An interview with

Alfio Quarteroni

Conducted by Philip Davis
on
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Interview conducted by the Society for Industrial and Applied Mathematics, as part of grant # DE-FG02-01ER25547 awarded by the US Department of Energy.

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ABSTRACT:

Professor Alfio Quarteroni discusses a range of topics in this interview with Phil Davis, including his work on finite elements, spectral methods, and domain decomposition. He began studying economic issues in a more professional school before switching over to study mathematics. Quarteroni received his Ph.D. from University of Pavia in Italy, completing a thesis on finite methods under Franco Brezzi. He finished it up in Paris, at the Laboratoire d'Analyse Numérique (now called the Laboratorie Jacques-Louis Lions), where he was encouraged to begin studying spectral methods. Professor Enrico Magenes, who dominated mathematics at Pavia, also exerted a strong influence on Quarteroni. Quarteroni observes how numerical methods have gone from obscure, rarely-used tools to common ones, although they are still not popular in some quarters of the engineering community. Quarteroni notes that low-order methods such as finite elements and higher-order methods such as spectral methods seem to be merging. Numerical methods are necessarily tied to experimental data. One of the challenges, especially in the airplane industry, is multi-objective optimization involving various fields of mathematics and physics. Quarteroni later became interested in domain decomposition, an area that became important with the rise of parallel computing but which are now important in a variety of fields, including the design of racing yachts. Quarteroni has been heavily involved in addressing various fluid dynamics problems central to the design of America's Cup boats, and his work recently helped boost the Swiss team to an unexpected victory.

He enjoys teaching both engineering and math students, which he currently does both at the Politecnico di Milano in Italy and at the Polytechnic Institute of Lausanne in Switzerland. The former, Quarteroni notes, is the best technical institute in Italy and the latter is one of the most international educational institutions in all of Europe, attracting students from across Europe and the world. Quarteroni also discusses prominent figures in Italian mathematics, including Mauro Picone, for whom the Picone Institute is named, Gaetano Fichera, and Ennio De Giorgi.

PHIL DAVIS: This is an interview conducted with Professor Alfio Quarteroni on June 22, 2004, held in the Applied Mathematics Department of Brown University. The interviewer is Phil Davis. The occasion of this interview as the conference, the International Congress dealing with numerical methods for partial differential equations in which Professor Quarteroni was a participant.

One of the things that I found remarkable about this conference, because I have been in the business for maybe sixty years or so, was the fact that when I started there were very few individuals in the whole world that were interested in research in numerical methods. There were very few books on the topic, there were practically no courses given in the various mathematical or even engineering departments, and now it’s quite different. Now we have almost 200 people attending this conference, and this is only on one particular topic of numerical methods. I wonder if you have any reaction to this change?

ALFIO QUARTERONI: Ah, that’s interesting. Well, what is perhaps the most surprising is the fact that this is a special conference on high-order methods, which are certainly not the most popular among the engineering community. And they were new thirty years ago or twenty-five years ago, after the pioneering works of some people – among them David Gottlieb and Steven Orszag – when these methods were used only in very special kind of applications, very special kind of environments, typically for achieving very high accuracy coding, very high accuracy for complex fluid dynamics, complex flow sets. Nowadays it seems that these methods have achieved a certain level of maturity in their flexibility and that people from engineering science, from physics, from technology, are more and more interested in it. I think this is a kind of general remark. Perhaps thirty years ago, or twenty, twenty-five years ago, numerical methods were used in applications to really produce qualitative behavior of phenomena. When they were used to get quantitative analysis, then this was on relatively simple partial differential equations. Today, the mission is to be able to get quantitative information also for very complex (mathematically speaking) equations: Navier-Stokes equations for complex fluids, or equations for wave propagation in heterogeneous media, or equations for semiconductor devices. When you address these kind of problems you really look for a variety, a plurality, of numerical methods which have different properties, different features, and that are in principle capable of extracting different kinds of information out of the numerical simulation.

DAVIS: When you started to describe this you mentioned that these higher-order methods are not popular in the engineering community. Could you say something about this lack of popularity?

QUARTERONI: I think that when I started on this subject....well, perhaps I should first of all make a few opening remarks about myself. I graduated from Pavia, in Italy, and I wrote my thesis in finite elements. This was in the mid 1970s. At that time finite elements were very popular in applications because of their superior capability of following some complex geometries or to cope with complex structure. I think that this has produced over the years a number of very important complex codes, which have then been made available to the industry, and I think that now they are dominating the scene. So I think I'm not wrong if I say that most codes which are available in the industrial market are based on finite elements or finite volumes, and both of these are low-order methods. Spectral methods are the more sophisticated methods which, in principle, are potentially superior, in the sense that they are capable of producing results with much more accuracy. But they have some intrinsic limitations – at least they used to have some intrinsic limitations – for instance, concerning the shape of the domains on which they could have been applied. Today, however, the gap is much thinner and people are trying to merge, essentially, two types of techniques and making them available for complex geometries and also for achieving high accuracy.

