Computation

AN EXEMPLARY APPLICATION

One of the many purposes of this journal shall be to offer its readers, in each issue, an important and authoritative article on the use of high-speed analog equipment to further some particular area of knowledge.

These treatments shall be contributed by outstanding thinkers in each field of application, so that we shall serve such fields fundamentally and not merely publicize the computor per se.

In this issue we are proud to present the first of a series on the dynamics of governing. The author, a recognised expert, is Dr. Henry M. Paynter, Assistant Professor of Hydraulic Engineering at Massachusetts Institute of Technology.

The Analog in Governor Design, I
A Restricted Problem
by Prof. Henry M. Paynter

A prime mover governor like other control mechanisms should possess the twin qualities of rapid response and stability. Ideal governors would operate in a power system so as to maintain constant frequency under changes in the electrical load. Of course this is not attained in practice, since all the governors operate from a frequency or speed *error* and it is only possible to adjust their settings so that the prime movers will reach a new equilibrium as rapidly as possible after a change in load. In the governing problem, then, the basic error signal is the change of prime mover speed which is detected by a flyball mechanism. This error signal is fed into a series of hydraulic valves and servos to produce a change in throttle valve or gate position of the prime mover. While for proper operation the governor assemblage must be sensitive to small changes in speed, and the control relays very quick, the response must not be such that the governor will overshoot the new equilibrium or even cause a steady hunting of the generating unit.

This tendency toward instability is remedied by introducing a stabilizing element into the governor, which in most American practice is a restoring mechanism. With no water inertia, in the case of hydro units, and without boiler lags in steam units, only slight restoration (feedback) between gate or throttle opening and pilot valve position is required

LANDSLIDE FOR LITTLE BLACK BOXES

BOSTON, June 12 — As anticipated, the votes of right-thinking technologists everywhere have elected and kept in office (and laboratory) the entire slate of GAP/R Computing Components, in triumph over the misrepresentations of unhappier rivals. From the theoretical abstractions of top-level planning right down to the grass roots of practical computing accomplishments, these faithful servants are ushering in a new epoch of analogical progress.

Shown here in the glow of victory are your friends and mine: A, C, and J.

COMPONENTS K3-A, K3-C, AND K3-J, FOR ADDING, PROPORTIONING, AND INTEGRATING

These incumbents and their functional co-workers will never confound or confuse you, nor otherwise equate your intelligence to zero. Climb on the mathematical bandwagon with this dynamic team, and join the crusade against ignorance and brute force!

Tabulated hereinafter are the three solid planks of the platform on which these candidates have been chosen:

To Subscribers

Any reader of this journal who is impressed by it so deeply as to want to receive future issues may assure that outcome by writing to The Lightning Empiricist, c/o Geo. A. Philbrick Researches, Inc., at the address given above.

To Contributors

The welcome sign is out to fellow empiricists. We shall be glad to consider items for publication, long or short, and will guarantee full credit (of course).

CONTENTS

1. BASIC SIMPLICITY — We feature individual component types which are chosen to afford the most fundamental sort of *unit operations*, linear & non-linear, thus enabling preparation of close-to-Nature block diagrams which may be embodied immediately in an actual computor of similar configuration. Such computing structures may be set up either from equations or from a physical situation directly, and will serve simultaneously the varying purposes of analysis, synthesis, the building of models, and the exploration of hypotheses. The modular construction of the computing units has been worked out to give compactness, flexibility, and convenience.

2. LIGHTNING SPEED — This property applies to overall computing time, and includes setup (or "programming") and parametric adjustment, as well as the *faster-than-thought* solutions themselves. Fast operation brings exclusive benefits in a number of ways. Typically, in problems where optimum adjustments for stability are sought, or if the criticality of a set of parameters is to be minimised, one must study the effects of experimental variation among many parameters; hardly a spot for a sluggish computor! Responses on the CRO may readily be traced or photographed (courtesy of Dr. Land) whenever a record is required for posterity. Note further that the CRO is capable of at least as much accuracy as a low-speed mechanical recorder.

3. PROGRESSIVE ECONOMY — Our policy is to get the most out of up-to-date electronics: staying ahead with the latest techniques and employing precision parts which are in large scale production. The GAP/R system of interchangeable components is adaptable to every budget, yet permits growth at any rate or to any extent. We have striven to avoid overemphasis on any one refinement at the expense of others, and have tried to show good engineering sense in *balancing the design* of our computors. The result is maximum useful analog accuracy, maximum computing value for your dollar, and minimum obsolescence with the coming of still newer methods. Indeed, a number of interesting innovations are now in preparation, and will be promulgated soon.

