## An interview with BERTIL GUSTAFSSON

Conducted by Philip Davis on 21 August, 2003, at the Department of Applied Mathematics, Brown University

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## ABSTRACT:

Gustafsson discusses his career in numerical analysis and applied mathematics in both Sweden and the United States with interviewer Phil Davis. Gustafsson's interest in mathematics began early. As six year-old boy living in rural Sweden, he discovered his talent in mathematics thanks to a perceptive schoolteacher who lived with Gustafsson's family. While lucky enough to have several good teachers during his schooling, Gustafsson believes that being a good mathematician requires talent as well. Although undoubtedly talented, however, Gustafsson believes his success in numerical analysis is mostly attributable to hard work and a little luck.

Gustafsson received his Ph.D. from Uppsala, where there was a tradition of prominent mathematicians, including Arne Beurling and Lennart Carleson. When Heinz-Otto Kreiss arrived at Uppsala in the early 1960s, Gustafsson was in his second year, had already developed a strong preference for applied mathematics, and so quickly took to Kreiss, his work with numerical analysis, and the new field of computing. His dissertation was on difference methods for hyperbolic equations, and Gustafsson has continued to pursue topics in the field of differential equations for most of his subsequent career. Indeed, Gustafsson considers himself lucky to have hit on a hot topic area with his initial research, and much of his early work consisted of applying his theoretical developments to practical problems. These included weather prediction, both in Sweden and at the National Center for Atmospheric Research in Boulder, Colorado and in a variety of problems in various fields during a one-year postdoc as an itinerant consultant searching for useful applications of differential equations in Swedish industry. Currently, Gustafsson is working with David Gottlieb on a variety of problems, which until recently included work in computer tomography.

Gustafsson, who often works with engineers, notes a certain tension between this group and mathematicians when the two work together, in which neither fully understands the challenges of the other's job. Part of the problem in working with industry is the question of proprietary versus open knowledge, and Gustafsson has encountered this issue often with his work at the Parallel and Scientific Computing Institute in Uppsala. Gustafsson believes that teaching while researching helps provide feedback for one's research, and so can be quite valuable. He suggests that incoming mathematics Ph.D. Students need a solid background in general mathematics, and notes that his program in Sweden requires students to have some experience in working with computers and to have taken a few courses in numerical analysis. PHIL DAVIS: This is an interview with Professor Bertil Gustafsson of Uppsala, Sweden, held on August 21, 2003, at the Department of Applied Mathematics at Brown in Providence, Rhode Island. This interview is made in connection with a conference just held here on Numerical Methods and Differential Equations supported by the Air Force Office of Scientific Research. This is a conference at which Professor Gustafsson was one of the principal speakers. I think I would like to start with a little bit of personal background. Can you remember when you, as a young person, first got interested in mathematics and recognized that you had talent in that direction?

BERTIL GUSTAFSSON: That was actually quite early. I remember that quite well, because it was when I was about six years old. In Sweden you start school at seven, and I happened to live in the same house as the teacher. I was living in the countryside in Sweden. It was a very small school and the teacher saw I was able to do addition and subtraction and things like that at that age and he was amazed that I could do this at this early age. So he gave me one of the books – you know in primary school you have these books that have one plus two equals question mark, and so on. He gave me that and I completed it before I started school, as I said we start at seven. So when I finally started school he gave me the book for the third grade. So that's when I guess I understood that I had some talent for mathematics.

DAVIS: This must have been a good teacher to recognize this and to encourage you and so on. So you were engaged in mathematics at a very early age. Where did you do your high school work?