DAVIS: When I see a triangularization of a airplane surface, can I assume that we're dealing with finite-element methods there?

QUARTERONI: Very often this is the case, in the sense that, in aerodynamics in particular, very often numerical solutions of Euler equations or of compressible Navier-Stokes equations are based on other finite elements or finite volumes. However, recently, (and we've seen several examples in this conference) even people working in high-order methods, like spectral methods, are trying to adapt these approach to two- or three-order domains. So I'm pretty confident that in a few years we could see very remarkable applications of high-order methods also in regions, complex regions, partitioned in triangles or tetrahedra. You might even see it in some cases now.

DAVIS: One of the...let me go back. Could you just say one or two words, for the sake of the people that are going to listen to this tape, about the distinction between the low-order method and the high-order method. Does it have to do with the power of h, or what?

QUARTERONI: Well, again, now the border is less clear cut than it used to be. Let me again go back to perhaps the most traditional perception of this concept among the engineering community. When you talk about low-order methods you talk, for instance, of finite volumes of finite elements, and you know from the beginning that you are building a method which has an intrinsic limit in terms of the order of convergence with respect to the grid-size h. You fix the polynomial degree *a priori*, and this is a limitation in the sense that the order is at most equal to the polynomial degree. If you hope to achieve better accuracy you have to basically refine your grid. On the other hand, higher-order methods, in principle, are capable of producing accurate results if the solution is smooth enough, but without changing the formulas and without changing the construction. So potentially they have the capability of producing high-order accuracy if the smoothness of the data will permit it. So you don't have to change the code, you don't have to make any other changes in the algorithms in order to be repaid by high-order accuracy. This is the basic distinction.

DAVIS: So you don't employ adaptive procedures?

QUARTERONI: Of course you can, but this is an extra feature. And this is exactly one of the new directions of investigation: using adaptivity in a high-order method and producing what people call the hp-version of finite elements. In that case you play with the two elements – the geometrical element and the functional element (that is the polynomial order) –.

DAVIS: So in terms of error, then, there would be *a posteriori* error estimates in this, rather than *a priori*?

QUARTERONI: That's correct. You can now try to exploit information on computed solutions in order to guess what is the actual behavior of the error without already knowing the exact solution, and use this piece of information locally on every element, on every substructure, in order to decide whether or not we have to refine or to de-refine.

DAVIS: I was thinking now of those interesting pictures of the triangularization of airplane synthesis. I know that there was a dream called the numerical wind tunnel, that the wind tunnels were going to be replaced by the computer programs and so on. Are we close to this, or not yet?

QUARTERONI: Well, perhaps we are closer than we used to be, but I'm not really sure that this will be an achievable goal because, as far as I know, most numerical methods, most advanced codes available in industry, when you try to apply them to a real complex problem you need to validate your code, you need to validate your method. And this stage of validation requires information on physical quantities that you can obtain by the physical wind tunnel, for instance, or by the tank tunnel if you are solving a problem of naval engineering or yacht engineering. So in many cases if you have that information, physical information, available from experimental data through wind tunnel or through the tunnel tank, these are very useful in order to improve the quality of numerical solvers.

DAVIS: So there is considerable discussion of the relationship between what is computed and what is observed, in terms of validation?

QUARTERONI: Yes, I think this is the problem. I'm not aware of a numerical method which can produce valid results, accurate results, on a variety of complex problems without exploiting to a certain extent information which are provided by experimental measurements.

DAVIS: The computer results that come from finite element methods, are they now employed in the design of future airplanes by airplane manufacturers?

QUARTERONI: Yes, I think so. And, actually, today what is very relevant in this type of industry is integration among different components of the simulation. You have the aerodynamic part, you have the structural part, you have the part for the propulsion, and you have to integrate all these things. A key word here is multi-objective optimization.

DAVIS: Elasticity or flutter?

QUARTERONI: Yes. Along with elasticity you have acoustic wave generation to prevent noise, you have radar data, and waves, which of course are propagated and interact with the external environment. So you have different fields of physics and engineering which are playing together, and the problem is how to combine them in an optimal way. So optimization is a key word now in this type of business and the single components are themselves part of a numerical simulation.

DAVIS: What techniques or strategies are used in this sort of optimization?

QUARTERONI: They are either standard classical methods – decent methods, conjugated methods, Newton – like algorithms – and genetic algorithms, when you have to construct for a global optimum, then genetic algorithms are being used.

DAVIS: Do they use any random search methods?

QUARTERONI: Random search can be used in order to activate the procedure, say, and then you can go with a deterministic or a genetic approach.

