered. From the illustrative example in Part II, where the equation for sinusoidal oscillations was dealt with, it should be evident how, for instance, the exponential portion of the E function in the ballistic equations is handled, and so nothing more will be aaid concerning that factor.

The equations for which Figure C-1 has been prepared are, in their usual form

$$\frac{d^2x}{dt^2} = -E\frac{dx}{dt} \qquad \left(E = \frac{e^{-hy}\,G(v^2)}{G} \right)$$

$$\frac{d^2y}{dt^2} = -E\frac{dy}{dt} - g$$

These can be integrated once with respect to time to give:

$$\frac{dx}{dt} = - \int_0^t E \, dx + v_{xo}$$

$$\frac{dy}{dt} = - \int_0^t E \, dy + v_{yo} - g \int_0^t dt$$

where the constants of integration, v_{xo} and v_{yo}, are the initial velocity components. The corresponding summation-difference equations may be written:

$$\frac{\Delta x}{\Delta t} = - \sum_0^t (E \quad x) + v_{xo}$$

$$\frac{\Delta y}{\Delta t} = - \sum_0^t (E \quad y) + v_{yo} - g \sum_0^t (\quad t).$$

In order to provide the argument for the function $G(v^2)$, it is necessary to build up values of v^2 as the work proceeds. In the present case this has been done in the following way; write:

$$v^2 = 2 \int_0^t \frac{d^2x}{dt^2} dx + 2 \int_0^t \frac{d^2y}{dt^2} dy + v_o^2$$

Now substitute the values of the second derivatives from the original equations into these integrals, and write the summation-difference version as:

$$v^2 = - 2 \sum_0^t (v^2 E dt) - 2g \sum_0^t (\Delta y) + v_o^2$$

Therefore the successive values of v^2 can be obtained by accumulating the values of $-2v^2Edt$ and $- 2g\Delta y$, after starting with the original value of v_o^2.

Figure C-1 indicates the sequence of interconnections necessary to carry out one step (or cycle) of the solution. Fifteen columns are used to stand for the fifteen accumulators to be used, and another column is used for the function of G generated by the function generator. Certain of the accumulators are cleared during or at the end of each cycle, and these may be called temporary storage accumulators, or merely storage accumulators. The remaining accumulators are originally provided with the initial conditions for the case to be solved, and these carry over from one cycle to the next the new initial conditions for the ensuing cycle.

Figure C-1 indicates the sequence of interconnections necessary to carry out one step (one cycle) of the solution. Fifteen columns are used to indicate 15 accumulators, and one column to indicate the function generator. Certain of the accumulators are cleared during or at the end of each cycle, and these may be referred to as storage accumulators. The remaining accumulators are originally charged with the initial conditions, and carry over from one cycle to the next the numbers which form the initial conditions for the ensuing cycle.

Operations are grouped together so as to obtain a compact operating cycle. All operations which take place simultaneously are indicated by the same group number; thus, 1A, 1B, 1C, and 1D are all initiated at the same time, and no operation in group 2 can begin until every one of the operations in group 1 has been completed.