GUSTAFSSON: High school, junior high, again I was born in the countryside and there was no high school in the neighborhood so I had to actually leave my home at twelve years of age to go to the nearest town where they had a high school. I lived with another family there because I couldn't commute at that time. But then, actually, it was a more general feeling that I was doing quite well in school, not only in math, but in languages and things like that. I didn't have any particular thing going in mathematics at that time. Then in senior high school, which was in another still bigger town, Norrkőping in Sweden, which is one of the largest cities, of course I went more into mathematics. But I actually became interested in physics then, in senior high school. And that's when my plans started forming in my head about going to university, Uppsala in this case, to study physics. I was fascinated by several things in physics at the time. I had a very good physics teacher there. This, I think, what we have said already now, points to this fact which believe in: that the teachers are very important to getting people interested in, if it's a research career or whatever, but to take care of the talent. It's very important that the teachers are good and know their topic.

DAVIS: Speaking of talent, do you have any feelings about the balance between mathematical talent as a genetic thing or as a thing that depends on environment and culture, teachers, family, etc.

GUSTAFSSON: Well I don't know. I've already told you that before, I mean, that I had some talent in mathematics – that has to be genetic. I mean, I was not in an environment up to six years age. [laughter] But later, certainly, I had a lot of influence from good teachers, and at a later stage from my teacher at the academic level, Heinz-Otto Kreiss. And the environment in general, if that is good, the environment sort of notices that you have this interest and encourages it. I think that's important as well, but of course it has to do with genetics from the beginning. That I believe.

DAVIS: So you got your doctor's degree from Uppsala?

GUSTAFSSON: Yes.

DAVIS: Who was your advisor in this work?

GUSTAFSSON: Heinz-Otto Kreiss.

DAVIS: Heinz-Otto Kreiss. Who were some of the other mathematical professors at Uppsala when you were there?

GUSTAFSSON: Uppsala had a math department that was essentially pure mathematics, only pure mathematics. We had the famous Lennart Carleson, who was a young professor at the time. I came to Uppsala in 1961, and by then he was already recognized but still quite young.

DAVIS: I met Carleson many years ago. In 1950 the International Congress of Mathematicians took place in Cambridge, Massachusetts. He was there and he already had a big reputation because he had done some...I forget what it was he had solved, I think it had to with the convergence of Fourier expansions. That was the thing – he solved that problem, but I never followed his career after that. He struck me at that time – of course, he was very young but still older than I was since I just had my Ph.D. degree at that time – he struck me as a bit eccentric. I don't know whether this was the case or not, but that was my feeling at the time. Who were some of the other mathematicians that were around?

GUSTAFSSON: He was a big star in Uppsala. The one who had retired when I came was [Trygve] Nagell. He was in algebra, number theory, and he was quite famous. And before him was Beurling –

DAVIS: Arne Beurling, oh yes.

GUSTAFSSON: Yes, but they had left already when I came. But there was this sort of famous...this atmosphere around the department at that time.

DAVIS: I met Beurling because I was a graduate student at Harvard after World War II, and he was there. He came there to work with Ahlfors, Lars Ahlfors. I think Ahlfors was a Finn, but Beurling was from Sweden, and he had a tremendous reputation in harmonic function theory and so on and so on. A formidable character as I remember him. I suppose in those years there were no courses in numerical analysis, is that correct?

GUSTAFSSON: Right, yes. And that's a very interesting point because I would never have gone on with a career in pure mathematics, I don't think I would have been able to do it either – it seemed to me to be too difficult, simply stated. But in my second year in Uppsala, the computers, people started talking about computers. I hadn't seen one, but here came this young person, Heinz-Otto Kreiss, who had immigrated from Germany to Stockholm, and Carleson picked him up to give a course in numerical analysis in Uppsala. This was in 1962 or 1963, I don't remember. So that's how it started. I was very interested in this. It sounded fascinating to me to do mathematics that could be applied, put on computers so that you really could solve problems. There was some fascination around computers at the time, so I just became very interested in this.

DAVIS: What was the subject of your doctoral thesis?

GUSTAFSSON: It was difference methods for hyperbolic equations.

DAVIS: So you were already into computation?

GUSTAFSSON: Yes, of course. But this was theoretical numerical analysis for sure that I did, and we did very little computation.