DAVIS: Let me change the direction of our discussion now and become a little more personal. When did you first realize your interest and your talent in mathematics or applied mathematics, engineering, whatever?

QUARTERONI: Well, I started...I was a student in Italy at the University of Pavia and I had to write my diploma thesis, what we call in Italian *laurea*. I went asking to Professor [Enrico] Magenes, who is a very well known analyst, for a subject. I didn't have any specific idea in mind; I was, roughly speaking, interested in applied mathematics, but I wanted to have a suggestion from him. He advised me to ask Franco Brezzi for a thesis in finite elements. Franco Brezzi at that time, it was 1975, was a very young brilliant mathematician and he was just back from a year spent in Paris. He made a very substantial contribution to the development of the theory of finite elements. At that time he proposed and proved what is now called the Brezzi-Babuska condition, or the Brezzi condition, for saddle-point problems. So Franco was interested in studying finite elements for saddle points, like those arising from linear elasticity or from Navier-Stokes equations. So he wanted me to work on this type of subject, which was fairly new at that time. I started with finite elements and then worked a couple of years with Franco Brezzi. Then I went, myself, to Paris to work with Professor Raviart and Professor Glowinski, who at that time were among the mathematicians who made substantial contributions to the theory of finite elements.

DAVIS: At which institution was this in Paris?

QUARTERONI: This was the what is today called the Laboratorie Jacques-Louis Lions, and that time called the Laboratoire d'Analyse Numérique, Université of Paris VI. Curiously enough, they both, Glowinski and Raviart, recommended that I study spectral methods. I was absolutely unaware of this subject. But they read papers, one paper by

Heinz Kreiss [tape malfunctions or is dropped] and by [David] Gottlieb. They gave me those papers and they insisted that I consider this alternative approach. So I started working on spectral methods at that time. It was very interesting because somehow having this kind of background in finite elements and in functional analysis, I could find, together with Claudio Canuto, an easy, straightforward way, to develop approximation theory for spectral methods, moving from finite element theory.

DAVIS: But before your university experience you must have realized that you were very good in mathematics. When did this realization occur? How old were you? Was this in high school or earlier than that?

QUARTERONI: Well, I have to admit that it was a kind of erratic path because I think I was fairly good in mathematics when I was in junior high and then high school. But I was not in a *lyceum*, I was in a professional kind of school. I found that I was interested in taking studies in economy at the university and this was my idea.

DAVIS: Become an economist or a businessman.

QUARTERONI: Yes, well, I don't know. I mean I was interested in the macroeconomy at that time. This was my interest, so I got this exam at the end of the high school in Italy, that we call the maturity [Diploma di Maturita] –

DAVIS: Matura –

QUARTERONI: Matura is the name they use in other countries, like Switzerland. So at that time the professor in the committee – there was a professor in mathematics or physics, I don't know because he was coming from another place – who strongly recommended that I continue with a more scientific subject. But I had never thought really of studying mathematics at the university, so this was a kind of sudden decision that I did not meditate very seriously, and it was kind of a challenge. So I went to the University of Pavia and I was admitted in one of the colleges there, but as a student in economy at the beginning. So after a couple of months I decided to turn to mathematics.

DAVIS: A few minutes ago you mentioned functional analysis, which is a fairly abstract subject. How important to numerical analysis are some of these general theorems...I'll say functional analysis and other abstract portions of mathematics?

QUARTERONI: I find it very important, especially if you want to carry out, let's say, stability and convergence analysis for numerical methods used for partial differential equations because, basically, you're working in subspaces of Hilbert spaces. You can derive the *a priori* stability estimate from the *a priori* analysis of the differential problem. You can really exploit that information coming from the analysis of the exact problem when dealing with the stability and convergence analysis of the numerical approximation.

DAVIS: Does the theory of complex functions – of one complex variable – play a role any longer in such questions, say in aerodynamics and so forth, as it did many years ago when the problems were only two-dimensional?

QUARTERONI: Well, I simply have to judge on the basis of my experience, so I'm not pretending to give a valuable answer in the most general context. In my case, I must say "no" unless when you use Fourier methods for Navier-Stokes or similar kind of equations.

DAVIS: In 3-D?

QUARTERONI: In some cases yes, when you use potential methods or boundary integrals, or the vortex method, then you might need to use complex analysis concepts, say. There are classes of numerical methods where you really need to know and stick with complex analysis, and some where you don't really need, strictly speaking, complex analysis.

DAVIS: I'm changing the subject again, back to your experiences as a student. Who would you say was your most influential adviser or professor when you were doing your work, your studies?

QUARTERONI: Well, when I was at the university there were a couple of professors who were very influential on me. One was Professor Magenes, and the other was Professor Brezzi.

DAVIS: What were their backgrounds?