The Lightning Empiricist

Wish us luck. We understand that a publishing venture like this is not automatically assured of success; and we are light-heartedly ignoring all the pitfalls. But light-heartedness is to be the tone in any case, whatever mistakes are made, and we shall do our best to sustain it.

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Incidentally, this present Organ is partly intended to replace the earlier "Progress Reports", which of course may not be remembered by the more recent GAP/R contacts. It will be noted that the new masthead claims only *aperiodic* publication.

★

For the convenience and edification of customers in the general Chicago area, we have engaged Everett Associates, 6744 No. California Ave., Chicago 45, as sole agency for the Midwest. Equipment for demonstration and technical assistance are available; ask for Jim Everett, or an associate.

In Canada, our representative is Ahearn & Soper, Box No. 794, Ottawa. Cognizant and willing are Messrs. John R. Foster and John D. Dure.

On the continent of Europe, we are now represented exclusively by CICE, pronounced "Cheecheh," which is short for Casa Italiana Commercio Estero, Via Umbria 7, Rome, Italy. The man to approach there is Sig. Cezary K. Bujalski.

★

A number of technical departments, in several universities, have been using the GAP/R Catalog & Manual as a text in courses involving analog methods, making us very happy. That document is now about a year old, and still "in print"; but it needs a few revisions to bring it up to the minute. We hope the adjoining column will fill the bill.

★

A consulting partnership has been formed under the name of Pi-Square Engineering Company, with headquarters at 400 Western Union Building, Boston 10, Massachusetts. It features the complementary experience and abilities of the two principals, Henry M. Paynter and George A. Philbrick.

Their chief area of advisory activity is in the application of computing aids to rational engineering analysis, particularly in dynamics and transient performance; for such studies, computing equipment is available through an exclusive arrangement. Each partner has continuing and major commitments aside from this joint venture, but no conflict of interests is indicated.

GAP/R CATALOG & MANUAL
Theme and Variations

This column is devoted to a pagewise modernization of the Catalog, including errata, revisions, and new developments.

Page 2: The discussion of "low-speed" applications of our computors was perhaps too restrictive. Since DC amplifiers are used everywhere in GAP/R computing equipment, the components will operate either fast or slow with equal facility and with comparable accuracy. For low-speed operation, the only changes required are in the characteristic times of the appropriate components. Such changes may be made by the user, or they will be performed at the factory for a small additional charge.

Page 5: The K3-A has been redesigned, with an appreciable improvement in accuracy, and an even greater improvement in stability. The input impedance is still at least one megohm.

Page 5: The K3-C now contains 5 12AX7 tubes.

Page 6: The K3-J, according to current plans will soon have a repertoire of characteristic times, including 1 second for typical simulative applications. This component will have improved stability and will boast an improved clamping circuit.

Page 8: The K3-B now contains 4 12AX7 tubes.

Page 9: The K3-Z now contains 4 12AX7 tubes.

Page 10: The K3-S adjustment procedure has been simplified, the two internal screwdriver adjustments having been replaced by one anterior screwdriver adjustment.

Page 10: The K3-T now contains 4 12AX7 tubes and 1 12AU7 tube. This component has also been improved in functional fidelity, and zero dial setting now yields zero sensitivity.

Page 11: The Wye Connector (WC), the Plug Adapter (PA), and the Cable Extender (CE) have all been discontinued. Signal Cables (SC) now feature simple telephone plugs, and the K3-4 Connecting Box is employed for all kinds of multiple connections.

Page 13: The Central Component (CC) now contains 16 12AX7 tubes and 2 12AU7 tubes.

Page 14: The Cabinet Assembly form of the Central Unit has been discontinued.

Page 16: K4 Components now contain from 8 to 21 12AX7, from 0 to 3 12AU7 tubes.

Page 18: The tube complement of the K4-MU Multiplier is now 16 12AX7 and 2 (Tung-Sol) 12AU7 tubes.

Page 19: The K4-FF now contains 18 12AX7 tubes.

Page 20: The K4-FG now contains 18 12AX7 tubes and 3 (Tung-Sol) 12AU7 tubes.