DAVIS: Do you consider yourself primarily a numerical analyst or a wider designation?

GUSTAFSSON: No, I consider myself as a numerical analyst, yes.

DAVIS: I've observed that you can divide numerical analysts into two rough categories: those that you might say are "theoramatic" or theoretical and those that are algorithmic. Now ,of course one has to be a little of both, but somebody like [Lawrence] Shampine for example, is more of an algorithmic numerical analyst. Where would you place yourself in this division?

GUSTAFSSON: Well, in the beginning, for sure, in theoretical numerical analysis. That was Kreiss's topic and I went in his footsteps, so to say. My thesis was about stability theory for initial boundary value problems – numerical approximations of initial boundary volume problems. Those were very theoretical papers in that thesis. So I

certainly started out being theoretical, even if it was very...I mean it was, of course, applied math – numerical analysis, as such, is classified I guess as applied mathematics.

DAVIS: Yes. So your Ph.D. work simply fed right into your later work?

GUSTAFSSON: Yes, it did.

DAVIS: You started out in essentially differential equations and it's something that you have pursued for many years.

GUSTAFSSON: Yes. I stayed in the partial differential equation area over my career, yes. And this thing about stability theory for initial boundary value problems is a little bit of a difficult area that not too many people go into. For several years I applied this theory to various problems. It was more of an application of the theory, but not many people could do this at the time. That was why I was somewhat successful in my career, because there was a need for this type of theorem, the application of this –

DAVIS: This is where you got close to bottom line computation, practical stuff on physics or engineering?

GUSTAFSSON: Yes.

DAVIS: What kind of applications did you get interested in?

GUSTAFSSON: The first one I became interested in was in metrology, actually, or weather prediction. I actually went to NCAR, National Center for Atmospheric Research, in Boulder, Colorado, before I had finished my Ph.D. by contacts that Kreiss had, because they were very interested at the time in how to pose boundary conditions for these fluid dynamics equations that they use for weather prediction. That was not very well known at the time. In particular, if you cut out a piece of the globe and you want to have a limited forecast for that limited area, how do you do the boundary conditions around there? How do you keep the computations stable? That was the first application that I became interested in. So I wrote a paper or two on that when I was there, for a year, before going back to Sweden again.

DAVIS: As I remember there was considerable emphasis on weather prediction in Scandinavia. There were some famous people in Norway, I believe –

GUSTAFSSON: Yes. Norway and Sweden.

DAVIS: Norway and Sweden both were in this field. You have obviously...you spoke of coming to the [United] States and working in Boulder, Colorado, and so on. You probably worked in different countries in your career. Is that correct?

GUSTAFSSON: Well, mostly the United States, except for Sweden, of course. I spent about five or six years, but just one year at a time, in this country. I was at this research institute, ICASE, on the East coast, that was sponsored by  $NASA^1 -$ 

DAVIS: Down in Hampton, Virginia -

GUSTAFSSON: Yes, right. And then I spent -

DAVIS: Did you meet David Gottlieb there?

GUSTAFSSON: That's where I met him, yes.

DAVIS: You met Gottlieb at ICASE.

GUSTAFSSON: Yes. That's a very good place to meet people, actually, at that time.

DAVIS: Was Milt Rose the boss at that time?

GUSTAFSSON: No, this was before his time.

DAVIS: Before Milt Rose, who was -

GUSTAFSSON: It was Jim Ortega.

DAVIS: Jim Ortega. He was a PDE fellow.

GUSTAFSSON: He was an administrator, essentially, at the time. But he did a very good job to establish this ICASE institute.

DAVIS: You know that my first professional job was at Hampton, Virginia -

GUSTAFSSON: No, I didn't know that.