QUARTERONI: Magenes is author, among many other things, of classical books in analysis of boundary problems for partial differential equations. So his background is in analysis of PDEs and in interpolation theory of functional spaces, for instance. At that time he was very influential in Pavia. I think that we all in Pavia felt like kind of pupils of Magenes. He was so influential with us that he was really our reference point for all of us. And Franco Brezzi was one of the developers of finite element theory. I had the chance to be one of his first students, if not the very first one. So he had a lot of time to devote to me at that time, and of course he has had a lot of influence on me.

DAVIS: Do you know anything about the numerical institute that was in Rome that used to be called the Picone Institute? Does it still exist?

QUARTERONI: Yes, it does. The institute in Rome is today called (in Italian) Istituto per le Applicazioni del Calcolo, or IAC – the Institute of the Applications of Calculus.

DAVIS: That exists still –

QUARTERONI: It still exists, yes. And I think this was one of the very first institutes in applied mathematics in the world, actually – certainly one of the very first in Europe.

DAVIS: Well, [Mauro] Picone probably started it in the 1920s?

QUARTERONI: Yes, I think it was in the mid-1920s.[1]

DAVIS: In the mid 1920s.

QUARTERONI: Yes.

DAVIS: I'm interested in this because I knew a man who was associated with it by the name of Gaetano Fichera.

QUARTERONI: Yes.

DAVIS: Do you know the name?

QUARTERONI: Very famous, yes. I met him once.

DAVIS: You met him?

QUARTERONI: Yes, he's another famous character –

DAVIS: About fifty years ago in the first (maybe the zeroeth) generation of digital computers, I did some work, very simple things, potential theory. I did this in Washington where I was working. Fichera liked this, and so he translated this into one of the publications of the Picone Institute, and I wanted to show you –

QUARTERONI: Ah great, historical kind of Calcolo. Yes, oh great.

DAVIS: And it has an introduction by Picone and so on –

QUARTERONI: Ah, it's piece of history then.

DAVIS: Yes, that's history, right. Very simple material in there, very basic –

QUARTERONI: Very impressive –

DAVIS: But in those days it was quite something to do, you know –

QUARTERONI: You know, of course, that Picone was a very brilliant person, as a pure mathematician, but he had a place for applied mathematics. When he was in the army in Italy he prepared the tables for shooting, you know for artillery, so the tables that are used for the artillery were prepared by

DAVIS: Prepared by Picone?

1 It was founded in 1927. Source: http://www.rm.iac.cnr.it/indexroma.htm, accessed 3-6-2006.

QUARTERONI: By Picone.

DAVIS: This was in World War One, probably? It must be.

QUARTERONI: Yes, perhaps yes. I guess so.

DAVIS: What did Fichera do? Fichera was here at Brown for –

QUARTERONI: Fichera passed away some years ago, I think about ten years ago.[2] Fichera is another very great name among the Italian mathematicians.

DAVIS: He did a lot of work with this kind of material.

QUARTERONI: Yes. He produced –

DAVIS: Linear stuff –

QUARTERONI: Yes, he produced one of the results for computing the eigenvalues of linear operators, among other things. That's been a branch of – well, some people in the Picone Institute are working on this subject. There are still people at the University of Rome who are working on the computational spectrum –

DAVIS: Computational spectrum levels. So you are now in Lausanne?

QUARTERONI: I'm actually part in Lausanne in Switzerland, and part in Milano – the Polytechnic of Milan [Politecnico di Milano]. So my typical week is a couple of days, Monday to Tuesday, in Milan and then I move to Lausanne where I have another group at the Polytechnic of Lausanne [Ecole Polytechnique Fédérale de Lausanne]. So I'm playing in these two different places, two different technical schools.

DAVIS: Can you drive from one place to the other?

QUARTERONI: Yes, I can. I often go by train, but I can drive; it's four hours. You have to cross the Alps – you've got the Alps in between.

DAVIS: Is there a tunnel there?

QUARTERONI: There's a tunnel, yes. You have to go 1700 meters, then you have this tunnel which is seven kilometers long, and then you are in Switzerland. Then you go downhill.

DAVIS: Over the years I assume you've done quite a bit of teaching.
QUARTERONI: Yes, I'm still teaching. I teach four courses per week, actually: two in Milan and two in Lausanne. I teach basic numerical analysis to engineering students. And

2 Fichera died in 1996.

I teach more advanced courses in finite element theory or in numerical approximation of partial differential equations, and some more advanced courses on modeling, for instance, problems in medicine, in particular modeling cardiovascular flows, which is one of my major subjects.

DAVIS: Does the teaching experience in any way feed back into your research?

QUARTERONI: Yes, absolutely.

DAVIS: How does it do that?

QUARTERONI: Well, I think in a couple of ways. The most obvious one is that when you teach high-level courses, you have the chance of meeting very bright students –

DAVIS: You have some brilliant students, I would guess.