BIBLIOGRAPHY
Selected & Annotated

Amplifying and modernizing the references given in the 1951 Manual

★ Franz ALT: Evaluation of automatic computing machines, *Product Engineering*, Nov. 1951. (Principal types of computors described & compared as to solution time, cost, applicability.)

★ Harold BELL and V.C.RIDEOUT: Precision in high-speed electronic differential analyzers, *Cyclone Symposium*, May 1952. (Good analysis and discussion, plus suggested improvement*.)

★ Han CHANG, R.C.LATHROP and V.C.RIDEOUT: Study of oscillator circuits by analog methods, *Proc. NEC*, v.6, 1950. (Solution of nonlinear oscillations in systems up to 4th order, with transients*.)

★ F.P.COZZONE: Organizing a computor center in the engineering department, *Prod. Engineering*, Jan. 1952. (Discusses size and type of such facilities*.)

★ A.A.CURRIE: The general purpose analog computor, *Bell Labs. Record*, Mar. 1951. (Describes operational structure.)

★ G.A.KORN and T.M.KORN: *Electronic analog computors*, McGraw-Hill, N. Y., 1952. (General & Instructive*.)

★ E.LAKATOS: Problem solving with the analog computor, *Bell Labs. Record*, Mar. 1951. (Discussion of applications: see Currie.)

★ H.G.MARKEY and V.C.RIDEOUT: Analog computor solution of a nonlinear differential equation. AIEE Misc. Paper 51-171, Apr. 1951. (This is on a high-speed machine*.)

★ H.M.PAYNTER: Methods and results from MIT studies in unsteady flow, *BSCE Journal*, April 1952. (Hydraulic and regulatory transients succumb to analog and graphical analysis*.)

★ H.M.PAYNTER: Electrical analogies and electronic computors for surge and water hammer problems, *ASCE Proceedings*, 1952. (Transients*.)

★ G.A.PHILBRICK and H.M.PAYNTER: The electronic analog computor as a laboratory tool, *Industrial Laboratories*, May 1952. (Concepts elementary and realistic, with profuse block diagrams and photographs. Read *causal* for "casual" throughout*.)

★ W.C.SCHAFFER: Application of analog techniques to control design for aircraft engines, SAE Symposium, Jan. 1952. (Abstracted in *Machine Design*, May 1952. Appraisal of needs and potentialities*.)

★ D.ZANOBETTI: Le calcolatrici analogiche ad alta velocità, *L'Energia Elettrica*, XXVII, 12, 1951. (Describes gear at University of Bologna; treating techniques and advantages of high-speed computing*.)

Refers to, or describes, GAP/R products.

Continued from first page

for stability. Actually, however, in hydro units, when the turbine gates begin to close on a decrease of load, for example, the water inertia opposes this operation and produces a temporary *increase* in output due to the over-pressure or head rise. This unstabilizing effect may be overcome by an increase in the restoring action, which generally creates a speed droop so large that it must be made temporary by interposition of a dashpot. Thus one is confronted with the problem of establishing the proper settings of the governor stabilizing mechanism to insure a rapid return to equilibrium with stable and non-oscillatory response.
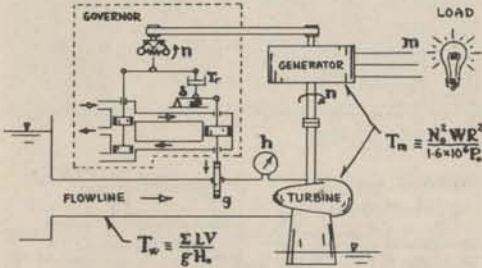
Schematic of Hydroelectric Plant

In the formulation of the simple governing equations outlined below, certain simplifying assumptions have been made which will be investigated in some detail in later instalments of this series. These may be listed as in Table I.

Subject to these assumptions, the dimensionless linearized equations for a dashpot-compensated governor may be written as follows, where $p = d/dt$:

$$\text{MACHINE ACCELERATION} \Big\} \quad T_m pn = (g + 1.5h - m)$$

$$\text{WATER ACCELERATION} \Big\} \quad -0.5T_w ph = (T_w pg + h)$$

$$\text{GOVERNOR RESPONSE} \Big\} \quad -\delta T_r pg = (T_r pn + n)$$

VARIABLES		PARAMETERS	
Relative Speed	n	Machine Inertia	T_m
Relative Head	h	Water Inertia	T_w
Relative Gate	g	Dashpot Time	T_r
Relative Load	m	Restoring Effect	δ