DAVIS: Yes. It wasn't called NASA in those days, it was called NACA, because space hadn't been invented yet.<sup>2</sup> This was during World War II, and I just had a bachelor's degree in mathematics. This was the first time that I came across computation. We were still doing some computation with slide rules and with some of the electric machines, adding machines. The fancy ones did multiplying, but you had to shift the thing over every time. [laughter] This was the first time I came across – we didn't call it that, but – the first time I came across the Fast Fourier Transform, because this was really invented by [Glenn] Bergland. Bergland made some stencils for twelve-point, twenty-four-point,

<sup>&</sup>lt;sup>1</sup> ICASE stood for Institute for Computer Applications in Science and Engineering. It closed on December 31, 2002.

<sup>&</sup>lt;sup>2</sup> NACA stood for National Advisory Committee for Aeronautics and existed from 1915 to 1958.

thirty-six-point discrete Fourier analysis. They were blueprinted in big sheets like this that went from here to here to here to here to here to here. And then – what a trick – in order to do it backwards you just followed the thing backwards, you see, because the inverse is the transform. Anyway, this is early history of numerical analysis.

GUSTAFSSON: Well, you asked about other countries, I will finish that. After that, I spent most of the time in this country, first at Caltech, and then at Stanford has been the place where I spent the most time.

DAVIS: Stanford. Who were you associated with at Caltech?

GUSTAFSSON: Well, that was again – Kreiss was there. He had moved from Sweden to Caltech. He was there, so that was the reason –

DAVIS: Do you know Jack Todd?

GUSTAFSSON: Yes, I know him. But he was, I think...wasn't he already retired at that time?

DAVIS: Was he retired at that time?

GUSTAFSSON: I don't remember that he was there, but Herb Keller -

DAVIS: Herb Keller was there. And Stanford, who was -

GUSTAFSSON: At Stanford I worked with Joe Oliger, at the time. He was also a student of Kreiss.

DAVIS: Gene -

GUSTAFSSON: Gene Golub. But he is in linear algebra, so I've spent a lot of time with him but we never did a paper together or anything like that.

DAVIS: Have you done consulting work for industry? I noticed some connection to Chalmers.

GUSTAFSSON: No, I've not really done any consulting work, but I've been in touch with industry quite a lot in Sweden. This may be a story that will interest you. Quite soon after I had my Ph.D. we got an external grant in Uppsala in Sweden, Kreiss had it. The idea was that I should travel around in the country – in Sweden, that is – and find out what type of problems did industry have that were of interest to them in the PDE area. So I was essentially traveling around looking for differential equations in industry, which lasted for almost a year. I wrote a report on that which became much read, actually, and that created a lot of contacts for me that have been important throughout my career.

DAVIS: Did you find, what shall I say, a certain tension between engineers and mathematicians and numerical analysts?

GUSTAFSSON: Certainly.

DAVIS: Can they talk to one another?

GUSTAFSSON: Yes they can. But you have – or you had, at the time, at least – to approach engineers, certainly, in a little bit of a careful way, because they don't trust mathematicians – in the sense that they think that mathematicians are good, of course, but they don't attack the right type of problems. On the other hand, mathematicians tend to think that engineers in industry are just applying well known theory of no interest to mathematicians. They think that's trivial.

DAVIS: Well, some people say the engineer simply is a person who takes some software off the shelf and uses it as he can in the problem. What about what comes out of the computer, the answer? What is the relation between that and what the engineer is interested in?

GUSTAFSSON: That is true, of course, even more now when you have all this software around.

DAVIS: Right. They just use software. Does the software – now, you spoke about this in your talk – does the software tell you how close you are to the theoretical solution?

GUSTAFSSON: I mean, there is a lot of work being done in that area now. Standards, of course, you don't know, that's for sure. But that's one direction of numerical analysis, I think, to try to find out, to make sure that your answers are right. But I think it is certainly not in the commercial software yet.

DAVIS: From an engineering point of view there's another question or ambiguity as to the way that the mathematical model represents what's going on in nature. Now you spoke of this just a few minutes ago, that your job was to find the boundary conditions in the weather when you take a limited portion of the globe. So then you have a combined problem of how good the math model is in reality and how good the computer answer is to the math.