QUARTERONI: Yes, absolutely. There are absolutely brilliant students in Milan, and there are some brilliant students in Switzerland as well. There's a very strong selection in Milano, the Polytechnic school in Milan, which is the most important technical university in Italy. So you really have the possibility of having first-rank students and then to have people who are capable of producing nice mathematics in an almost autonomous way. And the second point is that when you teach advanced courses on the subject which are close to your research interest you know that, depending upon the reaction of the students, you can understand if you have been taking the simplest way or the most complicated way. And you can get hints on how to improve your approach to –

DAVIS: You mean they're prepared to criticize the course?

QUARTERONI: Yes, absolutely.

DAVIS: But of course you're directing theses also. I mean, presumably...

QUARTERONI: Yes, yes. I now have eight students, eight Ph.D. students, six in Lausanne and a couple in Milan. So, certainly a good group of students.

DAVIS: What is the language in Lausanne? Is that French?

QUARTERONI: It's French, yes. It is French for the undergraduate courses, and then you may use English for the graduate courses.

DAVIS: English in the graduate courses. That's interesting.

QUARTERONI: Yes. Because today Lausanne, the Polytechnic Institute of Lausanne, is perhaps the most international school, university school I mean, in Europe. This might seem quite normal by American standards, but it is very unusual by European standards. We now have almost forty percent of undergraduate students who are not from

Switzerland, and two-thirds in the graduate courses which are not Swiss. And by European standards this is quite an exception.

DAVIS: Well this is a phenomenon here. You may have observed it with our students – that most, maybe half of our graduate students, are non-native born students. They're coming from, mostly from the Orient, but it seems to change from decade to decade. Some while back a lot of the students from India, the students from India don't seem to come now. We have lots from the Orient, from Japan, from China, that part of the world, also from the Middle East. This worries some observers in America about this lack of interest in going into fundamental science. The reason that they often give is that to go into science, to go into mathematics, is difficult and there is not much money in it, compared to other professions. What is the feeling in Italy or in Switzerland?

QUARTERONI: I think that it is quite diverse, in the sense that if...take Switzerland, for instance, which is for some reason, let's say it is not in the average. If you get a Swiss student in mathematics after his bachelor, his diploma – so after four years and half of studies – he can easily get a good job in an industrial environment and be paid twice what he would be paid if he continued with the Ph.D. program. Now, let's say in Swiss francs, we're talking about an average of eighty thousand per year, eighty thousand Swiss francs per year, in industry, and roughly thirty-six to forty thousand Swiss francs in the university if you are a Ph.D. Student. Now we have to divide by 1.5 to get dollars, roughly speaking. And the problem is that the industry is not really prepared in Switzerland, in Europe generally speaking, to then put into value someone with a Ph.D. degree. So if you decide to stay in university for a Ph.D. degree then you really have to be extremely motivated. It's not a matter of economical convenience, at least in mathematics or in engineering, because there are very few industries in Europe that are capable of giving any value to Ph.D. students. And this is good and bad. It's bad because, of course, the number of students who want to go on with the Ph.D. program is relatively low. It's good because those who decide to proceed are very motivated, so they are not driven by economical reasons, and this is why you can count on very good students.

DAVIS: Talking about the two-thirds of the twenty percent and so on. Do these young students have any problems in Switzerland with respect to immigration, visas and so on?

QUARTERONI: Well, first of all, most of these people come from European countries. There are people coming from the far east, and northern Africa, but these are very very few. Most of the immigration is from European countries. Now as you know Switzerland is not yet in the European Union (EU) –

DAVIS: It is in the EU yet?

QUARTERONI: It is not in the EU. I mean, today it is not in the EU. So you have control by immigration, but this is not troublesome, in the sense that while you are a student, you're officially just a student, it's no problem at all. And I would say that, in general, the environmental conditions are very, very good in Switzerland. So people come and they really enjoy staying. I don't think this is creating any social tensions or problems here.

Now I was saying before that this high percentage of foreign students that we have in Lausanne is quite normal for US standards, but is not normal at all in Europe. For instance if you take Italy, Spain or Greece, I think that the number of foreign students is extremely low. The Politecnico di Milano, as I've said before, is certainly the best technical school in Italy. Yet, the number of foreign students is really below one per cent.

DAVIS: That's negligible.

QUARTERONI: Absolutely negligible.

DAVIS: How do you explain the popularity of Switzerland?

QUARTERONI: Well, for different reasons I think. First of all it's the matter of language. I mean learning French for a foreign student is more attractive than learning Italian. Then you are better paid, and you have courses offered in different languages. And the universities are in general much better equipped. So the overall conditions are more favorable, I would say, because Switzerland is a rich country and because of the fact that Swiss people know that these people coming – the Swiss population is very low, it is about seven million people. They know that they have to attract people from the outside because this is bringing know-how, technological advancement, and so on and so forth. So the general atmosphere is very positive, compared to the whole of Europe, in this respect.