The governor is the "proportional plus integral" type with $1/\delta$ measuring the *sensitivity* and T_r the *reset* time. The feature which makes this problem different from conventional regulation problems is the effect of water inertia; the head change h counter to a change in gate opening g creates an unstabilizing term in the machine acceleration equation. The *relative stability* of any particular installation is therefore measured by the ratio of machine inertia to water inertia. Although the equations above are already dimensionless, it is possible to simplify them still further yielding a form in which the four defining parameters are

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TABLE I

Initial Assumptions	Later Considerations
1. Single hydro unit supplying an independent load	Parallel operation of both hydro and steam units
2. Small disturbances, so that the response laws of all components can be assumed linear	Systematic investigation of the most important nonlinearities
3. Electric load purely resistive with instantaneous voltage regulation, making the power independent of the speed and the torque vary inversely with speed, which is unfavorable for stability	Self-regulation effects of load on governing stability, including all coupling between the load and frequency
4. Turbine efficiency constant for small variations in speed, head and gate opening	Effects on regulation of actual turbine characteristics, including gate limits
5. Water and the flowline walls inelastic, so that only mass or inertia effects of the water are considered	Principal deviations in governor settings engendered by the elastic and resonant effects of the water and walls
6. Governor with no bounds, lags, hysteresis, dead bands, etc., and with a. very high gain between pilot valve position and gate servo velocity	Deviation from these assumptions encountered in actual governors

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reduced to two (independent) parameters. Thus, with $s = T_w p$

$$sv = (\gamma + \eta - \mu)$$
$$-s\eta = (-3\gamma + 2\eta)$$
$$-s\gamma = \lambda_1(1 + \lambda_2 s)v$$

VARIABLES	PARAMETERS
$v = (T_w/T_m)n$	
$\eta = 1.5h$	$\lambda_1 = T_w/\delta T_m$
$\gamma = g$	$\lambda_2 = T_w/T_r$
$\mu = m$	

In the above scheme the parameters λ_1 and λ_2 (which measure the two stabilizing components) completely specify the response of the governed unit to any particular load disturbance signified by μ. It is now possible to make a general analog study of this response merely by varying these two parameters over their practical ranges.
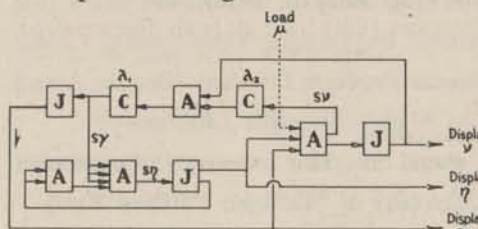

Block Diagram of Hydro Unit

An effective block diagram is illustrated for this problem. Note that it is possible to multiply by integer coefficients using adding components.

As mentioned at the outset it is desired to make the governing as stable as possible and to return the machine to equilibrium at synchronous speed in the quickest time. Accordingly, it is of interest to investigate the present configuration for: *(1) stability limits; (2) optimum transient response.*

One finds by direct experimentation using the computor that there exist two extreme stability limits on λ_1 and λ_2 as indicated on the diagram; the parameter λ_1 (measuring the restoring effect or primary compensation) cannot exceed unity ($\lambda_1 < 1.0$) while λ_2 (measuring the dashpot effect or the secondary compensation) cannot be increased beyond the limit two-thirds ($\lambda_2 < 2/3$). Between these extremes, a smooth curve relates the values of λ_1 and λ_2 at the stability limit, which can be determined by direct adjustment and reading of the coefficient components.

The second question, that of the proper settings for optimum response, may also be found by simple experimentation with the computer settings (which correspond to the actual governor settings). The illustration indicates the effects of λ_1 and λ_2 by response curves which are pictures taken from the oscilloscope screen of the computor. The origin of coordinates for each trace, indicates the corresponding values of λ_1 and λ_2. Thus it may be found quite rapidly that for best governor response, and the shortest transient time (with the above assumptions) the following values are read directly from the coefficient settings:

$$\lambda_1 = 0.40 \quad , \quad \lambda_2 = 0.17$$
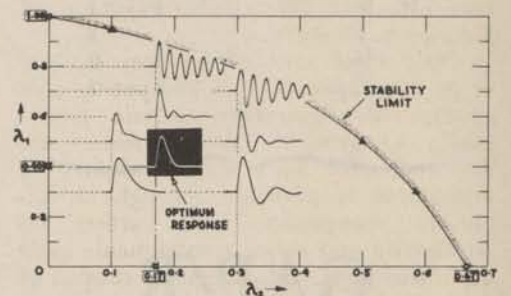
Effect of Governor Settings