GUSTAFSSON: Yes, exactly.

DAVIS: So many pictures are around in technical journals that come from very complicated computations, beautiful pictures. Do you have any idea as to what is the reality of these pictures.

GUSTAFSSON: Yes, I'm sure there are many of those pictures that do not show the correct results at all. They show nice pictures, but who knows if it's right or not. Many of those are...I mean, we've seen examples, I guess, even at this conference, about such things: the model is really not right from the beginning. And, of course, when it comes to weather prediction everybody knows that predictions are not right. [Laughter]

DAVIS: Well, in television, of course, they have the weather programs and they show the radar. They show where it's raining, and then you know that the weather goes in this direction and the rain will come here in a day or so. I'd like to know the extent to which they are running models or whether they simply look at the radar picture.

GUSTAFSSON: Of course, they are running models, yes, but in the end they use a lot of knowledge that is based on experience, as well.

DAVIS: Speaking of commercial software, there's a feeling among mathematicians, it's been around for centuries, that mathematics ought to be quite open and available to everybody. Of course, this was not exactly true in the sixteenth century when people like [Girolamo] Cardano tried to keep things secret, you know these formulas and so on.<sup>3</sup> But after that mathematics became quite open, until recently. Now you have material that is company confidential and you do not know what is in the software, it's protected in some sense. Have you run into this question in your work at all?

GUSTAFSSON: Certainly. Quite a lot, actually, because in Uppsala and Stockholm, jointly, we are running something called the competence center in scientific computing.<sup>4</sup> The idea is that we should work together with industry to solve problems that they are interested in. At the same time, there should be research so that Ph.D. theses can come out of it. Here, we encounter this problem quite often: the problem that a certain Ph.D. student is working on, and in the end, is sort of secret because the company wants to keep the information to themselves. Here we have to be very clear from the academic side, I think, that we should never enter into anything that cannot be published. Afterwards, of course, if the person wants to work for the company, fine. That's not a story, but as long as we are doing research at the universities, a public defense of a Ph.D. thesis and so on has to be open. And that has been a problem in a few cases, yes.

DAVIS: You know this is part of a wider problem called intellectual property. And you know that in this country, anyway, there's arguments as to what can be downloaded free of cost and so on. Of course, mathematics is one of the greatest of intellectual accomplishments, in my mind, and there has been no sense of the property. We all work with the property of other people and the accomplishments of other people. But I have a feeling that this tendency towards company confidentiality is going to get worse before it gets better.

<sup>&</sup>lt;sup>3</sup> The English version of Cardano's name is Jerome Cardan.

<sup>&</sup>lt;sup>4</sup> This is known as the Parallel and Scientific Computing Institute (PSCI).

GUSTAFSSON: I think so, too.

DAVIS: Of course, the lawyers want to keep the property rights. I don't whether the law has entered into mathematics yet, but it certainly has entered into every other area of intellectual accomplishment.

**GUSTAFSSON: Yes.** 

DAVIS: Looking over the things that you have done over these many years, is there something, some particular thing that you are particularly proud of?

GUSTAFSSON: Well, in a sense, I was lucky in the beginning, in that I was brought into a problem that was very hot at the time by my supervisor, Kreiss. That was this initial boundary value theory that he had developed to quite an extent, and I took part in finishing it, on the numerical side. And that has been, I mean if one looks at citations of my work that is, by far, the one that is most cited, this first type of work. I followed it up with some other theorems on convergence rates where the mesh size goes to zero and so on, that was built on this theory. So I don't know if I should be proud or not, but that certainly is the part of my work that is most well known, yes. And the other thing, may be not any particular mathematical result, but actually this thing that we are talking about: that I have been trying to bridge this gap between theoretical numerical analysis and applications. I've been trying to do that throughout my career, I think. At this conference, for example, you will note that there are some theoreticians and there are some others who are doing the real problems, and I think I am somewhere in between there. I just talked to Anthony Jameson yesterday and he classified me that way, that "you are somewhere in between."