DAVIS: Let me give you a story from fifty years ago. Fifty years ago I was working at the National Bureau of Standards, which was in Washington. It has now changed name to something else. This was about five, five six seven, years after the end of the war. It was in perhaps the first generation of digital computers, and maybe the zeroeth generation of digital computers, and there was an attempt to get together a group of people to examine the potentialities of the computer vis-á-vis scientific computation, to look into all kinds of methods. They assembled a group from the United States and from Europe, from England and so on. There was so many Swiss – this is the point of my thought – there were so many Swiss that were working with me, alongside there, that we made the joke that it was time to raise up the Swiss flag over the building in which we were in. And I'll tell you some of the names, first of all there was Henrici. Peter Henrici was there and I shared an office with him for a while, a very difficult man. And then there was Ostrowski, Alexander Ostrowski, from an earlier generation. And he changed over and did work on matrix theory. And there was Gautschi, Walter Gautschi, and [Eduard] Stiefel was there –

QUARTERONI: Yes, that's an impressive group.

DAVIS: Olga Taussky was there working on eigenvalues, and so on. She was from Austria, but anyway. I think things must have changed by now if you have these two-thirds of twenty percent, or whatever –

QUARTERONI: What they have kept is the attitude of – I'm talking about the Swiss students now – to leave after their Ph.D. because they know that if they want a good chance to come back to the country and to be employed as professors in the university they need a sort of baptism, it's a kind of moral outlook, they have to have had a substantial experience abroad. So it’s very normal for Swiss students who got their Ph.D. in Switzerland, then to find them as postdocs or assistant professors in other countries.

DAVIS: Let’s get back to some personal aspects again. Looking over the work that you have done what do you consider some of your main contributions to numerical analysis?

QUARTERONI: That’s difficult to answer this personal question because, to give an objective –

DAVIS: Kind of puts you on the spot. No, give me a subject heading.

QUARTERONI: Okay. Perhaps I should mention those subjects which have interested me more than other things. One that I mentioned before is the analysis of spectral methods. This was not my first subject because I started in finite elements with saddle-point problems, but this was perhaps the first subject that intrigued me a lot because there was a lot of room for finding new results in this emerging subject. I was lucky to get there at the right time, work with very good colleagues and coauthors, and set somehow some of the basic results for approximation theory for spectral methods. So this is one. The second thing that I was a very interested in, and impassioned me a lot, was domain decomposition for partial differential equations. I started in the mid-1980s, and the interest at that time was to try to extend spectral methods to complex domains, but then the theory was general enough and you can apply it to any kind of numerical approximation, finite elements or finite volumes, say. So domain decomposition is a technique which consists of taking the computational domain and subdividing it into sub-domains. Consequently, you have to split your original problem into subproblems which pertain to different sub-domains in order to solve them independently on independent computer processors. But since the physical solution is unique and global, you cannot split it on the sub-domains. So you have to conceive methods which are capable of first of all splitting and then patching the different contributions in a natural way through suitably devised interface conditions.

DAVIS: Have you worked actually on the geometry, the codes for the geometry of this or no?

QUARTERONI: I worked on the partitioning for the domain geometry – it was more into simple shapes, simple geometries. And then splitting partial differential equations into these different sub-domains. This was driven by the need of parallel computation problem. You wanted to take advantage of parallel computing by setting up a scalable algorithms, algorithms which are capable of taking the greatest advantage of parallel structure.

DAVIS: The programming for parallelism is rather difficult isn’t it?

QUARTERONI: Yes, yes. Because, of course, first of all you have to somehow parallelize your differential problem, your original problem, splitting it conceptually. And you need a new way of considering here that problem mathematically speaking. And then you have to set up algorithms which are capable of exploiting the specific characteristics of the computer on which you are going to work. But moving from that, it was clear to me since the beginning, that this was a potential tool for treating multi-physical problems – problems where when you have different kind of problems in different portions of the computational domain, or you have a different behavior of the solution in different portions of the domain. For instance, you have fluids in one part and solids in another part, then the domain decomposition method provides a natural environment to separate the problems, solving independently the two pieces of the physics. And then you need to gather the solutions in a coherent manner. You find these type of problems in many kinds of applications. When you consider multiple fluids, and I've been going through this type of application recently while working for the America's Cup to try to solve the fluid dynamics around a yacht where you have the hydrodynamics part, the aerodynamics part, and the free surface which is separating the air from the water. Or I've been using it extensively in the modeling of cardiovascular flows where you have blood flow in arteries and then you have the arterial walls which are compliant, which deform under the action of the blood flow. In that case you need to separate the computation in the fluid part, where you use Navier-Stokes equations, say, and the solid part (the arterial wall) where you use models for solid mechanics. So this type of concept is nowadays called heterogeneous domain decomposition, as opposed to homogeneous where you have the same kind of continuum media everywhere in the computational domain. So heterogeneous domain decomposition is, I think, a very interesting method which allows you to work with multi-physical problems. This is perhaps one of the subjects that I've been working on more and more extensively.