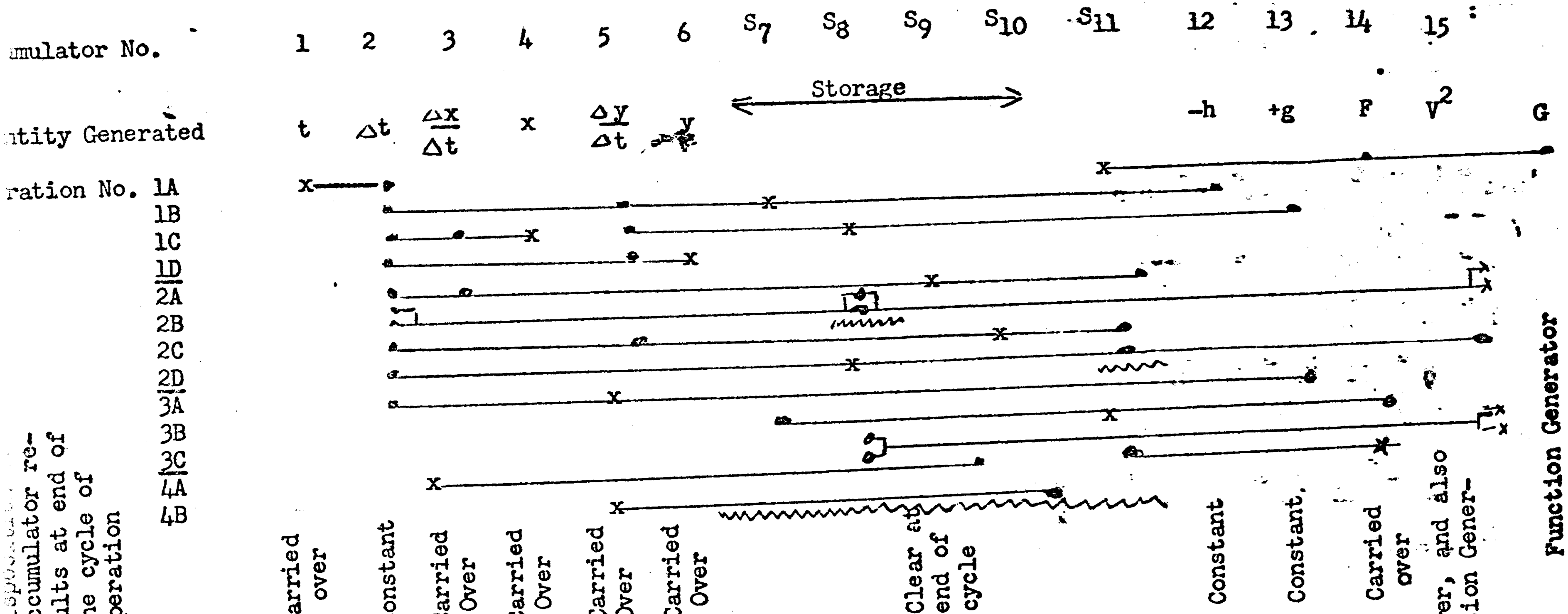
The symbol X is used to designate the accumulators which receive the result of any particular computation. A heavy dot indicates the accumulators which furnish the numerical values which enter into this result. When information is drawn from only one accumulator, the operation is one of either addition or subtraction. When information is drawn from two different accumulators, the operation is one of multiplication, in general. However, when one of the participating accumulators is t, the multiplication is considered to be a simple one, such as multiplication by 0.001, which can actually be accomplished by the process of addition after displacing the decimal point. Hence it is possible, by an extension of this idea, effectively to multiply the outputs of two accumulators, and include also the factor t all at one time. This is done, for instance in line 1B. Such an operation is counted as one multiplication. It should be noted that, when it seems desirable to use some increment in the independent variable other than a power of ten, a simple change of variables can be effected so as to bring the calculations back to this simplified form. It is then desirable to arrange that the recorder shall print the original variables, but such an arrangement will not lengthen the time of computation appreciably, if at all.

Although this arrangement requires eight multiplications (apart from those by t), nevertheless the various multiplications and additions are arranged so that the total time required for the calculation is that for three multiplications plus two additions. If 10^5 pulses per second is assumed for the pulse rate, then the time per step or cycle can be estimated, by the methods of Appendix B, as less than 20 milliseconds for 6-digit numbers, or less than 25 milliseconds for 8-digit numbers.

These estimates may possibly be too optimistic because they leave out of consideration the time required for the function generator to produce a new G value after it is furnished with a new value of v^2. If, however, the increments are small enough, there are several ways out of this difficulty; the time estimates just made are then valid.

At forty steps per second in a step-by-step integration, it is possible to make 4800 steps in a two-minute run. To this must be added the time required for transferring values to the recorder perhaps 50 to 100 times during the run. But this may be made as little as the time required for 100 additions, which is 10 milliseconds.