DAVIS: Well, this is a question that I raised a little bit earlier, the fact that between the algorithmic people and the theoretical people, and so on, and also the applications people. Have you ever – well you've done work, much work and original work, you've come out with things that are different and not known, better maybe, and so on – have you ever thought about how you get ideas? Where do the ideas come from? Before you answer, let me just tell you, you there's a famous book by [Jacques] Hadamard called the *Psychology of Mathematical Invention*, or something like this.<sup>5</sup> He tries to go to this question of mathematical creativity from the psychological point, and I don't think he gets very far.

GUSTAFSSON: I haven't read it.

DAVIS: Yes, well I think you would find it kind of disappointing, because there's no magic formula or something. You know the old story of [Henri] Poincare: Poincare got the idea when he stepped on a street car, then the idea somehow popped into his head. So do you have any feelings about where the ideas come from?

<sup>&</sup>lt;sup>5</sup> The actual title is *Psychology of Invention in the Mathematical Field*.

GUSTAFSSON: No, I don't really. I must say that in my case I'm pretty sure that if I looked back and tried to see where it came from, if I did something new it was simply through hard work. You are thinking about some problem and you are doing some calculus or maybe you are doing some experiments on the computer, and then I can remember a few cases where I saw some pattern here that led me to a new idea about how to do some things. But I would say it was through hard work and –

DAVIS: Was the computer important in seeing the pattern?

GUSTAFSSON: Yes. I would say in few cases yes, right.

DAVIS: So without the computer it would probably have been harder to derive the pattern?

GUSTAFSSON: Yes.

DAVIS: You've done, I imagine, quite a bit of teaching.

GUSTAFSSON: Yes.

DAVIS: As far as your career is concerned, what do you have to say about the balance of teaching and research. How important is the teaching for the research or the research for the teaching, that kind of question?

GUSTAFSSON: It's very important that you do teach when you're involved in research, that's for sure, at least I think so. The teaching, if you take it from my point of view as a researcher, gives me some feedback, actually, because the students, at least in Sweden – unfortunately we are sort of a shy people in Sweden, the students don't ask very many questions and so during lectures they are just sitting there, and that's one reason why I like it sometimes –

DAVIS: That must be students all over the world, probably – [Laughter]

GUSTAFSSON: But here I find a little more curiosity, I think, from the students, at least the few classes that I've had here in this country.

DAVIS: You've probably had very good students, was this at Caltech?

GUSTAFSSON: At Stanford.

DAVIS: Stanford. These are the top students, of course.

GUSTAFSSON: But certainly it is important, and you have to structure your thinking a little bit. In Sweden, as a full professor, you have quite a bit of freedom in teaching. If you want to teach some new topic, you just do it. And that's what I've been doing, trying to keep myself on a sufficiently high level on new topics. So in order to learn something myself, I will give a course in it, actually, many times.

DAVIS: So you actually have courses that are principally in numerical work?

GUSTAFSSON: Yes.

DAVIS: Numerical work, yes. Suppose that a young student comes to you and says he wants to do a doctoral degree in numerical analysis. What mathematical background would you suggest that such a student have before he enters?

GUSTAFSSON: He has to have quite a strong mathematical background in general: pure calculus, algebra, and all those things. And when they come to us for a Ph.D., having a Ph.D. in numerical analysis, we also require quite a ,lot of numerical analysis before that. The topic is very strong in Sweden, we have had very well known people like Dahlquist and Kreiss and others, and we have managed to get it into the curriculum, to get quite a lot of numerical analysis into the undergraduate courses. So we also require that at least two courses in numerical analysis before we accept them as Ph.D. students. But in general, I would say in mathematics...they have to be strong in mathematics for sure.

DAVIS: How important would certain abstract subjects be in mathematics?

GUSTAFSSON: Not so important I think. Well, it depends on what you mean by abstract, but –

DAVIS: Well, I mean things along the line of Bourbaki.