DAVIS: Let me go back to the America's Cup. Of course, we lost it a few years ago. It used to be that almost every year America won it. The boats were built here you know.

QUARTERONI: In Providence?

DAVIS: Not in Providence, but in Bristol, which is about twenty miles down the Bay.

QUARTERONI: I see. This I didn't know.

DAVIS: We watched it being built. The boats were famous. I don't know whether it has to do with the boat design or the crew or whatever, but maybe you should be a consultant to the design. The manufacture was in the hands of one family for years, called the Herreshoffs, and the family –

End of Tape 1, Side A – Start of Tape 1, Side B

DAVIS: I would like to get into it, but we don't have time, actually, so I will have to be selective. I would like to get your opinions on certain questions that you may or may not have thought about, which are rather removed possibly from the work that you do, actually. I noticed that you had put out a book called *Scientific Computing with MATLAB*. I'm very fond of MATLAB myself, I love it. I know the man that invented it, before he invented it, and so on. In the little bit of computing that I still do, I like to use it. One of the things that have occurred to people – there was a man in our department who was in PDEs, who is no longer here, and he raised this question. He said, since we are doing numerical methods, what is the utility of special functions any longer? Maybe special functions have no utility. At the same time, you know there's a big book on special functions that appeared about fifty years ago, in which I have a chapter, and so on. This is currently being updated and more special functions have been put in, and so on. I had no role in this, but I know about it. And the idea of the utility of special functions in today's computational environment is an interesting one. What is the role that they play?

QUARTERONI: I'm not sure I have the right answer. Certainly, there are still methods, very specific methods, which make use of special functions and need special functions, and –

DAVIS: You mean things like Mathieu functions, and that kind of thing?

QUARTERONI: For instance, yes. Or let's say methods which try to reproduce in a ad-hoc manner – a semi-analytical solution of equation or part of the equation is exploited. When you want to get your solution, a numerical solution contains a lot of information whenever it is possible.

DAVIS: This is a most explicit solution –

QUARTERONI: It is a most explicit solution. In those cases, of course, you want to get a special function. I think that this is perhaps not the most obvious approach followed today by engineers, but in some communities, of physicists, for instance, where what is dominant for them is the specific problem, and they know a lot about the specific problem, then they might have a lot of interest in approximating the whole mathematical solution or part of its components by special functions. It is somehow different than the approach that is followed in industrial environment, where you are looking for a method which is robust enough and flexible enough to treat all the possible kinds of applications. However, the trade off is that if you have a generic method which is capable of solving all possible problems, this generic method must be stabilized enough in order to not face difficulties when solving unexpected problems. On the other side, if you are really focusing on a specific problem, a specific application, your interest is not to use a generic method but to exploit at the best the basic information you may have on a problem. In those cases you may want to use a expansion in special functions that are specifically tailored on the application at hand.

DAVIS: Well, I think that's a good, so they're not going to go away so fast.

QUARTERONI: Ha, I don't think so. I don't think so because – you see this is going back to your first question on high order methods versus low order methods. Personally, I have to say that when I decided in the mid-1980s to go back to finite elements after having studied for five years only spectral methods, it was because I was convinced at the time that if you really wanted to face applications, you had to go to robust low order methods. Now since the cost of computers is decreasing dramatically, you can afford to solve problems with a higher order of accuracy, and in that case you get much better information. Let me just make an example. You were talking about the America's Cup before – but that's just an example – in America's Cup you wanted to create ingredients to simulate the shape of the free surface, because if you know the shape of the free surface with a good level of accuracy then you can predict much better the wave resistance and the viscous resistance, which are important components of the resistance to the motion of the yacht. You really have to strive to get the solution, the form of the free surface, up to at least one percent of accuracy. You do not want to leave anything out, which is less than one percent of accuracy, because we're really striving to gain very small digits of precision. In that case, you know that first order methods will never be able to produce this kind of result because they are too dissipative. Then you have to increase the order of the method. Today, you cannot avoid using second order method for this type of computation, but tomorrow you will need more advanced methods. So it is not a matter of computational cost. If you have to solve the problem, and solving the problem means solving it with a certain level of accuracy, you have to be prepared to use an expensive method, which is potentially more accurate, even though this requires more computation, more computer time.

DAVIS: This is interesting. It suggests that in the future it becomes a competition between my algorithm and the Australian algorithm. [Laughter] I don't know where the Cup right now. Is it in Australia or some other place?