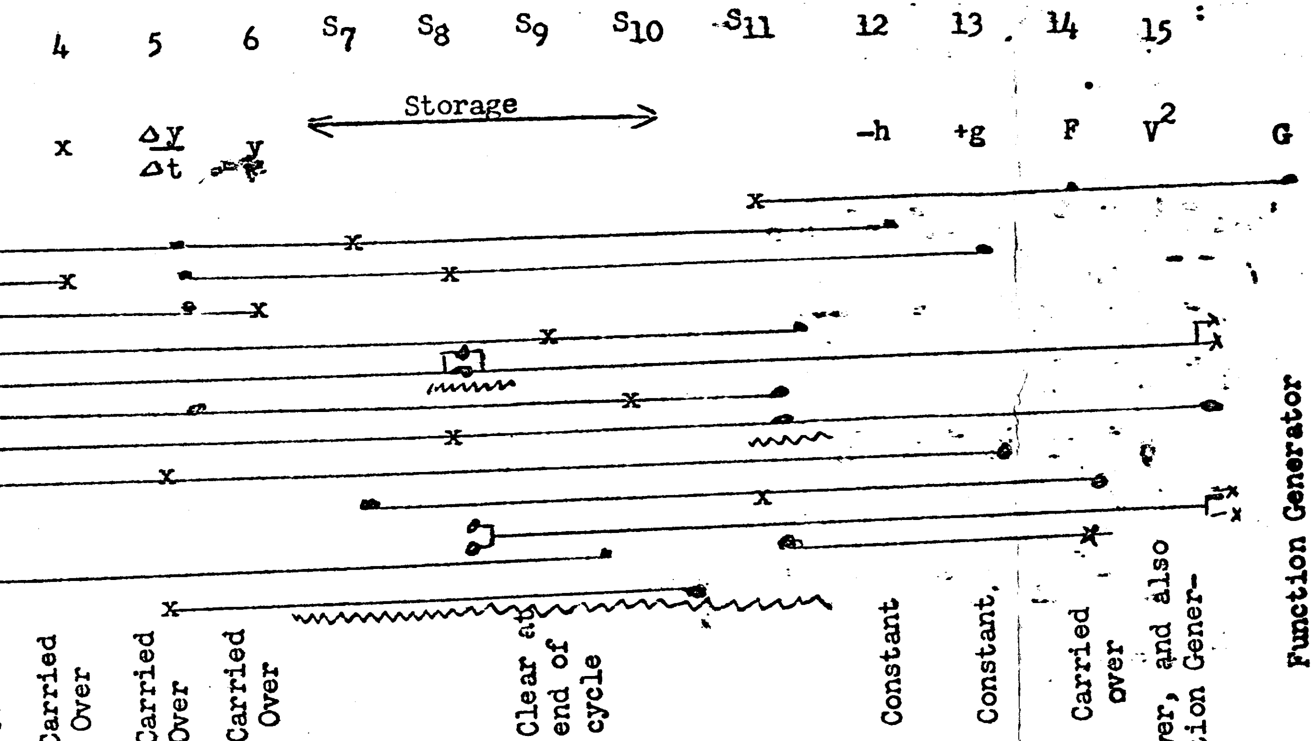
OUTLINE OF ONE STEP OF SOLUTION ON THE ELECTRONIC DIFFERENCE ANALYZER
of the EXTERIOR BALLISTIC EQUATIONS



tes: 1. Minimum total time for one step is time for 3 multiplications plus 2 additions.
2. By one cycle is meant the series of operations carried through for one value of t.
3. Multipliers not shown.

FIGURE C-1

STEP OF SOLUTION ON THE ELECTRONIC DIFFERENCE ANALYZER
of the EXTERIOR BALLISTIC EQUATIONS



e step is time for 3 multiplications plus 2 additions, series of operations carried through for one value of t.

FIGURE C-1

Operations Performed

Note: Heavy dot ● indicates participation in process ending in x.

$t_{n+1} = t_n + \Delta t$ $\quad S_{11} = E_n = G_n F_n$

$S_7 = -h\Delta y_n$

$x_{n+1} = x_n + \Delta x_n$ $\quad S_8 = g \frac{\Delta y_n}{\Delta t}$

$y_{n+1} = y_n + \Delta y_n$

$S_9 = S_{11} \Delta x_n = E_n \Delta x_n$

$V_n^2 - 2g\Delta y_n$

$S_{10} = S_{11} \Delta y_n = E_n \Delta y_n$,(

$E_n V_n^2 \Delta t = S_8$

$\left(\frac{\Delta y_n}{\Delta t}\right) - g\Delta t$ (Clear S_{11})

$S_4 = F_n S_7 = -F_n h \Delta y_n$

$V_n^2 = 2g\Delta y_n - 2E_n V_n^2 \Delta t = V_{n+1}^2$

$\left(\frac{\Delta x_n}{\Delta t}\right) - E_n \Delta x_n = \frac{\Delta x_{n+1}}{\Delta t}$ $\quad F_{n+1} = F$

$\left(\frac{\Delta y_n}{\Delta t}\right) - E_n \Delta y_n = \frac{\Delta y_{n+1}}{\Delta t}$

CLEAR S_7, S_8, S_9, S_{10}, S