GUSTAFSSON: No, I don't think that...if you take topology, number theory and things like that, it's –

DAVIS: Algebraic geometry -

GUSTAFSSON: I mean, I encourage them to take such courses if they do it afterwards, because I think it sharpens their thinking and so on, but I don't think it's that fundamental for doing their thesis –

DAVIS: What about some experience with physics?

GUSTAFSSON: That's good, that's very good.

DAVIS: What about some experience in the computational line, programming and so on?

GUSTAFSSON: Yes, sure. We require that, for sure. I think it is very important that they are good programmers when they start because that's one thing that maybe is neglected sometimes. Programming is taught at many universities nowadays at the more abstract level, but they have difficulties writing a simple program for doing a simple ODE software or something like that. That's really striking, and it's bad I think.

DAVIS: Well, we've covered many of the topics that I want to cover, there's one more thing that I would like to ask you before we say goodbye. What are the hard problems in numerical analysis that you think will be solved in perhaps the next ten or twenty years?

GUSTAFSSON: That is a very difficult question. There are so many hard problems, and it depends. There are many hard problems, how should I say, that are created within numerical analysis as such, you may want to improve a certain constant or something like that. But there is also the other side: that you want to be able to solve certain very difficult problems that come from physics, for example. We are seeing examples of that at this conference here, and to build a theory that really proves that you now have –I mean take Navier-Stokes equations, people have been solving them numerically for forty, fifty years and –

DAVIS: Navier-Stokes equations -

GUSTAFSSON: Navier-Stokes equations, yes. And you don't even know if there is a solution sometimes –

DAVIS: The no-existence-Stokes -

GUSTAFSSON: Right. [laughter] So there you have the tough problem where you get well paid if you solve that. And then you have, in the next step, the numerical part of it. We produce solutions, but what is the accuracy of it? Direct simulation of turbulence, if you take that as an example, that's impossible today for any realistic problem because the scales are so small that you have to resolve compared to the size of the main –

DAVIS: Questions of scales, different levels in turbulence? Do you think an engineer worries about whether for some of the equations, the solutions have been proved to exist?

GUSTAFSSON: No, they don't. [laughter] For sure not.

DAVIS: And they go ahead and get them anyway.

GUSTAFSSON: Yes, they do. But they do worry if the turbulence model, for example, that is involved, if that one is accurate or not; usually they are not.

DAVIS: You are working with [David] Gottlieb at the moment?

GUSTAFSSON: Yes.

DAVIS: What area are you working on?

GUSTAFSSON: Well, actually, we've been working on very different things even if we haven't written any papers together. The funny thing was that the problem popped up once for me in Sweden from computer tomography, you know this classical problem –

DAVIS: Tomography.

GUSTAFSSON: Tomography and these X-ray pictures, for example, if you talk about the X-ray all around and then you want to find out what is the real pattern inside the object that you are taking these X-rays from. And that's where I came to get David interested in this, because they are using Fourier transforms and he is so good in all these –

DAVIS: Spectral methods –

GUSTAFSSON: Spectral methods, yes. So that we did a lot of work on and we actually managed to publish in one of these journals for the specialists. We were not at all specialists ourselves, but that was a lot of fun.

DAVIS: That's a hot topic at the moment isn't it, tomography?

GUSTAFSSON: Yes, it is. But we have quit working on that, actually, because our method and the algorithm that resulted from it was N square log N algorithm but there are others that also have that property now. It's such a hot area, as you say, so we decided not to compete anymore. But we have more theoretical background, as mathematicians, and people in this area are usually from the engineering community.

DAVIS: Do you enjoy collaboration?

GUSTAFSSON: Yes, a lot. In particular with David. [laughter] We always have a lot of fun, work you might have seen on the blackboard here over the years.

DAVIS: Well, thank you very much. I've enjoyed it and I hope you have.

GUSTAFSSON: I've enjoyed also, it was a pleasure.

End of Interview