QUARTERONI: You know, the last winner was the Swiss team.

DAVIS: It's in Switzerland?

QUARTERONI: The winner was from Switzerland, yes, which is fun –

DAVIS: Right on the ocean.

QUARTERONI: Switzerland has only lakes. So the yacht that won is now on the lake.

DAVIS: On the lake. Well, are you working for the Swiss yachtsmen, and maybe that's why they're winning?

QUARTERONI: What I can tell for sure is that when we started working on this problem for the Swiss team, it was clear that there were several components playing together. Of course, there's the skipper, there is a crew which is really important, the sailors, and then there is the technology and the design of the boat. You want to optimize the shape of the boat, you want to optimize the shape of the different components of the boat: the hull, you have the keel, you have the bow, you have the winglets, and you have the mast and the sail. And you have a multiple regime during the regatta: you have the downwind leg and the upwind leg, and the aeronautical, hydrodynamical behavior of the boat in these two regimes is totally different. And the boat is the same – you cannot change the boat. So you have to strive for a form which is capable of optimizing the performance in both regimes. This is why there is so much numerical simulation and all sorts of experimental results which are needed in order to adopt shapes which perform better than –

DAVIS: I understand that the competitive boats now have computers on board.

QUARTERONI: Yes.

DAVIS: Does this come in with what we're talking about? Would this come in with the strategy –

QUARTERONI: Not really. What I was talking about was the phase prior to the regatta, the phase with which you have to end up with a design, you have to end up with a manufacture –

DAVIS: You are talking about the design stage.

QUARTERONI: Yes, this is for the design stage.

DAVIS: But during the race –

QUARTERONI: During the race you are not allowed to communicate with the external environment. But you have a computer on board which helps – there are sensors on the mast and on the sails, and these sensors are transmitting information to the computer. This computer, according to the information it gets, is capable of recomputing, in real time, the state of the deformation of the mast and sails. You know that these are very delicate ingredients. You cannot submit the mast, for instance, to extra stresses because it's a very fragile element. So this computer is capable of reproducing the geometrical shape, configuration, in real time, of some of the critical structural components of the boat, and then of suggesting to the skipper or navigator, or to a technician, say, how to maneuver in order not to pass some critical threshold.

DAVIS: This is now allowed?

QUARTERONI: This is allowed. This is absolutely allowed.

DAVIS: So this is rather sophisticated programming, I suppose?

QUARTERONI: Yes. And there are other things which are quite sophisticated, for instance the meteorological information. You know, it is really important to predict the state of the wind for the next couple of hours during the race, and you are not allowed to communicate with other people during the race. So typically what the syndicates do is to place boats in the harbor, in the space where the race would be with the boats on the sea, for hours beforehand, before the race. They stay and they get data, they elaborate data until the very end, let's say ten minutes before the start of the race. Now in these ten minutes, they have to elaborate all these data in order to produce predictions. Since solving the complete Navier-Stokes equations would be too expensive, they need to use cheaper methods in order to predict on the ground of very many data and without spending too much time in computation. This has called for new mathematical models, new algorithms, which are capable of producing theoretically good results in a much shorter time than what would be needed to solve complete meteorological equations.

DAVIS: That's very interesting, how a rich man's amusement has generated sophisticated numerical methods. I just want to take up more topic very briefly, and then we can put an end to this conversation, which has been very instructive to me, enjoyable, and I hope also to you.

QUARTERONI: Sure, a lot.

DAVIS: When I think of Italian mathematics I often think of Italian mathematics the way it was a hundred years ago. In that case, the Italian mathematicians were mostly interested in geometry, in differential geometry and a variety of obliques and so on. In those days there were national flavors to mathematics: the Germans have a certain mathematical flavor, the English have a mathematical one, the French have one, and so on, and the Russians have their own mathematical flavor. Do you think in this day of globalization, that all of these flavors are merging into a sort of unflavor? [Laughter]

QUARTERONI: Ah, tough questions. Well to tell you about the Italians. There is a school, I would say...you see I come mostly from the analysis, and what in this field is certainly visible is the school of [Ennio] De Giorgi on the calculus of variations that originated in Pisa from the studies of many mathematicians, but perhaps the most prominent mathematician of the past century in Italy was Ennio De Giorgi, who passed away some years ago.[3] This school of calculus of variations is probably one of the most visible fields of mathematics which are today cultivated in Italy. These techniques are now developed under different frames by pure and applied researchers in different countries.That's a good thing, after all, because we are not going to be over-homogenized in the process of the globalization of culture; we have to keep somehow our specificity. Everyone is injecting in this theory a specific flavor. I find this, actually, interesting and valuable.

DAVIS: Well, thank you very much.

QUARTERONI: Thank you very much. This has been a great pleasure for me.

3 He died in 1996.