"Standard" Assortments of Computing Components

Included in the 1951 Catalog & Manual, on page 15, is a suggested group of components referred to as the Basic Computing Assembly and identified by the symbol CA. This selection has frequently served as a logical first choice in cases where an expandable computor for general purposes has been decided upon. Although of course any appropriate combination may be selected from among the components available, we have become convinced that a useful purpose will be served if certain more advanced assortments are offered as "standardized" computor sets. The quantities of K3 Components for a total of four such sets are tabulated below under their respective identifying symbols. The CA assortment is identical with the earlier *Basic Assembly*. Assortment DA has been worked out for about twice the equation-solving capacity of the CA, and with somewhat broader dynamic scope. For control studies, involving *tardigrade* loops and stability problems, the FA assortment has been specially selected. Finally, Assortment GA offers all the capabilities of the lesser assemblages plus added capacity for general solutions of rather broad coverage. Along with this assortment, we recommend including one K4-FF and one K4-MU Component. Each assortment is considered to include one Model CC Central Component and all appropriate accessories. Each includes sufficient facilities for regulated power as supplied by one Model RS in all cases except GA, which uses two.

Formal quotations for these assortments or other selections will be promptly supplied.

Assortments

Components		CA	DA	FA	GA
	K3-A	3	5	4	8
	B		1	2	2
	C	5	10	6	14
	D		1	1	1
	E			1	2
	H			1	2
	J	3	7	3	8
	K			1	2
	L	1	2	4	3
	S			1	1
	T				1
	V		1	1	2
	Z			1	1

GAP/R

Prominent ★ GAP/R ★ Customers

The Foxboro Company
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Marquardt Aircraft Company*

Tracerlab, Incorporated

Vickers, Inc. (Division of Sperry Corp.)

U.S. Naval Underwater Ordnance Sta.

University of Missouri (E.E. Department)

*Indicates those customers who have given subsequent orders for extension of GAP/R equipment already purchased.

Triodes as *n*-th Power Elements

Analog experimenters may be pleased to know that many smooth nonlinear functions may be accurately ($\frac{1}{2}$ to 2%) obtained by a simple application of the plate characteristics of certain triodes.

For example, the plate current of a type 12AU7 as purchased from Tung-Sol varies nearly as some power of the plate voltage, the grid voltage being fixed. The proportionality coefficient and the power itself are functions of the grid voltage, and the law is remarkably faithful for exponents between 1.5 and 11.

By treating the above plate circuit as a nonlinear resistor, and placing it for example in the input or feedback path of an operational amplifier, one may compute powers or "roots" of an input voltage, the output likewise being a voltage. A circuit for the former case is shown in the adjacent figure.

$$i_p \approx m(e_g)e_p^{n(e_g)}$$

+300vdc   1M,1% i_p 5K e_g Δe e_2

$e_1 \approx e_p$

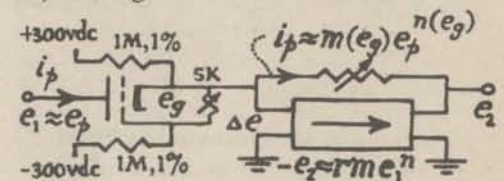-300vdc   1M,1% $-e_2 \approx rme_1^n$

Plate current is of course only positive, but this incompleteness is remedied without much trouble. For nonlinear *even* functions, one simply applies the absolute value of the input voltage, and for *odd* powers the triode plate circuits are connected "parallel-opposing". When the cathode terminals of these triodes are not at the inner input of the amplifier, somewhat more ingenuity is required in applying an invariant grid voltage.

It is evident that a large variety of special combined functions may be synthesized in this way, to a reasonable precision. Well-aged triodes, and quite constant heater temperatures, are all to the good.
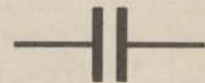
News of Multi-dimensional Functions

A number of mechanisms are currently well-known for the approximate embodiment of non-linear functions of a single arbitrary variable. So far as is known to us, however, only one of these is adjustable during usage, yet operates fast enough for high-speed computors. Furthermore, this particular method may be generalised to apply to functions of more than one variable; and fortunately the above properties are not sacrificed in the process. We should be interested to hear from users of analog computors who may have potential applications for this new technique, which has now successfully been reduced to practice.